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Mr H. J. B. Harding, Member, in the Chair

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Authors.

Structural Paper No. 47

THE ROOFING OF CLEADON RESERVOIR

by

*** Nigel John Ruffle, B.Sc.(Eng.), A.M.I.C.E.,**
and
Hugh Tottenham, M.A.

SYNOPSIS

The Paper deals with the design and construction over an existing service reservoir of a 160-ft-dia. reinforced concrete domed roof. The dome is believed to be the largest of its type so far constructed in Europe.

The history and mode of construction of the reservoir are described and the sources of pollution of water in open service reservoirs are discussed with special reference to the particular causes of trouble at Cleadon.

Various types of roof construction were considered and the reasons for the choice of a reinforced concrete dome are stated, followed by details of the foundation work, and provisions for ventilation, water-level indication, and access.

The principles of design of the dome are outlined with particular reference to the analysis of the "edge-effect" forces by an approximate treatment due to Geckeler. Among the assumptions entailed in this treatment, that of ignoring the variation in thickness of the shell near the edge is not true in this case, and comparison is made with

* Mr Ruffle is Engineer for New Works, Sunderland and South Shields Water Co. Mr Tottenham was, at the time of writing the Paper, a Senior Engineering Assistant with Twisteel Reinforcement Ltd.

the results obtained by a new method of solution in which variation of thickness, radius, and slope can be included. This alternative method is presented in Appendix I.

The various factors influencing the type and disposition of reinforcement are discussed.

The system of scaffolding, the shuttering, and sequence of concreting are described, and reference is made to the method of concrete control. Strain measurements and vertical and horizontal deflexions were taken during removal of the formwork and reasons are advanced for the unexpectedly small values of stress and deflexion which were recorded.

Finally, a statement is made on the cost.

Introduction

The Sunderland and South Shields Water Company's service reservoir at Cleadon (Fig. 1, facing p. 272) was brought into use in 1863, its mode of construction following the contemporary practice of using puddle clay as the waterproofing medium. The reservoir is circular in plan, with a floor consisting of puddle clay, about 18 in. thick, overlain by stone slabs bedded on a thin layer of sand. The encircling wall is of masonry, 17 ft high, and varying in thickness from 7 ft 0 in. at the base to 4 ft 0 in. at the top. Behind the wall is 2 ft of puddle clay and under the wall, providing continuity between the layers of clay in the floor and behind the walls, is 18 in. of puddle clay "reinforced" with gravel, known as "gravel puddle." The wall is of insufficient cross-section to withstand the outward thrust of the water, which is resisted by the clay puddle and the embankment behind. Weepholes through the wall prevent inward hydrostatic pressure when the reservoir is rapidly emptied. The capacity of the reservoir is 1,800,000 gal.

Roofing of the reservoir has been carried out in accordance with the accepted modern practice of totally enclosing service reservoirs, and the work has formed part of the Company's programme for covering its existing open reservoirs. Water in an open reservoir may be polluted by leaves or other extraneous matter blown by the wind, by birds, or by insect or vegetable growths in the water itself. At one of the Company's reservoirs where pollution had been caused by seagulls, wires stretched above the water at intervals of 25 ft in both directions were effective in discouraging the seagulls from visiting the reservoir, and the pollution ceased. At Cleadon, however, the trouble arose partly from chloro-phenol tastes produced by the action of chlorine on decaying vegetable matter and partly from the breeding during the summer months of gnats, the larvae of which in some instances found their way to consumers' taps. All these forms of pollution can be eliminated by the physical barrier and the exclusion of sunlight which result from roofing.

Description of Roof

The chief reason for adopting a single-span roof was to avoid the disturbance of the puddle-clay floor which would have been caused in the erection of columns for the support of a multi-span structure. The reservoir has in the past been affected by mining subsidence and, although watertight, has a tilt of $14\frac{1}{4}$ in. across one diameter.

The decision to roof with a dome followed naturally, but the choice of material had to be made. Aluminium, which offered rapid and economical erection, was rejected because doubts were felt about its resistance to corrosion—externally in a coastal climate and internally in an atmosphere conditioned by a large area of water having a residual chlorine content. Painting would be an unwanted recurring cost. A system employing prestressing and sprayed concrete was considered but it was felt that insufficient was known of the behaviour of prestressed concrete

under conditions of mining subsidence. It was also considered that, at the present stage in the development of sprayed concrete, a sounder and more lasting structure could be produced in conventional reinforced concrete, and it was the latter material which was chosen.

The dome, with a span of 160 ft 6 in., was founded on a reinforced concrete capping beam which was cast on top of the masonry wall of the reservoir, and which, by varying in depth from 1 ft 11½ in. to 6 in. corrected for the tilt of the existing structure. The weight of the roof is 640 tons and it transmits to the capping beam and thence to the reservoir wall, an additional dead load equivalent to 1¼ ton/ft run. At intervals of 5 ft pockets 18 in. × 12 in. × 6 in. deep were made in the masonry which, when filled with concrete monolithically with the capping beam, formed dowels to anchor the capping beam securely on to the wall. The beam was constructed in alternate sections each 30 ft long. Before the intermediate sections were cast, sheets of bituminous damp-proof-course were cut to shape, softened on the surface with a blow lamp, and stuck to the existing faces of concrete, thus forming joints which would allow contraction and a little expansion.

Freedom of movement for the ring beam of the dome was allowed for by laying on the capping beam a single thickness of bituminous damp-proof-course. As a result of experience with a barrel-vault roof, which had moved bodily on a similar sliding surface, a kerb was formed at the edge of the capping beam 1 in. distant from the face of the ring beam. Six weeks after removal of the shuttering from the soffit of the dome, the 1-in. gap was filled lightly with untwisted tarred yarn and sealed with a hot-poured rubber-bitumen compound.

Access into the reservoir is provided by four manholes in the shell, and at one of these is fixed a rigid ladder, 26 ft long, constructed of greenheart. Circulation of air takes place through an emergency overflow pipe in the reservoir wall and through a 16-in.-dia. ventilator at the summit of the dome. The ventilator is guarded by a copper screen and covered by a removable reinforced concrete slab, which respectively prevent the ingress of birds and leaves and block the direct passage of sunlight. The water level in the reservoir is indicated by a float-operated instrument fixed at a convenient point on the dome. Instead of the more usual stand-pipe stilling well, two wires tensioned between the roof and the floor were used to guide the float. Rain-water is carried in the gutter formed by the kerb of the ring beam and discharges to the drainage system *via* a single reinforced concrete hopper head.

Principles of design

The forces in a shell dome can be considered in two groups. Of these, the first consists of forces which lie within the surface of the shell and do not give rise to bending moments. These forces are generally termed the "membrane" forces and they predominate over the greater part of the shell. The second group represent a more complex force system including bending of the shell, and these are effectively restricted to a narrow zone near the edge of the shell; they are therefore generally referred to as the "edge-effect" forces.

Membrane forces

The membrane forces in a shell dome represent a statically determinate system. The derivation of expressions for these forces under several types of loading will be

found in the book by Timoshenko.[1] In the case of a uniform vertical load, which is the most important load in a structure of this type, they are given by:

$$T_1 = -gR\left[\cos\phi - \frac{1}{1+\cos\phi}\right] \quad . \quad . \quad . \quad . \quad . \quad 1(a)$$

$$T_2 = -gR\frac{1}{1+\cos\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1(b)$$

Where the following notation * is used (Fig. 2) :—

R denotes radius of shell
ϕ ,, angular co-ordinate of point, measured from the crown
g ,, uniform vertical load per unit area of shell
T_1 ,, hoop (circumferential) force per unit length (tension positive)
T_2 ,, radial force per unit length (tension positive).

It is thus seen that, for shells with a small angle of opening ϕ, the forces in the shell are everywhere compressive. The force T_2 at the edge will require a reaction from the ring beam which will cause the latter to be in tension. The elimination

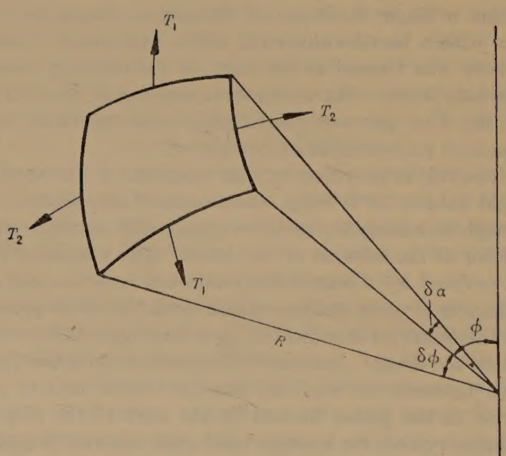


FIG. 2

of the discontinuity of stress between the shell edge and the ring beam necessitates the introduction of the "edge-effect" forces.

Edge-effect forces

A thorough treatment of these forces has been given by Love.[2] The exact solution is not, however, suitable for normal use since it is in the form of slowly converging series.

Several approximations have been suggested and several methods of approximate solution devised. That due to Geckeler[3] is probably the simplest and gives good results for the types of shell dome likely to be used in reinforced concrete. The approximations are usually introduced by ignoring terms in the exact differential equation, but by presenting them in this way their physical significance is obscured.

* A further list of the notation in general use throughout the Paper and Appendices is given on p. 278.

In Appendix II, the derivation of the Geckeler equation is given based on the following assumptions.

(1) Deformations caused by forces other than the circumferential tension T_1 and the radial bending moment M_2 can be neglected.
(2) The "edge zone," or the portion of the shell in which these forces assume appreciable magnitude, is sufficiently narrow for the change in inclination of the shell to be neglected.
(3) The edge zone is sufficiently narrow for the variation in horizontal radius to be neglected.
(4) The shell has constant thickness in the edge zone.

Using a notation for the forces, etc., as in Fig. 3 Geckeler's equation can then be derived:

$$\frac{d^4N_2}{d\psi^4} + 4a^4N_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Where
$$4a^4 = \frac{12R^2}{d^2}$$

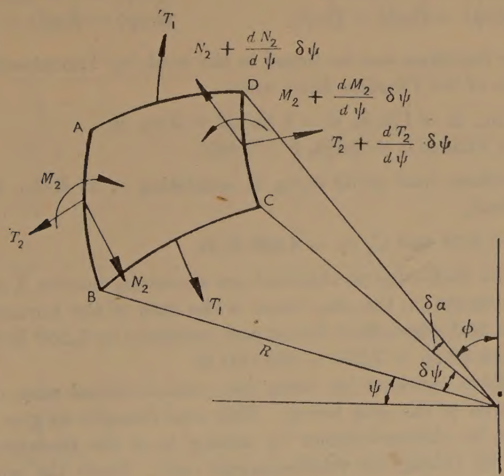


Fig. 3

The general solution for this is:

$$N_2 = C_1e^{-a\psi}\cos a\psi + C_2e^{-a\psi}\sin a\psi + C_3e^{a\psi}\cos a\psi + C_4e^{a\psi}\sin a\psi \quad . \quad (3)$$

Considering the case of a uniformly distributed load g, and introducing the notation*:

ϕ_1 denotes angle of inclination of shell at edge
A ,, cross-sectional area of ring beam
r ,, horizontal radius of shell edge
$b = Rd/A$
d denotes thickness of shell

* A further list of the notation in general use throughout the Paper and Appendices is given on p. 278.

and assuming that the ring beam is fixed against rotation, the constants of integration are:

$$C_1 = \frac{-gR}{a+b}\left(\cos\phi_1 + \frac{b\sin\phi_1\cos\phi_1 - 1}{1+\cos\phi_1} - \frac{\sin\phi_1}{a}\right) \quad . \quad . \quad . \quad 4(a)$$

$$C_2 = -gR\frac{\sin\phi_1}{a^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4(b)$$

$$C_3 = C_4 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4(c)$$

The forces in the shell are then given by:

$$N_2 = C_1\theta(a\psi) + C_2\xi(a\psi) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 5(a)$$

$$T_2 = -\cot\phi_1 N_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 5(b)$$

$$T_1 = a[-C_1\phi(a\psi) + C_2\psi(a\psi)] \quad . \quad . \quad . \quad . \quad . \quad 5(c)$$

$$M_2 = \frac{ad}{2\sqrt{3}}[C_1\psi(a\psi) + C_2\phi(a\psi)] \quad . \quad . \quad . \quad . \quad . \quad 5(d)$$

where $\theta(a\psi) = e^{-a\psi}\cos a\psi$ $\qquad \xi(a\psi) = e^{-a\psi}\sin a\psi$

$\phi(a\psi) = \theta(a\psi) + \xi(a\psi)$ $\qquad \psi(a\psi) = \theta(a\psi) - \xi(a\psi)$

Tables of these functions can be found in the book by Timoshenko[1] (p. 394).

The dimensions of the Cleadon dome are:—

$r = 80$ ft 3 in., $R = 170$ ft, $d = 3$ in., $A = 6$ sq. ft,
$\phi_1 = 28°\ 10'$; whence $a = 34{\cdot}24$, $b = 7{\cdot}083$.

Assuming a uniform load of 52 lb/sq. ft consisting of 37 lb/sq. ft dead load and 15 lb/sq. ft live load,

$C_1 = -406{\cdot}3$ lb/ft and $C_2 = -3{\cdot}536$ lb/ft.

The forces in the shell edge so obtained are shown by curves A in Fig. 4.

The horizontal thrust on the ring beam is the sum of the horizontal components of the membrane and edge-effect forces and amounts to 3,260 lb/ft. The tension in the beam is then $80{\cdot}25 \times 3{,}260 = 262{,}000$ lb.

These results were obtained by using the cross-sectional area of plain concrete as the effective area of the ring beam. This was thought to give a more realistic value than would be obtained either by adding to it the transformed area of the reinforcement or by taking the reinforcement only. Since the whole section is in tension, the addition to the calculated stresses of the tensile stresses which may arise owing to shrinkage might produce a tension sufficient to cause cracking of the concrete. At any crack the effective area would be that of the transformed reinforcement only. However, the bond of the concrete on the reinforcement would limit the effect of the cracks to a small zone and between these zones the full area of concrete and reinforcement would be effective.

The stresses in the ring beam are then:—

(*a*) uncracked section ignoring reinforcement, 304 lb/sq. in.;
(*b*) uncracked section with reinforcement, 207 lb/sq. in.;
(*c*) cracked section steel stress, 9,000 lb/sq. in.

From the graphs of the forces it will be seen that the zone affected is small and that the assumptions (2) and (3) are reasonable. Assumption (1) can also be shown to be valid. The amounts of strain energy associated with the forces T_1 and T_2 are proportional to T_1^2 and T_2^2, and the constant C_2 is small compared with C_1.

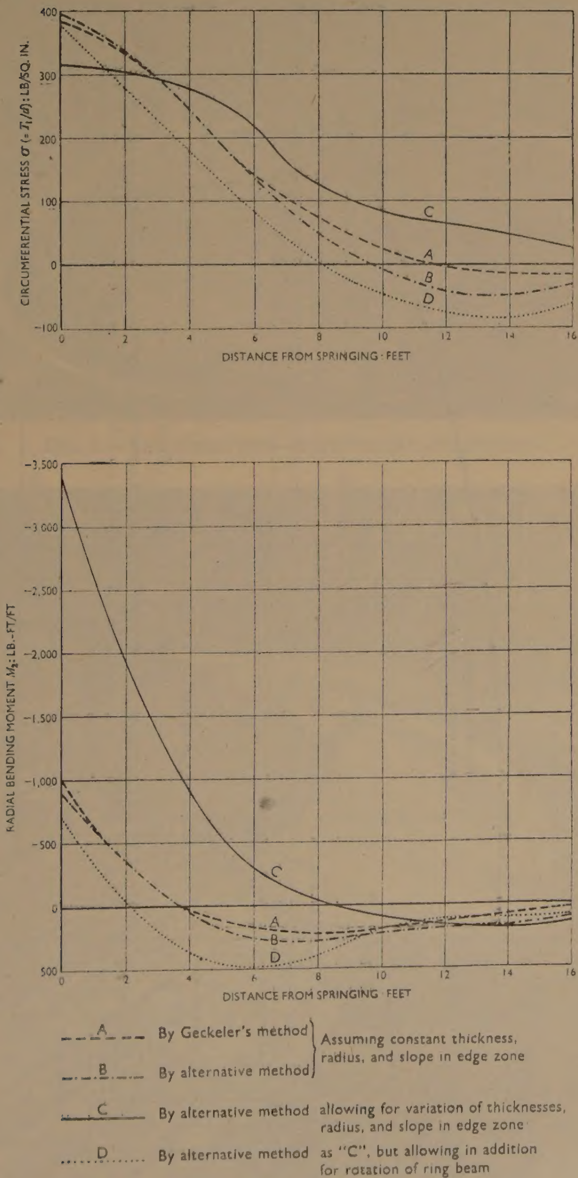
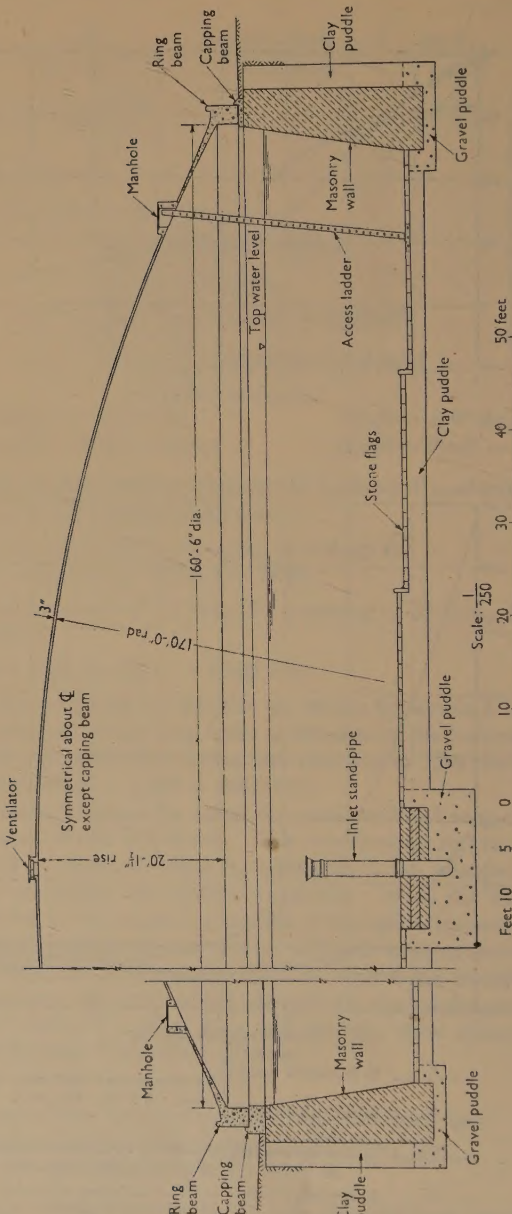

CIRCUMFERENTIAL STRESS σ (= T_1/d): LB/SQ. IN.
400
300
200
100
0
−100
0
2
4
6
8
10
12
14
16
DISTANCE FROM SPRINGING · FEET
C
A
B
D
RADIAL BENDING MOMENT M_2: LB.-FT/FT
−3,500
−3,000
−2,500
−2,000
−1,500
−1,000
−500
0
500
0
2
4
6
8
10
12
14
16
DISTANCE FROM SPRINGING · FEET
C
A
B
D
A By Geckeler's method
B By alternative method
Assuming constant thickness, radius, and slope in edge zone
C By alternative method allowing for variation of thicknesses, radius, and slope in edge zone
D By alternative method as "C", but allowing in addition for rotation of ring beam

FIG. 4



(a) PART CROSS-SECTION OF RESERVOIR AND DOME

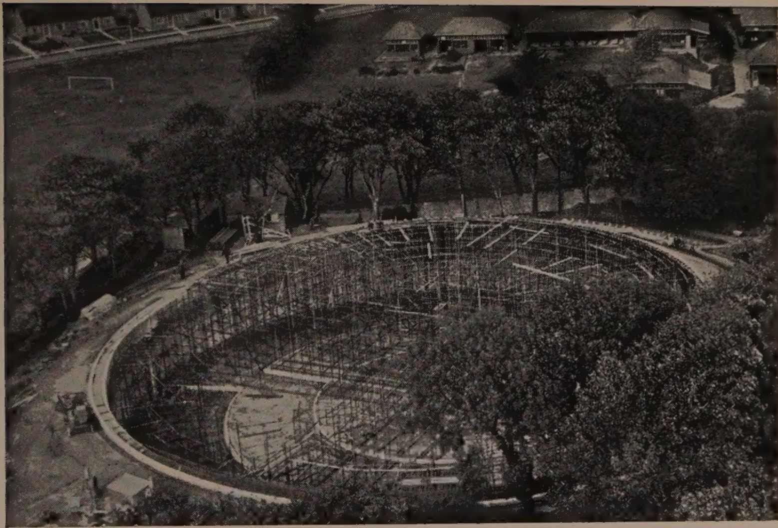


FIG. 1.—THE RESERVOIR—SCAFFOLDING COMMENCED

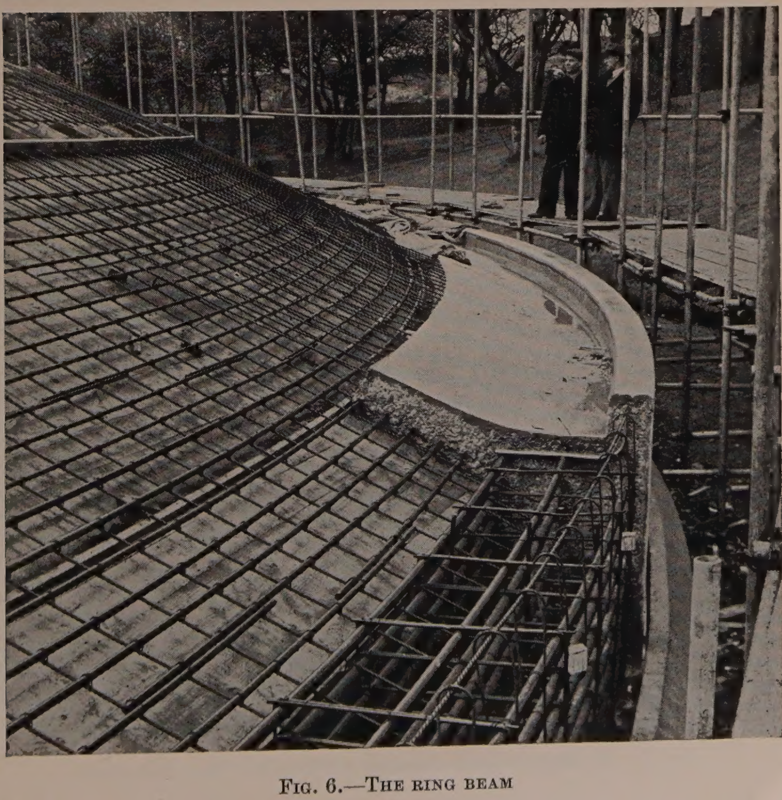


FIG. 6.—THE RING BEAM

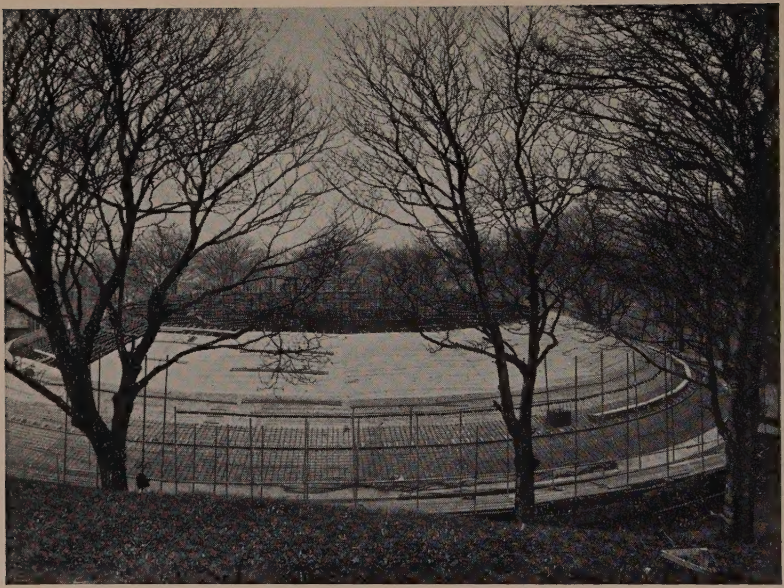


Fig. 8.—Shuttering in progress

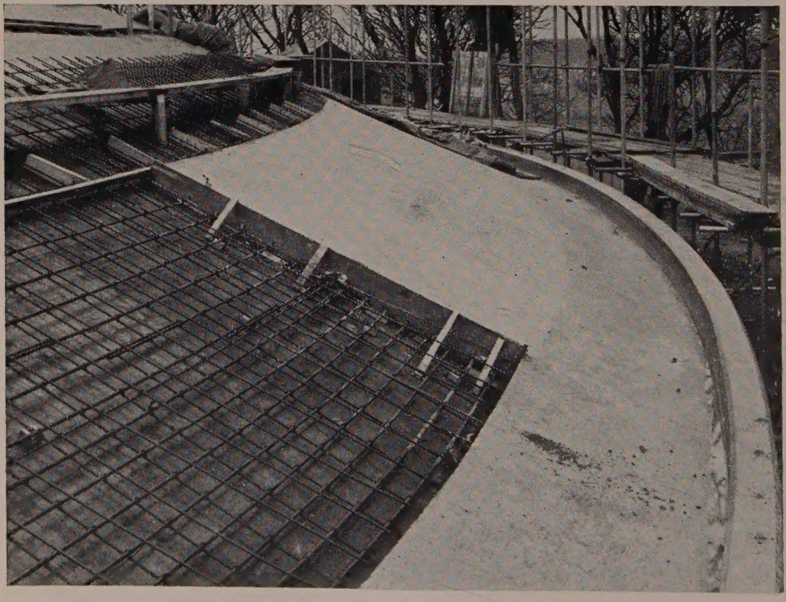


Fig. 9.—Shuttering and reinforcement near the ring beam

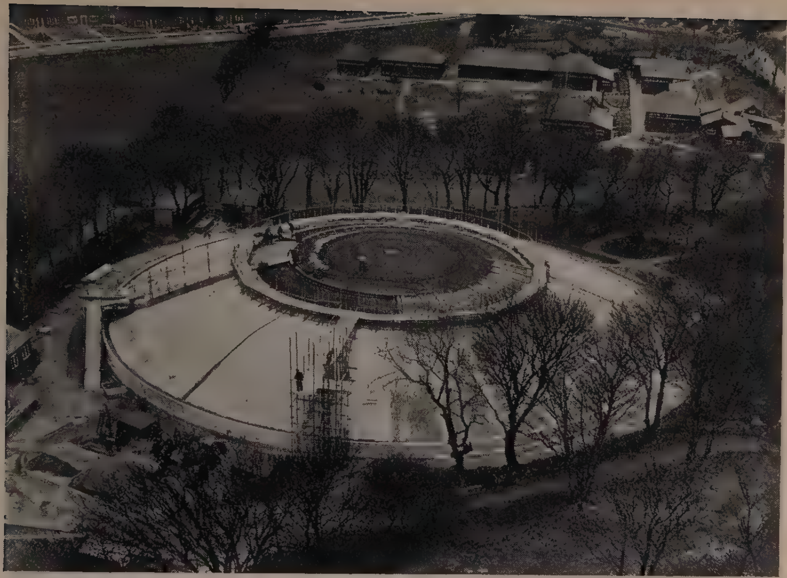


Fig. 10.—Concreting at an advanced stage

Fig. 11.—The completed structure

FIG. 12.—VIEW INSIDE THE RESERVOIR AFTER COMPLETION OF THE ROOF

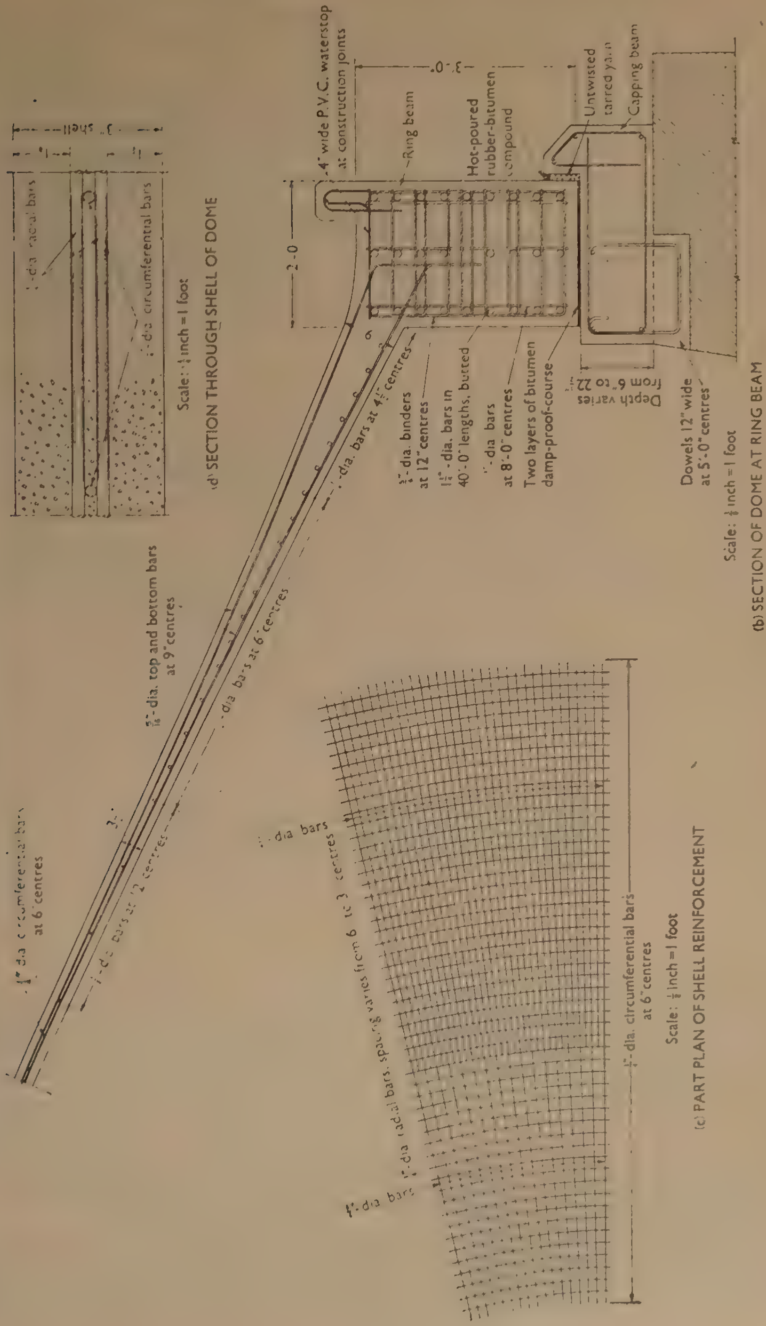

(a) SECTION THROUGH SHELL OF DOME
Scale: ¾ inch = 1 foot
3" shell
circumferential bars
radial bars
(b) SECTION OF DOME AT RING BEAM
Scale: ⅛ inch = 1 foot
4" wide P.V.C. waterstop at construction joints
Ring beam
Hot-poured rubber-bitumen compound
Untwisted tarred yarn
Capping beam
at 12" centres
40'-0" lengths, butted
at 8'-0" centres
Two layers of bitumen damp-proof-course
Depth varies from 6" to 22"
Dowels 12" wide at 5'-0" centres
at 9" centres
at 6" centres
(c) PART PLAN OF SHELL REINFORCEMENT
Scale: ⅛ inch = 1 foot
dia bars
at 6" centres

FIG. 5

T_1 is therefore of the order of aC_1 and T_2 of the order of C_1. Equation (2) could be obtained from consideration of minimum energy and the effect of ignoring T_2 would introduce an error of 1 in a^2, which in this case is of the order of 1 in 1,000.

Normally, as in this case, the edge zone of the shell is thickened to cater for the high values of the bending moments near the ring beam. Thus the fourth assumption is not really true. One method of allowing for this variation is to assume that the edge zone is made up of a series of concentric rings each of constant thickness. The general solution can then be found for each ring and the constants of integration obtained by consideration of continuity of stress and displacement. Since there are four constants in the general solution this process leads to four equations at each joint between the rings and two at the innermost joint. Thus for general purposes this method becomes unwieldy for variations of thickness such as are normally employed.

It was nevertheless desirable to estimate the effect on the forces, and more particularly on the radial bending moments, of the variation in thickness in the edge zone. The method of analysis which is described in Appendix I was accordingly devised. This method leads to only two equations for each joint and can be used for continuously varying sections. Assumptions (2) and (3) need not be made. It is essentially a finite difference method and is an extension of the method proposed by Vlasov[4] for the calculation of cylindrical shell roofs.

Curves B in Fig. 4 show the results obtained by this method and curves A show the results of calculations by the Geckeler method for comparison. Curves C show the results obtained when variations of thickness, radius, and slope are taken into consideration. It will be seen that all three solutions give similar values for stress at the springing but that the solution which takes into account variations of thickness, radius, and slope indicates the presence of larger radial bending moments.

Such large bending moments, however, would cause rotation of the ring beam, since it is not rotationally stiff in comparison with the thickened shell in the edge zone.* The effect of allowing for the rotation of the ring beam in addition to the variations of thickness, radius, and slope is shown by curves D in Fig. 4; it will be noted that rotation of the ring beam has relaxed the large bending moment indicated by curves C. It may therefore be concluded that, unless the ring beam is rigid, the practice of designing the edge zone as if it were of constant thickness does not lead to serious error.

Reinforcement

The reinforcement can be considered in three parts; the ring beam, the edge zone, and the shell reinforcement. Details of the arrangement of bars are shown in Fig. 5.

In the ring beam the principal reinforcement, which generally consists of twenty-six $1\frac{1}{4}$-in.-dia. M.S. bars, was spread over the whole cross-section (Fig. 6). In view of the low working stress and the high percentage of reinforcement it was not thought necessary to use deformed bars to give better crack control, even though large diameter bars were used. To hold the steel in position during concreting $\frac{1}{2}$-in.-dia. uprights were provided. To eliminate the crowding of the reinforcement the bars were butted, with the joints staggered.

The edge-zone reinforcement is provided principally to resist the hoop tension and radial bending moments arising from the edge forces. All the steel was assumed to take its working stress independently of the theoretical concrete stress. Whilst

* The radial moments will induce in the ring beam a state of pure bending, not torsion.

this may seem to make the calculations illogical, the alternative of using the modular ratio multiplied by the theoretical concrete stress is just as illogical and less economical.

The reinforcement for the shell proper, which consists of a central layer of circumferential bars ($\frac{1}{4}$-in.-dia. M.S. bars at 6-in. centres) sandwiched between two layers of radial bars ($\frac{1}{4}$-in.-dia. M.S. bars at centres varying from 3 in. to 6 in.), is provided to resist forces arising from a number of circumstances extraneous to the general design. These forces include those due to small concentrated loads, slight inaccuracies in setting out, local variations in the properties of the concrete and, finally, the temporary conditions obtaining during the release of the shuttering. These last forces were expected to be the most severe but, for reasons which will be given later, turned out to be insignificant.

Construction

Formwork

In the extensive scaffolding system to support the roof shuttering, 8,300 lengths of tube totalling 81,000 linear ft and 15,000 couplers were incorporated. The uprights were spaced at a maximum of 6 ft apart and horizontal bracing was provided at vertical intervals of 6 ft. The arrangement of the scaffolding near the soffit is shown in Fig. 7 and it is of interest to note that the system obviated the need for swivel couplings. In order to distribute the loading evenly over the reservoir floor, the scaffolding was erected on 9-in. × 3-in. timber sole-bearers laid flat and running continuously between uprights.

Timber shuttering was employed throughout, the dome itself being shaped with $\frac{5}{8}$-in. boards in 6-in. widths nailed on to 2-in. × $1\frac{1}{2}$-in. radial bearers, which were in turn supported from the scaffolding and adjusted for level by wedges (Fig. 8).

Concreting

Concreting of the ring beam (Figs 9 and 10) was carried out in alternate sections 30 ft long, and the concreting of the shell followed in concentric rings 9 ft wide, each ring consisting of a number of strips up to 60 ft in length. At least 2 days elapsed between the concreting of each strip and its neighbour so as to allow part of the shrinkage of the first strip to take place and to enable the stop ends under the steel reinforcement to be removed cleanly. The concrete was cured for 7 days after placing, by three layers of hessian which was kept continuously damp. Except for the outer 15 ft of the dome, where it thickens to 10 in. at the junction with the ring beam, the shell is of a uniform thickness of 3 in. and the placing of the reinforcement allowed a top and bottom cover of $1\frac{1}{8}$ in. The temperature of the water pumped from the Company's wells is almost uniformly 55°F and condensation is continuous during the cooler months of the year. Special attention was therefore paid to the maintenance of the correct cover to the reinforcement in order to minimize the risk of corrosion.

The concrete was mixed to a richness equivalent to that of a nominal mix of 1 : $1\frac{5}{8}$: $3\frac{1}{4}$; it was actually designed to contain 636 lb. of cement per cu. yd of finished concrete. Owing to the dearth of satisfactory material in the area, the aggregates were obtained from the River Swale at a place about 45 miles distant, the gravel being delivered in two grades—$\frac{3}{4}$ in. to $\frac{3}{8}$ in. and $\frac{3}{8}$ in. to $\frac{3}{16}$ in. Ordinary Portland cement was used and was batched by the bag; the aggregates were batched by volume. Inundation of the sand was employed to eliminate the effects of bulking

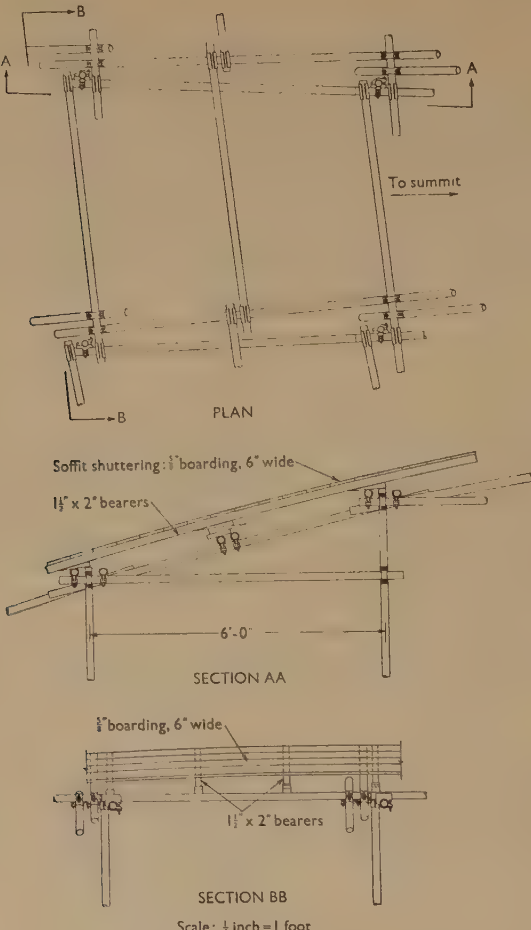


FIG. 7.—TYPICAL ARRANGEMENT OF SCAFFOLDING

and the workability of the concrete was controlled by the use of a compacting-factor apparatus, which had been used previously for the works of the Company. The results of the compression tests on 6-in. concrete cubes are given in Table 1.

The concrete was consolidated by hand. The surface of the dome was finished with a wood float and left with the characteristic rough texture which is so much less liable to craze than the polished surface which is produced by a steel float.

Removal of formwork

Striking of the shuttering was commenced 18 days after the final concreting operation, i.e., the placing of the portion around the ventilator at the crown. Starting at the centre, the scaffolding supports were lowered ring by ring and the shuttering was withdrawn. Before removing the support at any one ring, the

TABLE 1

No. of sets of 3 cubes	Average 28-day strength: lb/sq. in.	Average w/c ratio	Average compacting factor	Standard deviation between sets of cubes: lb/sq. in.	Coefficient of variation: %
20	4,260	0·45	0·91	407	9·5

wedges at the next ring were slackened with a view to reducing any local bending moments which might be induced. Check couplings were clamped on to each upright before release so as to restrict the initial deflexion to $\frac{1}{4}$ in. but, as the release of the supports progressed and it became evident that the deflexions would everywhere be much less than $\frac{1}{4}$ in., this latter precaution was discontinued.

During the removal of the shuttering the ventilation proved to be inadequate and the atmosphere became stiflingly hot and humid. The condition was appreciably relieved by the erection over two of the manholes of "wind-sails" such as are used for drying out the holds of ships.

The lowering of supports took 22 days and during that period strain-gauge readings were taken periodically at a large number of points on the surface of the shell and ring beam. Levels were similarly taken to detect vertical deflexion and measurements were made to fixed points around the perimeter of the reservoir to determine the spread of the ring beam.

There was little reward, however, for the elaborate arrangements made to determine the movements of the structure during release of shuttering. All the movements recorded were slight and appeared to be related only to the temperature changes. There was no measurable drop of the crown due to removal of the supports but movements of + and − $\frac{1}{8}$ in. during the period corresponded with temperature changes of about + and − 5°F.

Levels which had been taken at intervals during construction on the shuttering and on the finished shell concrete had also indicated no movement and it was concluded that, while the concreting of the shell had proceeded ring by ring, the structure had progressively released itself from the shuttering and the ring beam had gradually taken up the resultant tension. This view was supported by the observation, before the lowering of the supports, that the shuttering under the soffit of the dome near to the ring beam was "drummy" and that the 1-in. gap between the ring beam and the capping beam had closed by a little less than $\frac{1}{8}$ in. This value is consistent with the expected stretch of the ring beam assuming the concrete to be uncracked and assisting the reinforcement in tension.

Whilst in consequence there was no danger of cracks due to local bending over the temporary internal supports, the data obtained was not as valuable as it might otherwise have been. All too little is known of how the actual force distributions in such structures compare with the theoretical distributions and there is, in particular, an insufficiency of knowledge concerning the force distributions in conditions such as may occur when a shell roof is temporarily supported on part of the formwork.

Conclusion

The dome, which is believed to be the largest of its type in Europe, appears to be entirely free from cracks and promises to be a most durable structure (Figs 11 and 12). Construction was commenced in May 1953 and was completed in July 1954, the maximum number of men employed being thirty-two. The cost has amounted in all to £29,866 which represents a unit roofing cost of 29*s* 6*d*/sq. ft, the area covered being 20,200 sq. ft. The amount is made up as follows:—

	£
Design fees . . .	900
Reinforcement . . .	2,104
Construction . . .	25,586
Supervision . . .	1,276
Total .	29,866

Acknowledgements

The roof was designed by Twisteel Reinforcement Ltd in collaboration with the Sunderland and South Shields Water Co. The contractors were William Moss and Sons Ltd and the contract was executed on a time and materials basis.

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Notation

a	denotes	parameter $= \sqrt[4]{\dfrac{3R^2}{d^2}}$
b	,,	parameter $= Rd/A$.
d	,,	thickness of shell.
g	,,	uniform load/unit area.
$l_1 \ldots l_n$	,,	width of ring $1 \ldots n$.
r	,,	horizontal radius of ring beam.
$r_1 \ldots r_n$	,,	horizontal radius of hinge $1 \ldots n$.
A	,,	cross-sectional area of ring beam.
$A_1 \ldots A_n$	,,	cross-sectional area of ring $1 \ldots n$.
$\left.\begin{matrix} C_1, C_2 \\ C_3, C_4 \end{matrix}\right\}$	denote	constants of integration.
E	denotes	Young's modulus.
H	,,	horizontal radial thrust/unit length.
H^m	,,	thrust required on shell to maintain membrane condition.
H^B	,,	thrust required on ring beam to induce stress in ring beam equal to edge value of membrane stress in shell.
M_2	,,	radial bending moment/unit length.

$M_1 \ldots M_n$	denotes	radial bending moment at hinge $1 \ldots n$.
N_2	,,	normal (radial) shear force/unit length.
R	,,	radius of sphere.
T_1	,,	hoop (circumferential) tension/unit length.
T_2	,,	radial tension/unit length.
α	,,	angle at centre of sphere subtended by two points on circumference.
β	,,	parameter $= RD/l^2$.
γ	,,	parameter $= 2\left(\frac{3A}{dl} + 1\right)$
δ	,,	radial displacement.
θ	,,	relative rotation of rings at a hinge (for suffix notation see definitions).
ϕ	,,	angular co-ordinate measured from crown, also inclination of tangent at any point.
$\theta(\alpha\psi)$, $\xi(\alpha\psi)$, $\phi(\alpha\psi)$, $\psi(\alpha\psi)$	denote	functions used in the solution of edge problem by Geckeler's equation.
$\phi_1 \ldots \phi_n$	denotes	inclination of ring $1 \ldots n$.
ψ	,,	angular co-ordinate measured from edge.
ξ	,,	change of slope.
σ	,,	hoop (circumferential) stress.
$\sigma_1 \ldots \sigma_n$	,,	hoop stress at hinge $1 \ldots n$.

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APPENDIX I

AN ALTERNATIVE METHOD OF CALCULATING THE EDGE-EFFECT FORCES IN CONCRETE SHELL DOMES

The method described below can be applied to any shell dome which is circular in plan nd supported round the perimeter. The cross-section need be neither circular nor of onstant thickness. The initial assumptions are that deformations due only to circum-erential stress and radial bending moments need be considered, and that only a narrow one round the edge of the dome is affected.

asic principles of method

The shell is first considered to be under membrane forces only. To maintain this ondition a horizontal thrust H^m must be applied at the edge and the circumferential ress at the edge will be $\sigma_m (= T_1/d)$. The ring beam is then given an equal stress by pplying a horizontal thrust $H^B (= \sigma_m A/r)$, A being the cross-sectional area of the ring

beam. It is now necessary to determine the effect of removing these thrusts, i.e., applying an outwards force $H^0 (= H^m + H^B)$ to the structure as a whole.

For this purpose the edge zone is considered as a series of concentric rings connected by hinges (Fig. 13). Each hinge in turn is first given a radial displacement, the remaining hinges being radially fixed. This induces a hoop tension in the two adjacent rings which must be resisted by horizontal radial reactions at the hinges of these rings. The displacement will also cause a relative rotation between the rings at these hinges. These reactions and rotations can be determined from considerations of statics and geometry.

Next the hinges are considered as fixed in position and moments are applied at each hinge in turn. These moments will also give rise to horizontal reactions and relative rotations at the hinges.

At the external hinge the sum of the horizontal reactions induced must balance the applied force, and at all internal hinges the sum must be zero. For continuity there must be no resulting relative rotation at the hinges. These conditions give a set of equations which can be solved, giving the stress and displacement at each hinge.

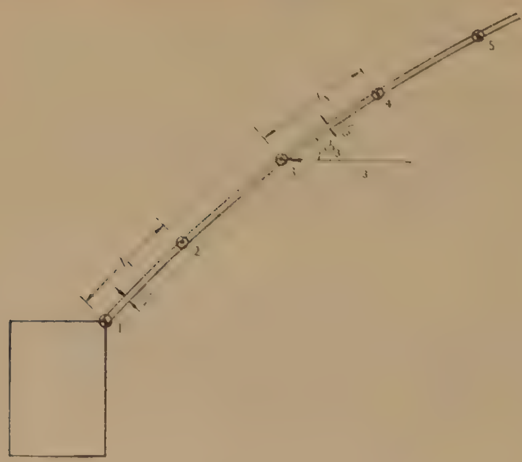

FIG. 13

Notation *

For the ring bounded on the outside by hinge n (Fig. 15):

l_n denotes width of ring.
d_n „ thickness of ring.
A_n „ cross-sectional area of ring.
ϕ_n „ angle of inclination of ring.
r_n „ radius of hinge n.
M_n „ radial moment at hinge n.
σ_n „ circumferential stress at hinge n.

For horizontal reactions:

$H^S{}_{mn}$ denotes horizontal reaction at hinge m due to stress σ_n at hinge n.
$H^M{}_{mn}$ „ horizontal reaction at hinge m due to moment M_n at hinge n.

For relative rotations:

$\theta^S{}_{mn}$ denotes relative rotation at hinge m due to stress σ_n at hinge n.
$\theta^M{}_{mn}$ „ relative rotation at hinge m due to moment M_n at hinge n.

* A further list of the notation in general use throughout the Paper and Appendices is given on p. 278.

The sign convention is such that positive values of radial bending moment produce tension on the upper surface, i.e., the opposite convention to that used for the Geckeler method.

For simplicity, each ring will be considered to be of constant thickness. Extension of the method to shells of continuously varying section is simple but leads to complicated algebraic expressions.

Details of method

(*a*) *Horizontal reactions due to hoop tensions*

Consider the structure shown diagrammatically in Fig. 14. Hinge 1 is given a radial displacement such that a hoop stress of σ_1 is induced at this hinge, hinge 2 and the rest of the structure remaining unaffected. The hoop stress in the ring beam will be σ_1 and the distribution of hoop stress in ring 1 will be as shown in Fig. 14. The tension in the ring beam is thus $\sigma_1 A$, and in ring 1 the total hoop tension is $\frac{1}{2}\sigma_1 A_1$.

These hoop tensions require horizontal radial thrusts to maintain equilibrium. The

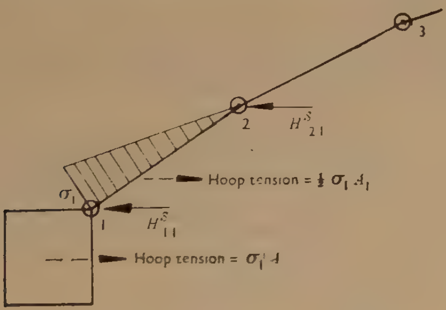

FIG. 14

thrust required to maintain the hoop tension in the ring beam is $\sigma_1 A/r$ and, since the centre of tension in ring 1 is at the third point, the thrust required at hinge 1 is $\frac{2}{3} \cdot \frac{1}{2} \cdot (\sigma_1 A_1/r_1)$. At hinge 2 a thrust of $\frac{1}{3} \cdot \frac{1}{2} \cdot (\sigma_1 A_1/r_1)$ will be required. Hence the reaction at hinge 1 is

$$H^s{}_{1\,1} = \frac{\sigma_1 A}{r} + \frac{1}{3}\,\frac{\sigma_1 A_1}{r_1} = \left(A + \frac{1}{3}\,A_1\right)\frac{\sigma_1}{r_1} \qquad 6(a)$$

and at hinge 2:

$$H^s{}_{2\,1} = \frac{1}{6}\,A_1\frac{\sigma_1}{r_1} \qquad 6(b)$$

Similarly the thrusts necessary to maintain equilibrium with the hoop tensions in rings $n - 1$ and n, when there is a stress σ_n at hinge n and zero stress at all the other hinges, will be (Fig. 15):

$$\left.\begin{aligned} H^s{}_{n-1\,n} &= \frac{1}{6}A_{n-1}\frac{\sigma_n}{r_n} && 7(a)\\ H^s{}_{n\,n} &= \tfrac{1}{3}(A_{n-1} + A_n)\frac{\sigma_n}{r_n} && 7(b)\\ H^s{}_{n+1\,n} &= \frac{1}{6}A_n\frac{\sigma_n}{r_n} && 7(c)\end{aligned}\right\}$$

(*b*) *Horizontal reactions due to radial moments*

Now consider the reactions which must be provided when moments are applied to each hinge in turn. A moment M_1 applied at hinge 1 in the direction shown in Fig. 16 will cause horizontal reactions at hinges 1 and 2 given by:

$$H^M{}_{1\,1} = -\,H^M{}_{2\,1} = \frac{M_1}{l_1 \sin\phi_1} \qquad (8)$$

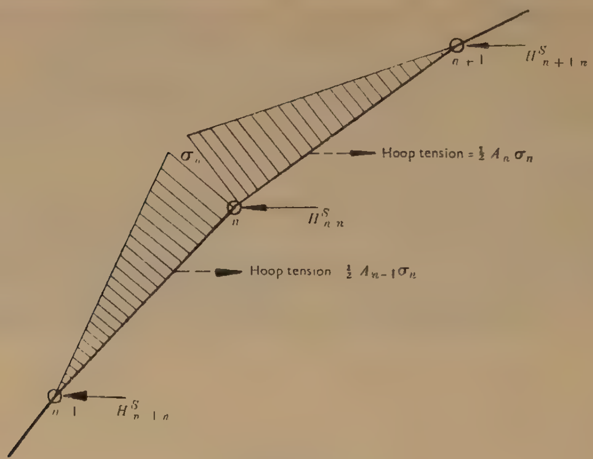

FIG. 15

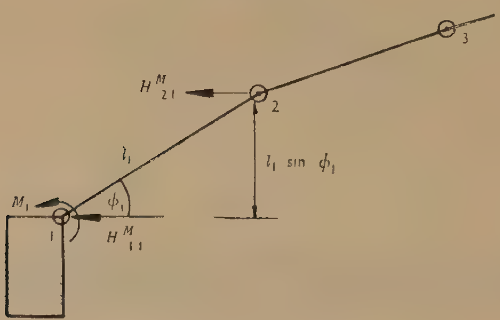

FIG. 16

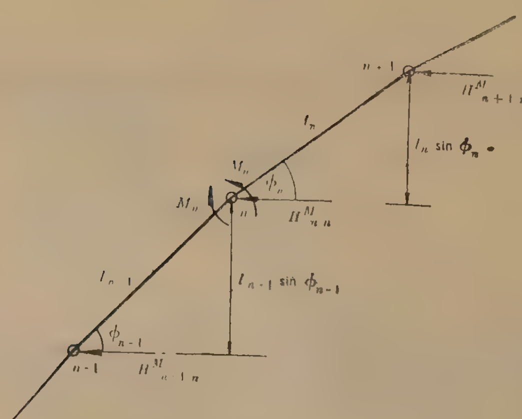

FIG. 17

Similarly at any internal hinge n a moment M_n (Fig. 17) will cause reactions given by:

$$H^M{}_{n-1\,n} = -\frac{M_n}{l_{n-1}\sin\phi_{n-1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 9(a)$$

$$H^M{}_{n\,n} = \left(\frac{1}{l_{n-1}\sin\phi_{n-1}} + \frac{1}{l_n\sin\phi_n}\right)M_n \quad . \quad . \quad . \quad 9(b)$$

$$H^M{}_{n+1\,n} = -\frac{M_n}{l_n\sin\phi_n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 9(c)$$

For equilibrium at the first hinge with an applied thrust H^0:

$$H^0 = H^s{}_{1\,1} + H^s{}_{1\,2} + H_{M1\,1} + H^M{}_{1\,2} \quad . \quad . \quad . \quad . \quad 10(a)$$

and at the interior hinges, since there will generally be no applied thrusts, the sum of the reactions must be zero. Hence:

$$H_n = H^s{}_{n\,n-1} + H^s{}_{n\,n} + H^s{}_{n\,n+1} + H^M{}_{n\,n-1} + H^M{}_{n\,n} + H^M{}_{n\,n+1} = 0 \quad 10(b)$$

Thus a set of n equations is obtained which connect the stresses and moments at the n hinges with the applied thrust. A further set of n equations linking the stresses and moments can be found from a consideration of the relative rotations of the rings at each hinge.

(c) *Relative rotations at the hinges due to hoop tension*

Owing to a stress σ_1 at hinge 1 the strain will be σ_1/E and the radial displacement of the hinge will be $\delta_1(= \sigma_1 r_1/E)$. Since there is no stress at the remaining hinges there will be no radial displacement at these hinges.

The radial strain is assumed to be zero so that the width l_1 of the ring is constant. Since hinge 1 moves outwards an amount δ_1 the remainder of the shell must move bodily

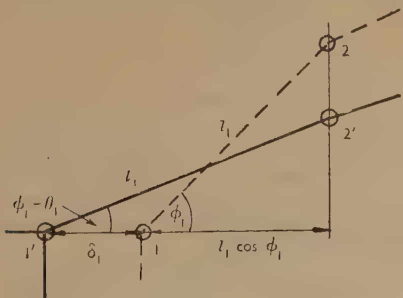

FIG. 18

vertically downwards (Fig. 18). Ring 1 will therefore rotate through an angle θ_1. A consideration of the geometry of the initial and final positions of the hinges shows that:

$$l_1\cos\phi_1 + \delta_1 = l_1\cos(\phi_1 - \theta_1)$$
$$= l_1(\cos\phi_1\cos\theta_1 + \sin\phi_1\sin\theta_1).$$

For small values of θ_1 the value of $\cos\theta_1$ can be taken as unity and the value of $\sin\theta_1$ as θ_1, whence:

$$\theta_1 = \frac{\delta_1}{l_1\sin\phi_1}$$

and

$$E\theta^s{}_{1\,1} = \frac{r_1}{l_1\sin\phi_1}\sigma_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 11(a)$$

The angle at hinge 2 will be changed by an equal amount but in an opposite sense.

$$E\theta^s{}_{2\,1} = -\frac{r_1}{l_1\sin\phi_1}\sigma_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 11(b)$$

At an internal hinge n the lateral displacement accompanying a stress σ_n is δ_n $(= \sigma_n r_n/E)$. Again, the remaining hinges have zero stress and thus no radial displacement. From Fig. 19 it is seen that at hinge $n-1$ ring $n-1$ undergoes a change of slope θ_{n-1} such that:

$$l_{n-1}\cos\phi_{n-1} - \delta_n = l_{n-1}\cos(\phi_{n-1} - \theta_{n-1})$$

whence, as before,

$$\theta_{n-1} = -\frac{\delta_n}{l_{n-1}\sin\phi_{n-1}}$$

and

$$E\theta s_{n-1\,n} = -\frac{r_n}{l_{n-1}\sin\phi_{n-1}}\sigma_n \quad \ldots \ldots \quad 12(a)$$

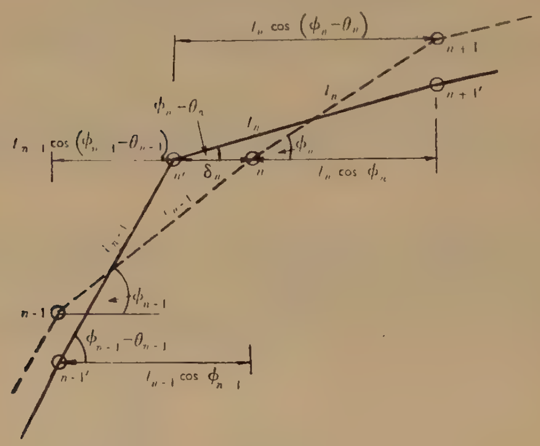

FIG. 19

In the same manner, at hinge $n+1$:

$$E\theta s_{n+1\,n} = -\frac{r_n}{l_n\sin\phi_n}\sigma_n \quad \ldots \ldots \quad 12(b)$$

At hinge n the total change of angle between the two rings will be the sum of the above rotations but in the opposite sense, whence:

$$E\theta^s_{n\,n} = \left(\frac{r_n}{l_{n-1}\sin\phi_{n-1}} + \frac{r_n}{l_n\sin\phi_n}\right)\sigma_n \quad \ldots \ldots \quad 12(c)$$

(*d*) *Relative rotations due to radial moments*

Since the radius of each ring is large compared with its width, the rotations produced by the application of moments to each hinge in turn can be determined by considering each ring to be a simply supported slab of span l_n and thickness d_n. When a moment M_A is applied at one end of a simply supported slab AB, of span l and moment of inertia I, the rotations at the supports are as follows:

At end where moment is applied: $\theta_A = \frac{2l}{6EI}M_A$

At end opposite to applied moment: $\theta_B = \frac{l}{6EI}M_A$

Whence for the first ring :

$$E\theta^M{}_{1\,1} = \frac{2l_1}{6I_1}M_1 = 4\frac{l_1}{d^3{}_1}M_1 \quad \ldots \ldots \quad 13(a)$$

$$E\theta^M{}_{2\,1} = 2\frac{l_1}{d^3{}_1}M_1 \quad \ldots \ldots \quad 13(b)$$

and at the internal hinges:

$$E\theta^{M}_{n-1\,n} = 2\frac{l_{n-1}}{d^3_{n-1}}M_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 14(a)$$

$$E\theta^{M}_{n\,n} = 4\left(\frac{l_{n-1}}{d^3_{n-1}} + \frac{l_n}{d^3_n}\right)M_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 14(b)$$

$$E\theta^{M}_{n+1\,n} = 2\frac{l_n}{d^3_n}M_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 14(c)$$

For continuity at each joint, ignoring the rotation at the edge due to membrane forces, the sum of the relative rotations must be zero. The following equations are then obtained.

At hinge 1:

$$\theta^{S}_{1\,1} + \theta^{S}_{1\,2} + \theta^{M}_{1\,1} + \theta^{M}_{1\,2} = 0 \quad . \quad . \quad . \quad . \quad . \quad 15(a)$$

At each internal hinge:

$$\theta^{S}_{n\,n-1} + \theta^{S}_{n\,n} + \theta^{S}_{n\,n+1} + \theta^{M}_{n\,n-1} + \theta^{M}_{n\,n} + \theta^{M}_{n\,n+1} = 0 \quad 15(b)$$

Equations 10(a) and (b) and 15(a) and (b) can now be solved to give the values of the circumferential stresses and the radial bending moments at the points considered as hinges.

Simplification for comparison with Geckeler method

In order to make a comparison of the above method with the Geckeler method, equations 10(a) and (b) and 15(a) and (b) can be simplified on the basis of assumptions (2), (3), and (4). Accordingly:

$$\begin{aligned} \phi_1 &= \phi_2 = \ldots\ldots = \phi_n \\ r_1 &= r_2 = \ldots\ldots = r_n = r \\ d_1 &= d_2 = \ldots\ldots = d_n = d \\ l_1 &= l_2 = \ldots\ldots = l_n = l \\ A_1 &= A_2 = \ldots\ldots = A_n = dl. \end{aligned}$$

Equation 10(a) then becomes:

$$H^0 = \left(A + \frac{1}{3}A_1\right)\frac{\sigma_1}{r} + \frac{1}{6}A_1\frac{\sigma_2}{r} + \frac{M_1}{l\sin\phi_1} - \frac{M_2}{l\sin\phi_1}$$

Multiplying this by dr/l and putting

$$S = \frac{6}{d^2}\sigma\,; \qquad H' = H^0\frac{dr}{l}\,; \qquad \gamma = 2\left(\frac{3A}{A_1} + 1\right)$$

$$\text{and} \quad \beta = \frac{rd}{l^2\sin\phi_1} = \frac{Rd}{l^2}$$

Equation 10(a) becomes:

$$\gamma S_1 + S_2 + \beta M_1 - \beta M_2 = H'.$$

Considering four hinges, the eight equations can be derived in a similar manner giving results as shown in Table 2.

TABLE 2

S_1	S_2	S_3	S_4	M_1	M_2	M_3	M_4	
γ	1	.	.	β	$-\beta$	.	.	$= H'$
1	4	1	.	$-\beta$	2β	$-\beta$	.	$= 0$
.	1	4	1	.	$-\beta$	2β	$-\beta$	$= 0$
.	.	1	2	.	.	$-\beta$	β	$= 0$
-3β	3β	.	.	2	1	.	.	$= 0$
3β	-6β	3β	.	1	4	1	.	$= 0$
.	3β	-6β	3β	.	1	4	1	$= 0$
.	.	3β	-3β	.	.	1	2	$= 0$

Since the equations for S and M so obtained depend only on H', β, and γ (the last factor appearing in only one coefficient), it is possible to obtain a solution for any particular value of β.

For $\beta = 2$ the width of each ring will be $l = \sqrt{\frac{1}{2}Rd}$ and the solution of the equations is:

$$S_1 = \frac{H'}{\gamma + 4{\cdot}02} \qquad M_1 = +\,1{\cdot}514 S_1$$

$$S_2 = +\,0{\cdot}534 S_1 \qquad M_2 = -\,0{\cdot}229 S_1$$

$$S_3 = +\,0{\cdot}032 S_1 \qquad M_3 = -\,0{\cdot}388 S_1$$

$$S_4 = -\,0{\cdot}125 S_1 \qquad M_4 = -\,0{\cdot}279 S_1.$$

Using these results for the analysis of the Cleadon dome, the dimensions of which were given earlier,

$$l = \sqrt{\tfrac{1}{2} \times 170 \times 0{\cdot}25} = 4{\cdot}61 \text{ ft}$$

$$A = 6 \text{ sq. ft, thus } \gamma = 2\left(\frac{3 \times 6}{25 \times 4{\cdot}61} + 1\right) = 33{\cdot}2.$$

From equation 1(b) the edge value of T_2 for the membrane condition is 4,715 lb/ft (taking $g = 52$ lb/sq. ft, as before).

This will require a horizontal reaction:

$$H^m = T_2 \cos\phi_1 = 4{,}160 \text{ lb/ft.}$$

The edge value of T_1/d from equation 1(a) is 86 lb/sq. in. and the horizontal radial force H^B required to give this stress to the ring beam is

$$\frac{86 \times 144 \times 6}{80{\cdot}25} = 925 \text{ lb/ft.}$$

Whence $$H^0 = 4{,}160 + 925 = 5{,}085 \text{ lb/ft}$$

and $$H' = H^0 dr/l = 22{,}100 \text{ lb.}$$

Thus $$S_1 = \frac{22{,}100}{33{\cdot}2 + 4{\cdot}02} = 594 \text{ lb.}$$

and since $$S = \frac{d^2\sigma}{6},\ \sigma_1 = \frac{6 \times 594}{0{\cdot}25^2 \times 144} = 396 \text{ lb/sq. in.}$$

from which the net tensile stress in the ring beam becomes $396 - 86 = 310$ lb/sq. in. and the net tension $310 \times 144 \times 6 = 268{,}000$ lb.

The bending moment at the springing

$$M_1 = 1{\cdot}514 \times 594 = 898 \text{ lb-ft/ft.}$$

The other values of stress and moment in the edge zone are similarly calculated.

APPENDIX II

DERIVATION OF GECKELER'S EQUATION

Derivation of Geckeler's equation

The assumptions on which this derivation is based were stated above. Axially symmetrical loads only are considered. The forces acting on an element of the shell are shown in Fig. 3. In order to determine the effect of edge forces the shell is considered to be free from surface loads. In view of this the sum of the vertical components of the forces acting on any horizontal section must be zero. Thus at any section:

$$T_2 \sin\phi + N_2 \cos\phi = 0$$

or $$T_2 = -\,N_2 \cot\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

Considering now the equilibrium of the element shown in Fig. 3, the sum of the radial components of the forces is

$$T_1 R\,.\,d\psi\,.\,d\alpha - \frac{dN_2}{d\psi}\,.\,d\psi R\,.\,d\alpha + T_2 d\psi\,.\,R\,.\,d\alpha = 0$$

where $d\alpha$ is an element of angle measured circumferentially.

On the basis of the first assumption the component of T_2 is to be neglected in comparison with the component of T_1 whence

$$T_1 = \frac{dN_2}{d\psi} \qquad (b)$$

(The variable ψ is measured from the edge of the shell, whereas ϕ is measured from the crown.)

The circumferential stress will be T_1/d and the corresponding strain T_1/Ed. The outwards displacement from the central axis will therefore be

$$\delta = \frac{T_1 r}{Ed} = \frac{R \sin \phi}{Ed} \cdot \frac{dN_2}{d\psi} \qquad (c)$$

The side AB of the element will be displaced by an amount δ and the side CD by an amount $\delta + \frac{d\delta}{d\psi} \cdot \delta\psi$ (Fig. 3). The side CD will thus move with respect to AB by an amount $\frac{d\delta}{d\psi} \cdot \delta\psi$ and since the length of the element is unchanged the displacement with respect to AB normal to the shell is $\frac{1}{\sin \phi} \frac{d\delta}{d\psi} \cdot \delta\psi$.

The rotation χ of the element is therefore:

$$\chi = \frac{1}{\sin \phi_1} \cdot \frac{d\delta}{d\psi} \cdot \frac{\delta\psi}{R\delta\psi} = \frac{1}{Ed} \cdot \frac{d^2 N_2}{d\psi^2} \qquad (d)$$

since the value of ϕ is assumed constant and equal to ϕ_1 over the edge zone.

The bending moment M_2 is given by:

$$M_2 = \frac{EI}{R} \cdot \frac{d\chi}{d\psi} = \frac{d^2}{12R} \cdot \frac{d^3 N_2}{d\psi^3} \qquad (e)$$

Finally taking moments about AB:

$$N_2 R \delta\psi + \frac{dM_2}{d\psi}\delta\psi = 0$$

$$N_2 = -\frac{1}{R}\frac{dM_2}{d\psi} = -\frac{d^2}{12R^2} \cdot \frac{d^4 N_2}{d\psi^4} \qquad (f)$$

This is Geckeler's approximate equation for the bending of spherical shells. Substituting in this $4a^4 = 12R^2/d^2$ the general solution (g) can be obtained:

$$N_2 = C_1 e^{-a\psi} \cos a\psi + C_2 e^{-a\psi} \sin a\psi + C_3 e^{a\psi} \cos a\psi + C_4 e^{a\psi} \sin a\psi \qquad (g)$$

Since the effect of forces at the edge will decrease as ψ becomes large, $C_3 = C_4 = 0$. At the edge where $\phi = \phi_1$ the horizontal force is given by:

$$H = T_2 \cos \phi_1 - N_2 \sin \phi_1$$

and, using equation (a),

$$H = -\frac{N_2}{\sin \phi_1} \qquad (h)$$

The remaining forces can be found by successive differentiation of N_2.

Putting

$$\theta(a\psi) = e^{-a\psi} \cos a\psi$$
$$\xi(a\psi) = e^{-a\psi} \sin a\psi$$
$$\phi(a\psi) = \theta(a\psi) + \xi(a\psi)$$
$$\psi(a\psi) = \theta(a\psi) - \xi(a\psi)$$

the resulting expressions for the forces are then as shown in Table 3. The edge values ($\psi = 0$) are also shown.

Solution for shell with ring beam uniformly supported round perimeter

For a uniform load g/unit area, the membrane forces[1] at the edge are:

$$T_1 = -gR\left(\cos \phi_1 - \frac{1}{1 + \cos \phi_1}\right) \qquad (j)$$

$$T_2 = -gR\frac{1}{1 + \cos \phi_1} \qquad (k)$$

TABLE 3

Quantity	Multiplier	General		At edge	
		C_1	C_2	C_1	C_2
N_2	1	$\theta(a\psi)$	$\xi(a\psi)$	$+1$	0
T_2	$-\cot\phi_1$	$\theta(a\psi)$	$\xi(a\psi)$	$+1$	0
H	$\sin\phi_1$	$\theta(a\psi)$	$\xi(a\psi)$	$+1$	0
T_1	a	$-\phi(a\psi)$	$\psi(a\psi)$	-1	$+1$
δ	$\dfrac{Ra\sin\phi_1}{Ed}$	$-\phi(a\psi)$	$\psi(a\psi)$	-1	$+1$
χ	$\dfrac{2a^2}{Ed}$	$\xi(a\psi)$	$-\theta(a\psi)$	0	-1
M_2	$\dfrac{ad}{2\sqrt{3}}$	$\psi(a\psi)$	$\phi(a\psi)$	$+1$	$+1$

and the corresponding displacements at the edge:

$$\delta_1 = \frac{T_1 R\sin\phi_1}{Ed} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (l)$$

$$\chi_1 = -\frac{2gR\sin\phi_1}{Ed} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (m)$$

The horizontal thrust on the ring beam will be $H_1 = T_2\cos\phi_1$.

The stress in the ring beam $\quad \sigma_B = T/A$ where T is the total tension,

or $\quad \sigma_B = T_2 R\sin\phi_1\cos\phi_1/A$.

The radial displacement of the ring beam will then be:

$$\delta_B = -\frac{\sigma_B R\sin\phi_1}{E} = \frac{gR^3}{EA}\cdot\frac{\sin^2\phi_1\cos\phi_1}{1+\cos\phi_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (n)$$

For no rotation of the ring beam the rotation due to the edge forces must eliminate the rotation due to the membrane forces or:

$$-\frac{2gR\sin\phi_1}{Ed} - \frac{2a^2C_2}{Ed} = 0$$

whence

$$C_2 = \frac{-gR\sin\phi_1}{a^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (p)$$

If the horizontal force necessary to give equal displacement of ring beam and shell edge is H, then the lateral displacement of the ring beam is:

$$\delta'_B = \frac{HR^2\sin^2\phi}{EA} + \delta_B$$

and of the shell edge is:

$$\delta'_1 = \frac{Ra\sin\phi_1}{Ed}(-C_1 + C_2) + \delta_1$$

Putting $\delta'_1 = \delta'_B$ and using the value of C_2 found above:

$$C_1 = -\frac{gR}{a+b}\left(\cos\phi_1 + \frac{b\sin\phi_1\cos\phi_1 - 1}{1+\cos\phi_1} - \frac{\sin\phi_1}{a}\right) \quad . \quad . \quad . \quad (q)$$

where $\quad b = Rd/A$.

The Paper, which was received on the 12th October, 1955, is accompanied by seven photographs and five sheets of drawings, from which the half-tone page plates and the Figures in the text have been prepared, and by the following Appendices.

Discussion

The Chairman opened the discussion by mentioning a few points which interested him. One was the slide showing trees growing out of the puddle trench; he felt it would interest Mr Tattersall, who had had experience of drying puddle at Chingford reservoir. The use of the puddle round the reservoir was similar to what had been done in gasholders round London in previous years. The reservoir had ended up by being almost like a gasholder, but the gasholder had a number of timber supports carrying the light steel dome when it subsided. It was interesting to see that at Cleadon a large number of supports had to be erected and a complete dome had been built in timber, as shuttering, only to be replaced by the permanent dome in concrete.

Some years previously the Chairman had investigated a reservoir at Easington for Mr Ruffle's Company where there had been some "rat-holes" and subsidences in the bottom, and there had been a difference of opinion on whether the cause was mining subsidence or cavernous limestone underneath. Mr McLellan had dealt with the matter by using a rubber lining, and the Chairman hoped he would describe his experience in the following discussion, since it was related to the problem of mining subsidence mentioned in the Paper.

Mr A. G. McLellan (Engineer and General Manager, Sunderland and South Shields Water Company) explained that the general picture in the Water Company's area of supply should be painted against a background of mining subsidence. Of the Company's twelve service reservoirs seven had been open reservoirs, and of those seven, six had been constructed in a similar manner to the one at Cleadon, using puddle clay as the water-retaining medium. They were in fact "Hawksley" reservoirs. They had done their job remarkably well over the years. The reservoir at Cleadon was slightly less than 100 years old. The idea had been that the puddle would move and take up any movement caused by mining subsidence, which had in fact occurred.

The first decision required had been whether to interfere with the puddle clay or to adopt some form of roof which had no intermediate supports. With a circular reservoir the second alternative led automatically to a dome, but not necessarily to a concrete dome. A metal dome might have been used. However, from the point of view of cleanliness and freedom from infection the concrete dome had much to commend it.

It could not be assumed that because mining subsidence had taken place it would not recur, or that because the coalmining authorities had said "It is extremely unlikely that we shall take coal out of that seam" any likelihood of their doing so was past. It was therefore necessary always to bear in mind the possibility of mining subsidence under structures in a coalfield until it was known that every seam had been completely worked out, or unless one took the expensive step of buying all the seams in the neighbourhood of each structure. That point had to be considered in relation to the expense of covering a reservoir.

The reservoir at Cleadon was important, especially at night, for balancing supplies to the South Shields area. In view of the considerations to which he had referred, the decision was taken not to puncture the floor of the reservoir. Whether or not that had been the easy way out might be proved during the current year, since in the case of the next reservoir to be covered the floor was to be punctured because no other kind of unsupported roof could be carried by the walls of the reservoir. In making good the punctured floor, however, expensive precautions would be necessary.

Could the Authors subdivide their figure for the cost of construction (about £25,000) and say how much of that had been due to the supports (the tubular scaffolding), and how much to the shuttering? Circumstances had made the work expensive. Only one use of the shuttering had been possible and it had been carefully and elaborately fixed. Even the question of suitable aggregate had been difficult—it was a bad part of the country for good aggregates. Therefore if the Authors would subdivide the cost of construction it would bring out the great cost of the shuttering and its supports.

The men on the job had taken an intense interest in the work, which had contributed markedly to the high degree of workmanship obtained.

Professor A. L. L. Baker (Professor of Concrete Technology, Imperial College of Science and Technology) observed that many good reasons were given in the Paper for using the dome, but probably one reason had been omitted, namely, the love of every engineer of designing and building a large-span dome.

Referring to the quality of the concrete, he said the standard deviation of the cubes had been good but the strength rather low. Presumably that had been due to the local aggregate, which was apparently rather poor. He asked how the reinforcement had been spliced in the ring beam and whether the joints were staggered so that not more than one splice occurred at any section, which was probably the ideal way of splicing the steel in a ring beam. Why had the ring beam not been prestressed? It seemed to have been a good case for prestressing. Would the Authors consider that the risk of cracking would be reduced or, if cracks did occur, that their width might be reduced slightly, if any kind of deformed bar had been used instead of ordinary smooth bars?

With regard to the detail design of the ring in relation to the shell, whilst he was sure that by longer and more careful study the answer would be apparent, would the Authors say whether or not the eccentricity of the horizontal reaction from the ring had been decided in such a way as to reduce the radial bending in the shell to the minimum? Presumably there had been some reason of that kind for the fairly high eccentricity of the horizontal reaction from the ring beam.

He had been interested in the method devised by the Authors and given in Appendix I of assuming hinges and using rotations as unknowns to determine the radial bending moments. Did the Authors consider that if the shell had been made thinner and therefore more flexible the moments would have been less and whether, with those calculations for the hinge rotations, the rotations could have been small enough to be easily taken up by local plasticity, so that it would not have been necessary to thicken the shell quite so much and yet a satisfactory solution obtained?

He noted that it had taken 22 days to lower the supports. In the case of a much smaller dome (60–70 ft in diameter), which Professor Baker had designed in South Africa, the shuttering had been lowered by loosening wedges, as the Authors had done, first of all making them hand-tight and then slackening them gradually to take the load off the shuttering. It had been calculated that the man who had to do the job had to walk several miles over the scaffolding. It was quite a big part of the operation of construction.

Mr W. E. J. Budgen (Chief Engineer (London), Twisteel Reinforcement Ltd) agreed with Professor Baker that every engineer obtained more satisfaction from designing a structure of the character of that under discussion than by designing something more humdrum. Usually the engineer's instincts in that direction had to be slightly subdued by the necessity to advise his client that such plans tended to be expensive. They had therefore been extremely fortunate when they had been asked by the Sunderland and South Shields Water Co. to design the structure at Cleadon in that that Company decided that the inadvisability of interfering with the puddle clay floor rendered a large-span structure desirable.

It had been thought at first that timber would not be available for shuttering and various schemes had been worked out by which the soffit could have been formed using flat steel plates. Fortunately, when the time came for construction timber had been available, and that enabled the soffit to be formed very accurately and thus maintain the calculated weight within very close limits. The good workmanship of the men on the site, together with adequate supervision, had resulted in a structure which was all that a reinforced concrete structure should be.

It had been realized that an almost unique opportunity would arise at one period, when the whole of the completed structure would be supported on shuttering subject to no loads and that that shuttering would gradually be removed. That seemed to provide a valuable opportunity to obtain some idea of the correlation between calculated and

observed stresses. With the co-operation of the Sunderland and South Shields Water Company a fairly extensive testing procedure had therefore been devised and many hundreds of strain-gauge readings had been taken. However, the effects of temperature, as had frequently been found previously when doing similar tests on shell roofs, defeated the object in view, and while it had been hoped that there would be many data to add to the Paper as a result of those tests, in fact those results which had been obtained were not felt to be worth including.

The tests had, however, taught two lessons. The first was that when contemplating building a large dome of that type it was advisable to build it during a period of rising temperature. The second was that whilst much of the elaborate analysis was interesting to the mathematically inclined designer, it was of little use so long as it neglected the important influences in reinforced concrete design of the shrinkage of the concrete and the effect of temperature.

Mr G. P. Manning (a Consulting Engineer in private practice), referring to the relative permanence of reinforced concrete and aluminium domes, asked whether either of the Authors could show him a 3-in.-thick reinforced concrete slab, constructed on a slope similar to the sloping part of the Cleadon dome, which had stood exposed to the weather for more than 15 years without showing some of its reinforcement. In other words, was the Authors' estimate of the permanence of their structure based on experience or on hope? If he had to choose on grounds of permanence alone between two structures, on the one hand a 3-in.-thick reinforced concrete dome designed and specified by the average British designer and built by the average British contractor, and on the other hand an aluminium dome of the Hamman type, with 16-gauge sheeting, he would certainly choose the latter.

The permanence of reinforced concrete depended largely on the correct positioning of the reinforcement, and the only way to ensure that was to design, specify, and provide special spacers for each particular job. The time-honoured method of shoving a block of sand and cement under the steel could not be relied on. Mr Manning showed a slide of some asbestos-cement spacers and remarked that it would be of interest if the Authors would give a sketch of the spacers designed and used for the work at Cleadon.

The cost of a Hamman-type aluminium dome of the size of the Authors' dome would be about £14,500, and its weight about 35 tons, compared with 640 tons for the Authors' dome. Would the Authors say how much the existing walls had settled under the new load?

Had the Authors considered eliminating the edge effects by prestressing the thrust ring? Mr Manning did not suggest that they should have prestressed it, but he would like to know if that had been considered and, if it had, the exact reasons for not doing so. He believed he was correct in saying that all the calculations in the Paper were based exclusively on loading symmetrical about the main vertical axis of the dome.

Mr Manning showed a slide of the first long-span aluminium dome ever to be constructed. The stresses in that dome under the partial loading shown were known; it carried about 25 lb/sq. ft over half its area. Would the Authors say what stresses that type of loading would induce in the dome at Cleadon? How much of the upper surface of the dome had actually been shuttered? The Authors said that the reinforcement in the ring beam was "butted." Did they mean butt-welded?

Mr Manning showed a slide of the Hamman dome nearest in size to that described by the Authors. It was 151 ft in diameter and cost about £12,600. The Hamman type of dome could be constructed without emptying the reservoir and without putting any supports in the water.

Mr C. A. Serpell (Deputy Engineer, Sunderland and South Shields Water Company) said that on p. 266 three possible causes of pollution were given, which in fact constituted the three main reasons for covering the reservoir at Cleadon, but there was another reason which had occurred to him. The Authors mentioned that the water which entered the reservoir from the well had a fairly constant temperature of 55°F. Now the Water Company had a number of burst mains every year, and in the months of December,

January, and February the frequency of those bursts was 100% above the average. From that it was deduced that when there was a period of frost an increase in the number of burst mains could always be expected. One possible explanation was that the water in the open reservoirs cooled, and then cooled the main, and that that cooling of the main was just sufficient to cause the fracture. With the thermal protection afforded by the roof the water should be kept at a more constant temperature, and it was hoped that maintenance of an even temperature would reduce the number of burst mains. Unfortunately, sufficient information was not yet available, because the problem was masked by the vagaries of burst mains due to mining subsidence, so that it would be some time before a definite conclusion could be reached on whether or not the covering of the reservoir would have that added advantage.

The Authors had referred to the choice of material, which lay between aluminium, prestressed concrete, and the shell concrete which had in fact been adopted. For the aluminium roof there had of course been a magnificent precedent in the form of the Dome of Discovery, but that had been intended from the beginning to be a temporary structure. It was felt that the possibility of corrosion had not been fully determined, and that there would be some risk of the corrosion of a permanent structure in aluminium. The prestressed concrete design which had been considered had been visualized as being made with pneumatic mortar, and they had been uneasy about it on two counts. First, no data were available on the behaviour of prestressed concrete under conditions of mining subsidence; secondly, from a number of jobs which had been inspected there had been a doubt about the consistency of pneumatic mortar as applied to a large structure of the type in question. The estimated costs of those alternatives being at that time roughly comparable, it had been decided to adopt the shell concrete roof.

The Authors referred to the additional dead load on the wall, equivalent to $1\frac{1}{4}$ ton/ft run. With all those reservoirs which had been roofed over by a single-span roof, whether it was a dome or a roof over a rectangular reservoir, the crux of the problem was the strength of the gravel puddle on which the wall was founded. That was a question which had been considered in every case. At one of the larger reservoirs which they were at present investigating the extra loading which would be put on the gravel puddle had decided them against the single-span roof; it would mean taking too great a risk. The dome, of course, was ideal in that it loaded the wall uniformly all round, whereas with a barrel-vault or rectangular roof there were concentrations of load at certain points round the perimeter which made the problem of the bearing strength of the puddle clay rather more complex.

The Authors referred to the freedom from cracks of the structure. Mr Serpell considered it could fairly be claimed that for its size it was remarkably free from any kind of cracking.

Mr F. Tattersall (New Works Engineer, Metropolitan Water Board) remarked that it was not the first time that the Sunderland and South Shields Water Co. had adopted rather an unusual solution to a problem. Another of their reservoirs had been rubber-lined, which was, he believed, the only one of its kind.

A dome had many apparent advantages for a structure of the kind in question. He had recently been considering a somewhat similar job, with a span of about 120 ft, but a dome would be difficult to justify economically on the basis of the costs given at the end of the Paper. The costs were very high and, judging by the experience of the Metropolitan Water Board, were perhaps $2\frac{1}{2}$ times as high as the average cost of a flat slab roof which was much more heavily loaded, being asphalted and covered with 18 in. of earth.

The Authors said that the roof became free of the shuttering during construction. If that happened would it not be possible to construct a dome of the type in question by shuttering it in stages, as would be done in the case of an ordinary flat roof? If it were shuttered in rings, by the time that two or three rings had been constructed the first ring would be free of the shuttering, which could then be taken off and used for the next ring. Any further information which could be given on the proportion of the cost represented by the shuttering would be welcomed.

The Chairman had mentioned the drying out of the puddle clay which had occurred at Chingford. The Metropolitan Water Board had had a great deal of experience of puddle. In one case there had been trees about 40 ft from a puddle wall, with 3-in. roots going right through the puddle wall. The effect of the roots on the puddle was that they extracted water from it and the puddle shrank and dried. In addition, the roots might provide a direct path of leakage.

He would not consider the fact that otherwise they would have to interfere with the puddle floor as a reason for deciding to construct a dome. More than one of the Metropolitan Water Board's reservoirs, which had been constructed basically in a similar way to that at Cleadon, had leaked and had had to be repaired. Many of their reservoir roofs were covered with puddle, but most of those roofs now leaked and were having to be repaired. In those cases, of course, the puddle had been exposed to drying. He knew of cases where puddle used in the floors of reservoirs had deteriorated, but even where that was not so he would not consider that a sufficient reason for deciding upon any particular type of roof construction. Mr Tattersall was interested to find that the Sunderland and South Shields Water Co. thought otherwise.

He would like to know the nature of the ground underneath and outside the Cleadon reservoir. In many of the cases where the Metropolitan Water Board had puddle there was London Clay underneath and it did not really matter whether the puddle leaked or not. If there were narrow bands of puddle outside the walls they would not now regard that as a good form of construction.

What was the real function of the weepholes provided in the walls at Cleadon? If their purpose was to let water in to relieve hydrostatic pressure, where did the water come from? Did it come out of the puddle, or between the puddle and the wall? In any case, the wall should be capable of withstanding the hydrostatic pressure.

On the question of settlement, another method worth considering would have been to make the roof entirely of precast members and then asphalt it. Asphalting was quite cheap on a job of that kind and did not add more than about 1*s*/sq. ft to the price. It should have been possible to deal with settlement quite satisfactorily with a roof constructed entirely of precast members and then asphalted.

Referring to the question of temperature, Mr Tattersall said the Metropolitan Water Board had areas which were supplied by river water, where the temperature fell almost to freezing-point in the winter, and also areas supplied by well water at a constant temperature of 52°F. The water in Cleadon reservoir would probably not be cooled very much, and he would be surprised if in future the Sunderland and South Shields Water Co. had any trouble with burst mains if their water was initially at 55°F. In the Metropolitan Water Board it was the practice to mix well water with the river water where possible because the higher temperature was quite enough to prevent bursts occurring on any substantial scale.

The Chairman remarked that the question of lining and of watertightness was always interesting. Mr McLellan had used rubber in his repairs. At that time I.C.I. had been advocating polythene or one of the plastic sheets which could be ironed together. The Chairman did not know whether that had yet been used in such cases.

Mr R. S. Jenkins (Partner of Ove Arup & Partners) remarked that circular domes had been talked about for many years, and he believed that there had been some German methods of introducing spiral curves at the edge in order to bring the edge stress to match that of the ring beam without departing from the membrane theory. The work described seemed an obvious case for a prestressed concrete ring beam, and he also would like to ask why that had not been used.

With regard to Mr Manning's contribution, it seemed to Mr Jenkins that the case for a light dome had been very powerful. The Authors stated that under the wall the puddle was reinforced with gravel, but the weight of the dome must surely be increasing the pressure on that region considerably, and only time would show whether it was going to stand up to it. Another good reason for an aluminium dome was that it was cheaper.

Mr Manning had said that he had worked out equations 1(*a*) and 1(*b*) in the Paper many years ago. Mr Jenkins pointed out that they had been worked out long before Mr Manning's time, but he thought that the Authors had been quite right to put them down, and also to give the next equation although it often appeared in structural theory, because they were all part of the Paper and of the necessary background.

Mr A. G. McLellan, speaking again, referred to Mr Manning's remarks about cover to reinforcement and the use of aluminium for domes. Mr Manning had said that allowances must be made for what could be obtained from workmen when designing cover. Mr McLellan suggested, on the other hand, that it was possible to get within $\pm \frac{1}{8}$ in. That had been done on the Cleadon roof and in the case of at least six other works which they had carried out recently. In one case the contractors had said that they were asking for machine-tool precision in the placing of reinforcement, but they had insisted on it and found that the men took a greater interest and did it satisfactorily. The contractors had agreed afterwards that it was not a costly business.

Mr Manning had shown an interesting slide of the interior of an aluminium dome, which showed exactly what Mr McLellan did not want to see inside the dome at Cleadon, with all the nooks and crannies to harbour insects, etc., which it was undesirable to have near a potable water supply. The Company's Mill Hill reservoir, which had been lined with rubber, was not entirely germane to the present issue but, since the Chairman had referred to it, Mr McLellan would say that although it had been expensive it was undoubtedly economically justifiable.

Reference had been made to the possible use of polyvinyl chloride instead of rubber. The Water Company had experimented for many months with P.V.C. in all thicknesses and in all sizes and shapes. Under workshop conditions a good joint could be obtained by hot ironing, but that could not be done under the damp site conditions, where it was necessary to think in terms of several thousand square yards. The Research Department of I.C.I. had, at the time, tried to find a solution to the problem of making a continuous lining by the welding of the joints. Not being able to do it with P.V.C. the Water Company had turned to rubber, and the net result had been that to line the reservoir, which had been on the point of being written off as an asset, had cost £70,000. A new 12,000,000-gal reservoir might well cost about £240,000. That was a defence against any charge of extravagance in that respect.

The Chairman, from knowledge of the Mill Hill reservoir, had asked about the "rat-holes" and what had been done about them. There had been no rat-holes under the reservoir which they had lined; the floor had been punctured with about 200 borings to establish that. The so-called "rat-holes" were under the reservoir yet to be repaired and they had been caused by the scouring away of lenticular pockets of sand in the clay foundations by the escape of water through the cracks in the structure.

Mr Bernard Whitteron (a partner in the firm of Howard Humphreys & Sons, Consulting Engineers) said that the question of the insulation of the roof had been mentioned, and he wondered if the Sunderland and South Shields Water Co. had any information on changes of temperature, because such information was difficult to obtain.

Had the question of an aluminium or concrete domed roof created any difficulties in obtaining planning permission? He understood that all such proposals had to be submitted to the County Planning Officer, who might perhaps suggest that a dome would look rather incongruous.

In Fig. 5 there appeared to be no distribution bars to the top steel. If that was so it would be interesting to know the reason. On p. 274 the Authors said that "To eliminate the crowding of the reinforcement the bars were butted, with the joints staggered." Mr Manning had already referred to that point and Mr Whitteron would welcome fuller details.

Lastly, he assumed that no waterproofing was applied but that reliance was placed entirely on the density of the concrete to obtain satisfactory watertightness.

Mr G. P. Manning, speaking further, said he had omitted to point out that when designing aluminium domes the first thing he had done had been to put adjusting screws on both ends of all the members, with a ball joint. He did that before working out any sections, stresses, or spacings. That was the best way to deal with edge effects and problems of that type, namely, to avoid them.

Professor A. L. L. Baker, in a further question, asked the Authors whether they had considered using a flat slab roof, probably with columns spaced at about 15-ft centres, the roof being 8 or 9 in. thick and the columns supported on a similar raft in the floor. That form of construction had been used during the 1939–45 war for tanks where there had been considerable relative settlement, and a tank of that type was very flexible.

*** **Mr B. P. Pritchard** (Engineer, Preload Ltd) observed that the use of a dome roof led to a substantial saving in concrete (or pneumatic mortar) and reinforcing steel, compared with a more conventional flat-slab or beam-and-slab construction. With the present shortage of steel, the latter economy should prove attractive. Furthermore, if the dome was intended as a covering to a water-retaining structure, as in the present case, other advantages ensued, namely:—

(1) The air space between the water and the curved underside of the dome provided a good insulating medium, eliminating the usual earth cover associated with flat roofs.

(2) The absence of roof columns ensured an even distribution of pressure on the floor and subsoil of the structure—an important factor in the watertightness and efficiency of that floor.

There remained only the economy of the roofing medium, and the figures quoted in the Paper were hardly encouraging. In that connexion it was interesting to record the case of a dome of similar proportions, built as the roof to a 2,000,000-gal service reservoir at Paisley, Scotland. The dome diameter was 143 ft with a rise of 17 ft 10½ in. Dome construction was begun about 2 months after the Cleadon commencement and was completed in less than 6 months, compared with Cleadon's 15 months. The cost per square foot was approximately half that of Cleadon. In fact, the whole reservoir structure was completed for less than the cost of the Cleadon dome. To emphasize that point, it should be stated that the structure consisted of a 143-ft-dia. reinforced pneumatic mortar floor, and an 18-ft-high, 143-ft-dia., 9½-in.-thick prestressed pneumatic mortar wall, with its foundation, besides the dome and dome ring.

The main shell of the Paisley dome was of 3-in.-thick pneumatic mortar, reinforced with wire mesh as B.S. 123. The peripheral dome ring, 2 ft 0 in. deep by 1 ft 0 in. wide, was monolithic with the reservoir wall. Both dome ring and wall were of pneumatic mortar and were prestressed circumferentially by a proprietary wire-winding system. The dome ring was prestressed by 170 wires, each carrying a design tension of 1·21 ton. It should be noted that the Paisley dome had to be designed for a live load of 40 lb/sq. ft compared with the 15 lb/sq. ft used at Cleadon.

The Authors stated that a system employing prestressing and sprayed concrete (pneumatic mortar) was considered and then rejected. That was unfortunate since both prestressing and pneumatic mortar offered special advantages over normal reinforced concrete in dome construction.

Pneumatic mortar was especially suited to the lightly loaded smaller diameter domes where the very small design thickness required could be satisfactorily placed. Several 2-in.-thick domes, up to 64 ft dia., had been constructed in Britain in the past few years, and it was hard to imagine their construction in reinforced concrete, especially in terms of compaction and construction joints. Pneumatic mortar, of course, obtained excellent compaction from its high-pressure placing and was well adapted to the forming of efficient

*** This and the following contribution were submitted in writing after the closure of the oral discussion.—Sec.

construction joints. It had the further advantages of easier placing, lighter construction loading on the underlying formwork, and a low shrinkage characteristic. The factor of high strength, also present, was incidental, since only moderate stresses occurred in those dome shells.

Prestressing, *via* a peripheral dome ring, had become a very common feature in shallow dome construction. Many hundreds had been prestressed by the wire-winding system alone, mainly in North America. Its advantages over the reinforced concrete peripheral ring dome could be listed as follows:—

(*a*) The direct stresses for a correctly designed prestressed dome were always compressive. Cracking was eliminated, and full use could be made of the most advantageous property of the concrete—its high compressive strength. Dome-ring proportions were smaller, especially in the large diameters.

(*b*) Edge moments in a prestressed concrete dome were considerably less than with its reinforced concrete counterpart, owing to the balance between the dome self-weight and a part of the prestressing reaction. Edge thickening and bending reinforcement were consequently reduced.

(*c*) Another feature of some importance was that prestressing at the peripheral ring tended to lift the shell off its supporting formwork, which simplified form stripping. In comparison, the normal reinforced concrete dome required special attention at stripping. Care had to be taken to avoid dangerous loading conditions when the formwork was being lowered and removed. The shell had to be let down as uniformly as possible.

A further point made by the Authors concerned the lack of knowledge of the behaviour of prestressed concrete under conditions of mining subsidence. Of course, if that was meant to apply to domes, the lack of such information extended to reinforced concrete in a similar manner. Theoretically, at least, the advantage once more lay with the prestressed solution, which was more resistant to edge disturbance than its reinforced concrete counterpart. A degree of subsidence resistance could easily be incorporated by the winding of extra prestressing wires.

Subsidence implied a removal of part of the support under the edge ring, and several prestressed domes had been specially designed and built for a similar loading condition—where the support was provided by columns. A striking example of that type of structure had just been completed in Havana, Cuba. That dome, 6 in. thick, had a diameter of 294 ft and was supported on a combined beam-dome ring 5 ft 3 in. deep by 3 ft 3 in. wide, wound with nine hundred 0·141-in.-dia. wires, each carrying a 1,650-lb. design tension. The structure was supported on twenty-four columns, giving clear dome-edge spans of 39 ft. However, the dome edge and ring had been designed to span twice that distance, in the event of subsidence rendering any column ineffective.

Mr R. M. Davies (Aluminium Development Association) remarked that the Paper demonstrated very effectively that the dome was still of great practical value today as a roof structure, although it had originated in ancient times.

The advantages of using aluminium as a structural medium at Cleadon might not have been fully realized—an aluminium alloy dome for the reservoir could have been erected more quickly than the reinforced concrete dome and at lower cost.

Mr Davies was surprised at the statement on p. 266 that aluminium was not used for the project because doubts were felt about its resistance to corrosion. With regard to external conditions, suitable aluminium alloys correctly installed had very good resistance to maritime atmospheres and would provide an indefinite life without maintenance and without painting. That had been shown by the tests of the American Society for Testing Materials and also by experience both before the 1939–45 war and since.

The present extensive application of aluminium alloys in shipbuilding and the fact that small boats made from aluminium alloys as long ago as the early 1930s were still in use, although unpainted, should dispel doubts about how the appropriate alloys performed

even under severe marine conditions. The results achieved in that field showed that the corrosion resistance could be of a high order.

With regard to the inside of the dome, the amount of chlorine present in the water was, of course, extremely small and the concentration in the atmosphere would be insufficient to have any appreciable effect on the aluminium alloy, particularly if the interior was well ventilated. In the case of reservoir domes already in service, there was no evidence of any interaction having taken place.

The use of aluminium alloy domes was not confined to reservoirs, and roofs of that type had already been used with success in industrial areas for covering oil tanks and sugar storage vessels.

It was possible that the Authors might not have had experience of the durability of aluminium for roofing purposes. One example was the cupola of the Church of San Gioacchino in Rome which was covered with aluminium in 1897; another was the Rheinbahn Station, Dusseldorf, which was roofed with aluminium in 1929. In both cases, the metal was found to be in good condition when examined in recent years. Another example of durability, perhaps better known, was the cast figure of Eros in Piccadilly Circus, erected in 1893. That had been examined several times and after the coating of dirt had been cleaned away no pitting had been found on the metal surface.

It was perhaps not unreasonable to expect that an aluminium dome for a reservoir would have a similar expectation of life.

With regard to considerations of weight, the load imposed on the wall surrounding the reservoir by an aluminium-alloy dome would be a very small proportion of the load imposed by the reinforced concrete structure described. That would seem to be a useful attribute since the load on the gravel puddle below the masonry wall would consequently be kept to the minimum, with reduced risk of disturbance and leakage. In the event of mining subsidence repair work would be rendered easier and, because of the inherent elasticity of the metal, the dome itself would not be so liable to fracture as one of concrete.

The construction of the reservoir dome in reinforced concrete appeared to have been a lengthy process entailing elaborate scaffolding and considerable usage of timber for shuttering. The use of scaffolding was not, of course, entirely eliminated in the erection of an aluminium dome, but a much smaller amount was required.

The experience which had been gained with aluminium alloys in past years, and recent developments in their use, had now made it possible for aluminium to be employed with advantage in a wide range of applications. Unfortunately, there had been occasions in the past when technical advice in the selection of alloy had not been sought and simple precautions against bimetallic action had not been taken. In some of those instances the material had not had the success that it should have had and the impression created had been undeserved.

Mr Ruffle, in reply, said that the trees to which the Chairman had drawn attention were not so close to the reservoir as they perhaps appeared to be in the photographs; none were nearer than 20 ft from the wall. Although it was well known that the roots of trees had caused trouble elsewhere, no harmful effects had so far been found at Cleadon.

In reply to Mr McLellan and to Mr Tattersall, the subdivision of the cost of the construction was as shown in Table 4.

He agreed with Professor Baker that the average strength of the concrete was low. He was not sure why, but it had been about 400 lb/sq. in. lower than he had expected. It had been known from other jobs in the district that the average value would in any case be considerably lower than that indicated by the curve in Road Note No. 4. The strength requirements for the design had nevertheless been met.

Mr Manning had raised the question of the permanence of aluminium compared with that of concrete. Mr Ruffle was confident that if the concrete were inspected after 15 years there would be no signs of rusting. The cover on the concrete had been very carefully maintained by, as Mr Manning had surmised, the use of blocks of sand and cement. (They were shown on one of the photographs.) There had been constant checking and inspection during the concreting operations.

TABLE 4

	£	£
Scaffolding (including sole bearers, barrow runs, etc.)		6,212
Shuttering: Materials	1,703	
Labour	6,336	
		8,039
Steel fixing		2,544
Concreting: Materials	2,241	
Labour and plant	4,518	
		6,759
Other permanent work		2,032
Total cost of construction		£25,586

No settlement of the walls had been detectable on a recent check.

The concrete had a compacting factor of 0·91, which had made it possible to place the concrete on the slope of the dome quite satisfactorily and to compact it adequately without top shuttering.

Mr Tattersall had compared the cost unfavourably with that of a slab roof and had suggested that it was 2½ times the cost of a flat slab roof. In Mr Ruffle's experience that was not so. Recent tender prices suggested that the cost was perhaps 1½, not 2½, times the cost of a slab roof. Nevertheless, he was not prepared to suggest that the structure would be suitable for a new reservoir. At Cleadon it had been used for the reasons given, which were not necessarily applicable to other reservoirs.

The puddle clay in the reservoir had been in good condition, despite its age. It was absolutely essential that the puddle should remain in good condition because, unlike the reservoirs referred to by Mr Tattersall, Cleadon reservoir was founded on magnesian limestone. If failure of the puddle occurred, it would quickly lead to very serious leakage. In point of fact the reservoir had withstood successfully, without leakage, considerable movement caused by mining subsidence.

The weep-holes through the wall were intended to keep water away from the back of the wall during emptying of the reservoir, so reducing the inward thrust. The cross-section of the wall was such that it would not be safe from either sliding or overturning if there were full inward hydrostatic pressure. In a copy of the specification, dated 1860, there was a clause headed "Drain-holes": "That at every 6 feet in length, or thereabout, and every 2 feet in height, or thereabout, a hole of the height of a course of stones, and three-fourths of an inch wide, shall be left entirely through the side-walls for the purpose of draining off any water which may happen to lodge between the wall and the puddle." It was conceivable that a certain amount of water could reach the back of the wall via the interstices and joints in the limestone masonry.

Mr Jenkins was right in saying that the loading on the gravel puddle had been considerably increased. The gravel puddle was in good condition, and the increase of 20% in the loading, applied after the many years during which it had stood and reached equilibrium with the loading of the wall itself, was not likely to cause any trouble.

In reply to Mr Whitteron, the Authors had no information on the insulating effect of the roof. Undoubtedly it would help to maintain the initially constant temperature of the water. Conversely it was to be expected that the water would tend to keep the temperature of the concrete in the shell lower in summer than it would be otherwise and rather higher in the winter, thus reducing the effect of the seasons, the time of day, and the weather on the expansion and contraction of the dome.

No criticism of the dome had been made by the planning authority who had stated that town planning approval was not required for that type of development. The dome was actually in a sheltered position and was not prominently in the public eye. But apart

rom that, it was difficult to imagine that anyone would take exception to the dome, vhich was entirely functional and yet not unpleasing in shape.

Mr Whitteron was correct in assuming that the density of the concrete was relied on to naintain the watertightness, and Mr Ruffle was sure that no rainwater seeped through. The underside of the dome was often wet, but that was due to condensation.

Professor Baker had suggested a flat slab roof constructed on a raft above the puddle lay. That had been considered, but Mr Ruffle had thought that it would be difficult to nsure reasonably even settlement on the soft puddle. Furthermore a concrete raft over he floor would render the puddle inaccessible in the event of severe ground movement ecessitating its repair. The stepped floor of the reservoir (Fig. 5a) would complicate the onstruction and it was probable that the cost would be more than that of the dome.

Mr Tottenham, in reply, referred to the suggestion by Professor Baker, Mr Manning, nd Mr Jenkins that a prestressed concrete ring-beam might have been advantageous. There were, of course, many advantages to be derived from prestressing, the principal one eing the elimination of hoop tension for normal loading conditions. However, the ffects of variations in temperature between the shell dome and the ring-beam would not e reduced by prestressing and experience with other forms of shell construction indicated hat they might be of considerable importance.

Moreover, it was felt at the time that insufficient data were available concerning the ffect of mining subsidence on prestressed concrete structures and that there were two ood reasons for adopting ordinary reinforced concrete. The first of them was that he distribution of steel in the ring-beam and the shell would give good crack control in he event of uneven settlement. With a prestressed ring-beam good control would be chieved only if either there were additional unstressed reinforcement distributed throughut the section, or the prestressing wires were distributed and had a good bond to the oncrete. Distributing the wires would be complicated and good bond difficult to ensure.

The second reason was that settlement of a portion of the ring-beam might, in the case f prestressing, cause the neutral axis to move so far to the compressed face of the beam hat the prestressing force would cause bursting of the concrete.

Professor Baker had asked whether the reason for the fairly high eccentricity of the shell n the ring-beam was to reduce the radial bending moments. Mr Tottenham agreed that y raising the springing level the line of action of the resultant of the forces T_2 and N_2 assed more closely to the centroid of the ring-beam, and thus reduced the tendency of he beam to twist. The main considerations were, however, practical ones. Had the pringing level been lower there would have been an upstanding portion to the ring-beam which would have complicated the casting of the beam. Also the draining of the rainvater from the dome would have necessitated an opening in the beam which would have een undesirable.

Professor Baker had also raised an interesting point concerning the possibility of educing the edge moments by allowing for a small amount of plastic rotation in the edge one. The plastic rotations would, indeed, be small, even if the edge-zone moments were educed to zero. The method of analysis outlined in Appendix I of the Paper permitted he "plastic" analysis of that type of structure. For example, the first four equations of Table 2 could be solved by taking the moments as zero, and checking the rotations at the inges by using equations (12). The value of E, which determined the strain due to hoop ension, should, for safety, be taken as that of concrete. Alternatively, the moments ould be given some definite value to suit the reinforcement detail, the first four equations olved, and the rotations then checked. In the case of the Cleadon dome, however, such method would not have been advisable, for cracking was to be minimized and the absence f insulation on the outer surface of the dome would give rise to daily thermal movement which, if associated with plastic deformation, might prove harmful. Should a smaller ome be required where external insulation were to be provided it would be interesting to esign it by some such plastic method and to observe its behavour.

In reply to the requests by Professor Baker, Mr Manning, and Mr Whitteron for more

details of the ring-beam reinforcement, the main bars were simply laid end to end, the positions of the ends being staggered within each of the three vertical rings of reinforcement. Thus the ends of the bars forming one hoop of reinforcement were effectively "spliced" by a bar in the same vertical band but some distance away. It was thought that that system would give a maximum chance of consolidating the concrete around the reinforcement. It was not considered necessary to provide distribution reinforcement in the top of the dome near the springing.

There were methods of eliminating edge-effect forces, as suggested by Mr Jenkins, by using a special shape at the shell edge, but those forces were eliminated only for vertical loads. Temperature stresses would not be removed. Mr Manning had suggested that the provision of ball joints and adjusting screws at the edge of the shell would have the same effect. That would be merely a form of prestressing and would only have the desired effect for one particular load. Mr Tottenham could not agree that the best way to deal with edge-effect forces was to avoid them. To him those forces were merely the effect of continuity between dome and ring-beam and were in no manner harmful. They were, in fact, of exactly the same nature as end moments in continuous beams and frames, and presented no greater difficulty in their analysis.

The calculations in the Paper were, as Mr Manning had suggested, entirely devoted to the effect of loading symmetrical about the main vertical axis. In the actual design temperature effects had also been considered. Partial loading had not been investigated, since the self-weight of the dome constituted a large proportion of the design load. Those interested in the analysis of domes under partial loading should refer to the paper by Reissner[7] and the report on model tests by Voss and Peabody.[8]

Mr Manning had also expressed doubts regarding the durability of a 3-in.-thick concrete slab cast on a slope such as that at Cleadon. There were in fact many reinforced concrete shell roofs, both in Britain and on the continent where weather conditions were more severe, which were less than 3 in. thick and were cast with slopes greater than 28°, and yet after more than 15 years showed no signs of corrosion of the reinforcement.

The lifting of the dome from the shutter had led to an interesting speculation by Mr Tattersall. He had suggested that it might be possible to cast a dome in a series of rings and as each ring became free removing the formwork and re-using it again nearer to the crown of the dome. Mr Tottenham did not think that experience of only one dome would be sufficient to justify such a procedure. The reason for the lifting of the dome did not seem to have been convincingly explained as being due to any one particular factor. It might well be that a slight change of circumstances, such as a falling temperature or a long dry spell, might be sufficient to cause the dome to remain seated on the formwork. If that were to occur then the supporting of the thin shell on the ring-beam and another ring of scaffold near to the crown might cause severe local deformations and cracking. It was to prevent such an occurrence that the method of striking of the formwork adopted had been devised.

It would probably have been possible to construct the dome in segments, leaving the segments propped but removing and re-using the formwork for the next segment. The saving in cost on the timber, however, would probably have been outweighed by the added cost of removing and re-erecting the staging for each segment.

The Authors, in joint reply, thanked Mr Pritchard for the details he had given of the dome at Paisley. Reasons for the decision not to adopt the system of construction employing pneumatic mortar had been stated in the Paper and amplified by Mr Serpell in the discussion. Those reasons were qualified in the Paper by "at the present stage in the development of sprayed concrete" and although the choice of material had been influenced by the inspection of some completed structures, the Authors did not wish to imply that there would not be subsequent improvements in the technique of sprayed concrete. The Authors were not yet convinced, however, that the excellent compaction referred to by

[7] References 7–9 are given on p. 301.

Mr Pritchard could always be obtained, nor were they satisfied as to the superiority of sprayed concrete in forming construction joints.

A weakness to which the wire-winding system was subject was the liability of the final sprayed cover on the wires to break bond with the concrete placed earlier, owing to the differences in stress and rate of shrinkage. It was worth noting that the only case of corrosion noted by Hill[9] in his report to the 2nd Congress of the Fédération Internationale de la Précontrainte had occurred in a horizontal cable around a tank which was protected by a gunite coating. The Authors could not agree that the risk of serious damage by mining subsidence was less with a prestressed than with a conventional reinforced concrete design.

It appeared that Mr Pritchard's conception of subsidence was one of local settlements which would result in intermittent support of the ring-beam. Previous experience at Cleadon, however, had shown that owing to the considerable depth of the coal seams, movement, if it occurred, would more likely consist of a large-scale tilting, possibly accompanied by "hogging" of the site as a whole. The Authors were not in any event attracted by the idea of constructing the dome on a series of columns. The advantage of the existing uniform loading of the reservoir wall would be lost and greater expense would be involved.

The case for using aluminium had been ably stated in the communication by Mr Davies. Aluminium was the first material to be considered by the Water Company, which had been in touch with a firm specializing in such work. They had been informed by the aluminium manufacturers that the alloy which it was proposed to use was susceptible to inter-crystalline attack under conditions such as might obtain under the roof and they had been advised that painting, comprising zinc chromate primer followed by two coats of aluminium paint, should be carried out. There had been at that time a lack of information concerning the behaviour of aluminium in the somewhat onerous conditions of roofing a reservoir.

The use of aluminium would undoubtedly have made for speedy and economical erection though it was felt that the advantage was with reinforced concrete when the lower cost of maintenance, the smooth soffit, and the greater insulation value were taken into account.

Additional references

7. E. Reissner, "Stresses and small displacements of shallow spherical shells". J. Math. & Phys., Feb. 1946, p. 80.
8. W. C. Voss and D. Peabody, "Thin shelled domes loaded eccentrically". Proc. Amer. Soc. Civ. Engrs, vol. 73, 1947, p. 1173.
9. A. W. Hill, "Report on present practice regarding grouting and anchorages in prestressed concrete in Great Britain". Paper No. 8, 2nd Congr. Fedn Intern. Précontrainte.

Correspondence on the foregoing Paper is now closed.—Sec.

WORKS CONSTRUCTION DIVISION MEETING

26 January, 1956

Mr A. C. Hartley, Member, Chairman of the Division, in the Chair

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division was accorded to the Authors.

Works Construction Paper No. 31

CONSTRUCTION OF 60-IN.-DIA. OUTFALL SEWER FOR MORECAMBE AND HEYSHAM CORPORATION

by

John Kenneth Brooks, M.I.C.E., and John Stobo Davidson Brown

SYNOPSIS

The Corporation are engaged upon the construction of a new system of sewerage; part of the work comprises a 5-ft-dia. outfall sewer, and the Paper describes the construction of the seaward length. This length starts at the Promenade, passes under the sea wall and extends, under fully tidal conditions, 4,160 ft out to sea.

The Paper covers various methods considered, and the method finally adopted.

The short promenade section under the sea wall was carried out in a steel sheet-piled trench 30 ft deep, and presented interesting problems in maintaining the safety of the wall during the breaking through, and until finally made good.

The beach section—the landward half of the tidal work—was carried out by working from beach level and laying the pipe in a sheet-piled trench, access being by a ramped timber gantry from promenade level.

The seaward half of the tidal work was constructed by driving a 2,000-ft-long working gantry with the rail level at O.D. and carrying tracks of 4-ft-8½-in. and 2-ft gauge. From the gantry half-tide cofferdams were driven in which the pipe was laid.

The plant used consisted of six 5-ton steam cranes, piling equipment, various locomotives, wagons, pumps, mixers, etc., all of which had to be moved to and from the beach according to the tide. This involved careful marshalling and arranging of the units, since two and sometimes three places were being worked at one time, and the units varied depending on the operation in hand.

Introduction

The Corporation of the Borough of Morecambe and Heysham are engaged on the reconstruction, expansion, and improvement of the sewerage system of Morecambe.

In 1948 the Authors' Company prepared and submitted proposals for construction and an estimate for an outfall sewer designed by the Corporation's Consulting Engineer.

The design called for the laying of 5-ft-dia. steel pipes, each 25 ft long, with Johnson couplings, partly haunched and partly encased in concrete, although this was later amended to a pipeline fully encased by a minimum thickness of 12 in. of concrete.

The invert level at the promenade end at approximately −6·00 O.D. is 31 ft below promenade level, and 18 ft below beach level; the line falls at a gradient of 1

* Mr Brooks is a Director of Harbour and General Works Ltd, London, S.W.1.
Mr Brown is Engineer-Agent for Harbour and General Works Ltd, London, S.W.1.

in 735 to an invert level of $-12{\cdot}00$ O.D. which is approximately beach level at the outlet.

After leaving the promenade, the line extends 1,000 ft in a north-westerly direction to a hatch-box chamber with a 20°-bend to the west, and thence straight for an additional 3,200 ft to the terminal point (Fig. 1). Throughout this length the beach slopes gently and the top of the pipe commences to rise above beach level at approximately chainage 2,100.

Opposite the outfall sewer Morecambe Bay is about 9 miles wide, generally shallow, and open in aspect to the south-west. The tidal range at ordinary springs is 28 ft, i.e., from $-13{\cdot}0$ to $+15{\cdot}0$. At neap tides the range is $10{\cdot}5$ ft, from $-4{\cdot}5$ to $+6{\cdot}0$.

The tides are greatly affected by wind from a southerly to westerly direction, when the respective levels are several feet above those predicted.

Sandbanks in the bay give considerable protection from the sea during the lower half of the tide, but for the upper half the site is fully exposed.

Sanction to proceed was given in 1951 and preparatory work commenced in October of that year. The permanent work started early in 1952 and was completed in February 1955, the remainder of the year being taken up in removal of temporary works.

Methods of construction considered

Consideration was given to a number of methods of carrying out the work. The selection of the most suitable could be a matter only of individual judgement, governed by economics and materials available at that very difficult period.

It was proposed that the short length under the promenade and seawall should, if possible, be carried out in tunnel between a shaft on the promenade and a cofferdam on the foreshore. Difficulties were foreseen, but the advantages were sufficiently great to make the attempt worth-while.

For the main work on the foreshore, which consisted of sand and soft clay overlying gravel, the following methods were considered.

(*a*) *Floating plant*

A large amount of floating plant would have been needed such as barges, tugs, etc., with the attendant complications of moorings and a large risk of loss and damage.

Floating plant could not have been used for the shore end of the sewer and working difficulties would have occurred in the grounding of craft in the correct position on every tide.

(*b*) *A high-level cofferdam with a working gantry at* $+20{\cdot}00$ *O.D.*

The scheme of a high-level cofferdam had advantages because plant could be run out to work at all times except during storms, and working periods at the sites would be at their maximum. In bad weather the plant could be brought to safety on the promenade.

Disadvantages were the cost and the amount of materials required for 4,000 ft of gantry, ranging from 12 ft to 32 ft in height; the timbering to support cofferdams of such a height would have been heavy and the damage from scour considerable.

(*c*) *A gantry and cofferdams at a level of* $+9{\cdot}00$ *O.D.*

The gantry and cofferdams would have been just above high water neaps to enable running out the plant at an early point on an ebbing tide.

FIG. 1.—BOROUGH OF MORECAMBE AND HEYSHAM

FIG. 4.—EXCAVATING IN TYPICAL BEACH SECTION COFFERDAM

FIG. 5.—CRANES DRIVING, EXCAVATING, AND EXTRACTING IN BEACH SECTION

Fig. 6.—Result of storm on piles not fully driven

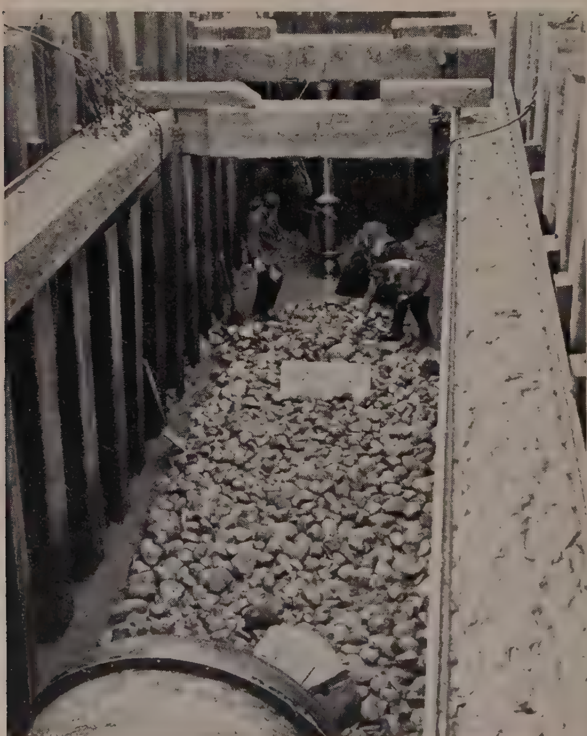

Fig. 7.—Rubble blanket over bad bottom in beach section

Fig. 8.—Pitching precast pile in gantry section

Fig. 9.—Completed pipe near the end of the outfall

Fig. 10.—Gantry section, August 1953

Fig. 11.—General view of the site, April 1954

This scheme had many advantages, but the main objection was the cost of a gantry ranging in height up to 21 ft for a length of 4,000 ft.

(*d*) *An isolated high level staging at about* +20·00 *O.D.*

This staging would have had cofferdams at suitable levels, the whole being removed and re-erected on the leap-frog principle as the completed line extended seawards.

Although there would have been a saving in materials, the objections were that the plant would be very exposed and difficult to service. Floating plant would be required for easy access, and the varying height of the staging above beach level would cause complications. It was also doubtful if time and labour would be saved.

(*e*) *Various other schemes*

Other schemes such as dredging a channel and sinking caissons containing the pipe section did not appear feasible or justified. The possibility of working on the beach with caterpillar-tracked plant was discarded on account of excessive time required in travelling and the risk of plant becoming bogged.

(*f*) *Method proposed*

The scheme put forward in general was to construct a sewer within steel-piled cofferdams with the aid of suitable plant carried on a 4-ft-8½-in-gauge railway track alongside. Materials were to be fed to the work on a parallel 2-ft-gauge track. During high water the plant would be stored on the promenade, and an access gantry sloping down to the beach would be erected.

For the first 2,000 ft the tracks would be laid at beach level with the cofferdam piles driven to about 3 ft above beach level.

For the seaward section a 2,000-ft-long gantry would be built with rail level at O.D. From this cofferdams would be driven with the tops also at this level.

The method would allow work to proceed during the lower half of neap as well as spring tides, and appeared to permit the maximum economy on temporary works.

At the time of the proposal timber was practically unobtainable, but a reasonable amount of steel was available, so the gantries were designed using a steel structure carried on concrete piles.

The track on the beach was to be laid on concrete sleepers or a slab and, if necessary, on short piles where ground conditions were bad.

Methods adopted

General

The initial problem to be solved was the provision of licences for steel and timber. The proposed method would have required about 1,250 tons of steel and 50 standards of timber.

By the time work started conditions had changed; steel had become difficult to obtain and after protracted negotiations allocations were obtained barely sufficient for half the sheet-piles and rails, but sufficient to enable work to commence.

Reconsideration was given to all the problems involved and many of the temporary works were redesigned.

A certain amount of softwood had become available, and greenheart, although expensive, was more easily obtained and did not require a licence. Accordingly the access gantry and about one-third of the seaward gantry were constructed with greenheart piles and Columbian pine caps, runners and bracings. As the work

progressed, Columbian pine became more readily available and it was used to complete the temporary work. Steel supplies eventually also improved, and sufficient was obtained for the cofferdams.

Clearance of the promenade began in October 1951; this was followed by construction of the temporary buildings which, owing to the exposed site and the time they would be in use, were built of brick laid in lime mortar. The buildings and the yard layout were completed by March 1952.

The access gantry to the beach was completed by June 1952, with the aid of a 17 RB as a crane, and the piles were driven by a No. 7 McKiernan-Terry hammer; after this track-laying on the beach was continued while the permanent work on the promenade section was carried out. A view of the site in April 1954 is given in Fig. 11.

Promenade section

The promenade section was a short length of 64 ft starting from an existing manhole (Fig. 2, Plate 1). A 3-ton derrick with 90-ft jib was erected in January 1952, and covered the full extent of the section.

Owing to the existence of old sea walls and other obstructions, it was decided to open up a trench, and to use 12-ft piles, of sufficient depth to allow the clearance of the obstructions. Within this trench 40-ft Larssen No. 2 piles were then driven and excavation proceeded to a depth of 32 ft by grab and crane-skip.

Below the top filling material used in building up the promenade is the original beach of sand and gravel which continues down to formation level, where fine running sand is encountered.

The water level was affected by the tides, but the draw through the bottom was not very fast, and was controlled by a 5-in. electric pump, running 24 hours a day. Steel pipes were laid and surrounded with concrete.

This construction was used to within 10 ft of the coping of the sea wall.

To save breaking out the sea wall it was desirable to lay the pipe in a heading as originally planned, but experience of the ground showed this would be difficult. Advice was sought on chemical consolidation, and dewatering was considered, but neither was thought likely to prove successful.

A short trial was made by driving a heading under the sea wall, but the bad ground conditions were found to continue. Further ideas of tunnelling were then abandoned and preparations were made to continue the sheet-piled trench through the sea wall.

The sea wall consists of precast blocks laid on 12 in. of reinforced concrete at a slope of 2 to 1 and surmounted by 5 ft of coping. The two main risks were a washout behind the wall during a storm, and loss of ground under the sloping wall slab, causing subsidence.

The method adopted was to break out the precast blocks for the width of the trench, leaving the slope still protected by the 12-in. slab of concrete. A narrow width of this slab was then broken out on each side of the trench, to take the sheet-piling.

To prevent scour under the adjoining sea wall, the piling was pitched, driven, and made good on the outside with quick-setting concrete. Sheet-pile diaphragms were also driven to prevent a wash-out occurring behind.

When this piling had passed the toe of the sea wall, excavation proceeded. All walings and struts were bolted to the piles to prevent displacement. Pipes were

laid in the trench, the diaphragms being lifted and replaced after the concrete surround was placed.

In making good the process was reversed. Within the sheet-piles sand and gravel filling was placed in layers, trimmed to slope, and blinded with 6-in. concrete. Following tides consolidated the filling.

When this filling had reached the underside of the coping, the piling was withdrawn, working from the toe of the wall towards the coping. The space left by the piles was made good with concrete at each tide. During this period new precast blocks to match existing blocks had been cast on site. After a period allowing full settlement of the filling, the new blocks were laid in the gap and made good.

Ground lost was negligible. A slight settlement of the coping was attributable to the earlier loss of ground when making the tunnel attempt.

Beach section

Railway track

As mentioned previously, the access ramp was constructed to carry standard and 2-ft gauge track, and tracks were laid on the beach while the work described above was being carried out (Fig. 3, Plate 2).

The beach, which consisted of a fine silty sand with about 50% passing a No. 100 sieve, varied considerably in softness. It was found that if the track was laid on the beach, even when well ballasted, it was subject to movement and damage by wave action, particularly during a storm.

The difficulty was overcome by excavation to 12 in. below beach level, and the placing of a 6-in. layer of crushed stone on which the sleepers were laid. The excavation was filled in with stone so that only the rails projected above the beach. This proved satisfactory and the tracks stood up well to storms and did not suffer badly from scour. Furthermore, it was not until 1955, 3 years later, that any trouble was experienced in the building-up of sand over the track. It was, of course, necessary to keep a gang of five or six men continuously on maintenance.

Cofferdams

The cofferdams were constructed very much as originally proposed, and at first consisted of Larssen No. 2 piles 30 ft long, with tops driven to about 3 ft above beach (Fig. 4*). All later piling, however, was of No. 3 section which stood up to conditions much better.

The two rows of piles were driven approximately 10 ft apart. At 50-ft intervals —two pipe lengths—a diaphragm was driven between two junction piles in the line of side piles.

Pipes were laid to within 10 ft of a diaphragm; the 10-ft space was used as a pump sump. When the next length of 50 ft had been excavated and bottomed up, the diaphragm was removed for a width of 7 ft giving, at this stage, a cofferdam 100 ft long. When the next pipe was laid and surrounded in concrete, short piles were pitched and bedded on the concrete surround to reform the diaphragm.

Since the cofferdams became flooded at each tide, the amount of water to be pumped was a major consideration. The above system allowed the whole excavation and the laying and concreting of every other pipe to be carried out within a 50-ft cofferdam. Only when laying and concreting the intermediate pipes which crossed a diaphragm was a 100-ft length of cofferdam open.

To avoid delays in threading the 25-ft-long pipes between closely spaced struts,

* Figs 4–11 are photographs, between pp. 304 and 305.

compound girder walings 31 ft long were used strong enough to require strutting only at their ends, and timber walings for the remaining 19 ft.

For a short length beyond the sea wall, the formation was in red marl and allowed the full diaphragm to be extracted, but after about 300 ft there was only gravel and sand at formation level, and to stop the follow-through of water under the completed pipe, the diaphragm piles were cut off at a little above formation level in every other diaphragm; these bottom sections were left in to maintain the seal. As and when piles shortened from those damaged became available, the full diaphragm was extracted and the short lengths were driven below invert level.

Quick pumping was essential because of the short time available for work on each shift. Except for the first few hundred feet, when neap tides did not flood the dams, the working period varied from 9 hours near the wall to 4 hours farther out, and averaged about 5 hours.

Sykes 8-in. self-priming centrifugal pumps were mainly used and had an output of 90,000 g.p.h. At the start of a shift the pumps were the first items of plant to be taken out, and were placed by crane in position on timbers across the dams. A quick-action connexion was devised and to save time flexible suction pipes were left in position during high water. The total time for setting-up and pumping varied from 30 to 50 min, depending on depth of invert and other factors.

An important factor was the amount of water in the pipe already laid. For instance, the water contained in 1,000 ft of pipe is about 125,000 gal. To avoid pumping this water, a pipe stopper was designed consisting of a ring made of angle in three sections, and bent to the internal diameter of the pipe. These were fixed to the pipe by studs. A circular door in two parts was then bolted to the angles. The ring, fitted before the pipe was laid, was caulked with yarn and the door was bolted to the ring against a rubber lining. The rings and doors were made up in salt-resisting alloy to reduce weight for ease in handling.

The stoppers were placed every 100 ft. As soon as a pipe with ring attached had been laid and encased in concrete and so would not float when closed, the stopper 100 ft back was removed and the door fixed to the forward ring. The spare ring was taken ashore and fixed to the next pipe. This limited the extra pumping to the volume of 100 ft of pipe.

The piling was driven by a No. 7 McKiernan-Terry hammer, handled by a Smith 5-ton crane; steam was provided either by a No. 16 Spencer-Hopwood boiler mounted on a railway wagon, or by a locomotive fitted with a suitable reducing valve.

Excavation, some of which could be done in the wet without pumping, was by grab. The dams were bottomed up by hand, and the excavated material was loaded into tipping skips, and tipped at the side of the trench.

The pipes, each weighing 4 tons, were laid on precast concrete cradles so that the concrete surround could be placed without a horizontal joint.

The dams were backfilled by grab, and the piles extracted by a No. 80 Zenith extractor (Fig. 5).

A feature of the subsoil in the Morecambe area is its frequent change of nature in a relatively short distance. This section started in red marl and later strata were coarse gravel, running sand, and peat. At about chainage 550 at a depth of between 10 and 15 ft below beach was a 2 to 3 ft layer of impacted gravel with many 2 cu. ft boulders, and occasional large boulders of 1 to 2 cu. yd. This caused great difficulties in pile driving and much damage to piles.

It was at this stage that the change was made to Larssen No. 3 piles. Progress became very slow and the method was changed to driving an outer cofferdam using

short piles driven to the hard layer. After excavating through this hard ground and removing the boulders, 30-ft piles were then pitched and driven inside the outer dam to the full depth.

It should be emphasized that piles once pitched had to be driven home during the shift to save damage from wave action.

On one occasion this was not done and an unexpected worsening of the weather caused both panels to become badly bent (Fig. 6).

The hard layer continued to chainage 1,000, when it changed again to heavy red clay for a short length, followed by clay and sand to chainage 1,600. In part of this length a fine silty sand, which was unstable and tended to boil, was encountered at formation level. Pairs of 9 in. × 9 in. hardwood piles were driven at 12-ft-6-in. centres; on these were placed 12 in. × 6 in. cross-heads to carry the pipes. The silt was taken out below formation, and the surface was blinded with about 12 in. of cobbles before laying and concreting the pipes (Fig. 7).

The ground then reverted to clayey sand and little difficulty was experienced for the remainder of this section.

On all the gravel and silt lengths considerable water came up through the bottom. The dams were widened sufficiently to accommodate a drain on either side of the pipeline. The drain was formed with perforated steel pipe which was removed for re-use after the sewer pipe had been laid, concreted, and shutters stripped.

At points where the diaphragm seals occurred, a 6-in. stoneware pipe was cast in, and later plugged.

Concreting

The pipes were encased in concrete with a minimum cover of 12 in. The concrete mix was approximately 1 : 2 : 4 adjusted to give maximum density and a low water/cement ratio. Portland cement was generally used but sometimes 417 cement when early flooding of the cofferdam could be expected.

There were to be no horizontal joints and vertical joints were limited to one per pipe length.

Easy setting of shutters was essential, together with a continuous and speedy operation of concreting. To obtain this, a square surround was adopted instead of one with the top half semi-circular, which would have involved extra labour and time in setting the top shutters. This saving of delay in the middle of a concreting operation more than compensated for the extra concrete placed, particularly because a tide's work could have been lost.

The side shutters were of 12-ft-6-in. × 7-ft steel panels, which were handled by crane and set on levelled blocks on the formation. They were strutted off the piling and had temporary 12-in. spacers to the pipe.

All concrete was vibrated, but longitudinal flow was kept to a minimum by careful positioning when placing each batch. The water content was increased to assist transverse flow under the pipe.

The method of placing had to allow for controlled grading of the mix, maximum rate of output, and owing to the frequent use of quick-setting cement a short time between mixing and placing. This made it undesirable to run mixed concrete out from the promenade. A weighbatching plant was set up in the promenade yard and ½ cu. yd mixings were discharged into jubilee wagons and run out to the site in sets of six.

The mixing plant consisted of a 21/14 mixer mounted on a 10-ton flat railway wagon with an extended loader down to beach level. Alongside the mixer on the

wagon was a 600-gal water tank which was filled on the promenade at the beginning of the shift. A belt-driven water pump on the mixer raised the water to the feed-tank above the mixer. Coupled to the mixer wagon was a covered box car with sufficient cement for the day's work.

These two wagons were drawn by steam locomotive to the site of concreting operation. The 2-ft-gauge track had been laid so that the jubilee wagons would tip direct into the loading hopper of the mixer.

A lay-on type switch was used to form a siding to take empty wagons so that a diesel locomotive, having delivered a set of full wagons, then returned to the batching plant with a set of empties.

The cement was moved in bags from the storage wagon to the mixer by a short roller conveyor. The concrete was discharged into 1-cu. yd bottom-dump skips which took two mixings; these were handled by a 5-ton steam crane and deposited around the pipe.

Each pipe required about 28 cu. yd of concrete and the arrangement allowed this to be supplied in about $2\frac{1}{4}$ hours, which was as fast as the placing gang could work, and allowed time for the concrete to form its initial set before the cofferdam was flooded.

The method of working adopted was for one crane to pitch and drive cofferdam piles continuously. Behind this were two cranes which worked as a team and shared the duties of excavating, pipe laying, concreting, backfilling, and extracting.

The second crane also assisted in finishing the driving when necessary. The rate of pile driving was the governing factor in the general progress of the work.

The gantry section

The gantry section was commenced in December 1952 with the construction of the seaward gantry. During 1952 all the materials necessary were accumulated to open up the section, which was 2,000 ft long, and a construction gang was formed for the driving of 12 in. × 12 in. timber piles, and constructing the roadway on top of them (Fig. 3, Plate 2).

A No. 7 McKiernan-Terry hammer was used and after the initial piles had been driven the subsequent piles were kept to line by a frame of timbers bolted around the piles already driven.

The piles were capped with 16 in. × 16 in. timber, on to which were bolted 16 in. × 16 in. runners to carry the standard-gauge track. The caps were cantilevered out on the north side to carry the 10 in. × 5 in. longitudinal sleepers on which the 2-ft gauge track was laid. The piles, which were driven to carry approximately 30 tons, varied in length up to 32 ft.

The work proceeded on both neap and spring tides, although it was only during springs that many of the bracings could be fixed.

Although the gantry was not completed until March 1954, as soon as the construction had advanced sufficiently, a start was made in February 1953 at chainage 2,080 on the cofferdams for pipe laying. The same procedure was adopted as on the beach section, in regard to length of dams and the use of diaphragms.

The pumping plant was reinforced by a No. 9 Pulsometer steam pump, which was left in position during high tides.

Excavation became a relatively small item, and was in fact less than that shown on the longitudinal section, owing to scour round the leading end of the cofferdams.

In some cases hard filling was placed in the dam to build the formation up to the

underside of the concrete surround. The sand, however, soon made up again around the dam as it progressed ahead.

Concrete piles

The outer half of the outfall, which gradually rises out of the beach until invert is at beach level, is constructed on concrete piles. The piles have the function of acting as an anchorage against any uplift caused by wave action on the pipeline.

The contract had called for some type of cast-in-situ piles, with a bulb base. However, it was not found possible to interest any of the firms specializing in this type of pile, and it was decided to use precast piles.

The design specified two pairs of piles per pipe with a resistance to uplift of 20 tons per pile.

The piles, of 13 in. × 13 in. section, 25 ft long, were cast in the promenade yard, and after maturing were taken to the site using the 2-ft-gauge track, pitched in guides fixed to the walings, and driven by a No. 7 hammer (Fig. 8).

The ground was tested for resistance to uplift by driving a fifth pile in the middle of a group of four, and then jacking off a framework set on the four outer piles, with two 35-ton hydraulic jacks acting against a cross-head attached to the centre pile. The upward pressure was measured by gauges attached to the jacks.

This test was applied at intervals varying from 25 to 200 ft. If the test pile showed signs of lifting at a load less than 20 tons, additional piles were driven under each pipe to make up the deficiency so that there were either three or four pairs of piles per pipe. This increased number was maintained until a satisfactory test was obtained.

The first few test piles were jacked out, removed, and re-used, but so much trouble and loss of time was caused by water welling up through the hole left by the pile that all other piles, once the resistance had been measured, were left in; eighteen piles were tested, two resisted more than 30 tons, nine between 20 and 30 tons, and seven less than 20 tons.

It was observed that resistance to uplift developed after several days, and in one case increased from 10 to 30 tons, when the pile was retested 14 days after being driven, the first test having been done the day after driving.

Since it was not practical to wait this length of time, the above-mentioned precaution of driving additional piles was adopted.

After driving and testing, the pile heads were broken down and the reinforcement was spliced to bars passing over the pipe.

Concreting

The gantry was not wide enough to allow a mixing plant close to the point of placing as was possible on the beach section, and to make it so would not have been economical.

A permanent back-shunt off the standard-gauge track was made at approximately chainage 2,000 for the mixer and cement wagons. All mixing for the gantry section was done at this point.

Gauged aggregates were taken out to the mixer as described. The concrete mixer discharged into 1-cu. yd bottom-dump skips set on 2-ft-gauge flat bogies. These were conveyed by diesel locomotive to the point of placing where a crane off-loaded them and placed the concrete in position.

A rate of output of between 11 and 14 cu. yd/hour was maintained up to about half-way point on the gantry, using two locomotives, hauling 1 cu. yd per trip.

This output dropped to about 8 cu. yd/hour at the end, and to maintain that the locomotives took 2 cu. yd per trip.

Weather conditions and the state of the beach track were factors with an immediate effect on outputs, but a greater slow-down was caused by darkness. Artificial lighting was provided by portable electric generators and paraffin vapour lamps, but despite them, efficiency at night fell considerably below that obtained during the day. For instance, after dark about 5 minutes was added to a round trip of the concrete skip, which in daylight would have taken about 10 min.

On this part of the coast the neap tide low water occurs at midday and midnight; the more favourable low water of spring tides is in the morning and evening. Thus, in winter, the spring tides were worked largely in darkness whilst in summer with long daylight hours, conditions were more favourable. Owing to the care needed in moving about on the gantry the effect was more noticeable here than when the beach section was being worked. Accordingly in June 1953, the whole effort was concentrated on the gantry section by opening a further section at chainage 2,680 (Fig. 10). This continued until September when the outer gang, which had reached chainage 2,900, was brought back on to the beach. The inner gang, however, continued on the gantry and reached chainage 2,680 in November 1953, although the actual connexion was not made until February 1954 to delay the moving of a brickwork plug built into the pipe at chainage 2,680.

This gang then went forward to the outer part, and working throughout winter and summer, reached the end at chainage 4,160 in November 1954.

The gang which had returned to the beach section in September 1953 continued on this length throughout 1954, and made the connexion at chainage 2,080 on the 9th February 1955. This completed the pipework, and the sewer was put into service within a few days.

A modification in design was made for the final 300-ft-length of pipeline which is above beach level and liable to scour. In addition to the concrete bearing piles, the cofferdam piles were also left in place as a further protection should scour occur. The concrete surround to half way up the pipe was cast against the cofferdam piling and the surround for the top half was of circular section.

The cofferdams were modified so that they were only approximately 9 ft wide. On completion the piles were cut off at half-pipe level (Fig. 9).

The method of concreting was changed and one pour was made up to half-pipe level, and the following day circular shutters were set and the top surround placed; this was the only length where a horizontal construction joint was used.

Working time was affected because the steel piles, which had been used a number of times, were leaking badly through the interlocks. This meant that when pumping started at the beginning of the shift the water level inside was reduced only a little faster than the fall in the tide.

As the tide fell and the submerged area of the dam walls was reduced, the pumps were able to empty the dams at an increasing rate. A 6-in. Pulsometer steam pump was used, with a capacity of 40,000 g.p.h., and was left in position during high water; in addition an 8-in. centrifugal self-priming diesel pump with a capacity of 90,000 g.p.h. was conveyed each tide and connected up to a suction pipe left in position in the dam being worked.

On neap tides the sea did not leave the dams. The leakage was reduced by sealing the interlocked joints with ashes and sawdust, which proved effective for the forming of the bottom and driving of concrete piles. As soon as the concrete, against the piles, was vibrated the seal was broken and water poured in and made concreting

npossible. As a result, concreting was limited to those spring tides when the sea vel fell to near invert level.

It seems that the only alternative would have been to have used new piles, and ossibly to have concreted without vibration.

larker beacon

To comply with shipping regulations, it was necessary to provide a temporary ghted beacon at the outer end of the work. This was constructed of steel scaffold-ng, 15 ft square at the base, tapering to 5 ft square at the top, which was about 0 ft above beach level. Vertical 15-ft-long tubes were jetted into the beach and n these the tower was built. A platform was constructed at 36 ft above beach on hich an acetylene gas lantern was fixed, and the gas cylinder was renewed about very 3 months.

The beacon proved very stable and withstood many storms. It would have lasted e entire job had it not been hit by a heavy channel buoy which had broken loose om near Barrow-in-Furness during a storm. Although listing badly and with a umber of tubes bent the tower functioned for several months. Eventually another orm caused it to collapse. It was then replaced by a timber post attached to the antry, which by then had been completed.

On completion of the pipework a permanent beacon was erected at the outer end f the gantry. It was constructed by driving a steel-piled box, and excavating by rab under water to approximately —16·00 O.D. Concrete piles were then driven, nd finally the box was pumped out for bottoming up and concreting. This work ould be carried out only at spring tides.

The steel superstructure 36 ft high was assembled in the promenade yard in two alves, bottom and top, each being taken out on a flat wagon, placed by crane, and olted in position.

General notes

Demolition of the seaward gantry commenced on the 31st March, 1955, was com-leted on the 3rd September, and was followed by removal of the tracks, which ork is in progress at the time of writing.

lant

The major items of plant consisted of:—

- Six 5-ton Smith steam cranes
- One K.44 mobile crane
- Two steam locomotives
- Ten standard-gauge railway wagons
- Four No. 7 McKiernan-Terry hammers
- Two No. 80 Zenith extractors
- One No. 16 Spencer-Hopwood boiler
- Three 8-in. Sykes pumps
- One 6-in. Pulsometer pump
- Three 3½-ton 2-ft-gauge diesel locos
- Twenty 2-ft-gauge tipping wagons
- Six 2-ft-gauge flat wagons
- Two 21/14 concrete mixers
- One 40-cu. yd batching plant
- Various pumps, compressors, vibrators, etc.

Materials

The principal materials used were:—

Steel piles	650 tons
Timber	41,000 cu. ft
Rails	170 tons
Concrete	4,000 cu. yd
Coal	2,000 tons

The quantity of piling taken against the total area of cofferdams gives an average use of about 5, but many piles were used 10 or 12 times, others being damaged on first driving, used, and left in diaphragm or held static for long periods at certain points such as the hatch-box chamber and where junctions were to be made.

The 650 tons of piling were disposed as follows:—

355 tons left in as protection to pipeline and in beacon foundation
55 tons left in diaphragms
20 tons sold for re-use
220 tons sold as scrap

650

Three-quarters of the timber is expected to be recovered. The rails after 4 years' immersion have suffered little. The sleepers and connexions in the 2-ft-gauge track which had been treated by immersion in a bitumen solution remained in good condition, but about 25% which had not been so treated was badly corroded.

Labour

The maximum number of men engaged on the work was 85, most of whom were recruited locally. The work was arduous but the men worked well under difficult conditions. There was very little time lost from absenteeism or unfavourable weather. Even during bad weather such as occurred during 1954 the work went on except when gales made it impossible to work, or sometimes even to stand, on the gantry.

Out of consideration for local residents, working time was limited to the hours of 4 a.m. to 10 p.m. Sunday was very rarely worked except for servicing of plant.

For 4 days during spring tides there were two shifts and for the remainder, one shift, the time of starting and stopping varying each day. It was found that 4 days of double shifts was the practical limit of endurance, owing to broken time and irregular hours of sleep and meals. The hours worked per week varied from 28 hours in five tides during unfavourable neaps, to 60 hours in ten tides during certain springs, the overall average being about 43 hours per week. A small yard gang worked normal hours on casting piles and other preparatory work.

Programme

The operations were programmed a week ahead, but were sufficiently flexible to cater for varying conditions of weather and tide. The duties of each gang were posted up so that all the men knew what was expected of them day by day. This it was found, added greatly to their interest in the work and increased efficiency.

An important part of the supervision was the marshalling in the yard of the various items of plant so that they went out along the single track in the correct order for the work of each shift. The order was different daily according to the operation

CONSTRUCTION OF 60-IN.-DIA. OUTFALL SEWER FOR MORECAMBE AND HEYSHAM CORPORATION

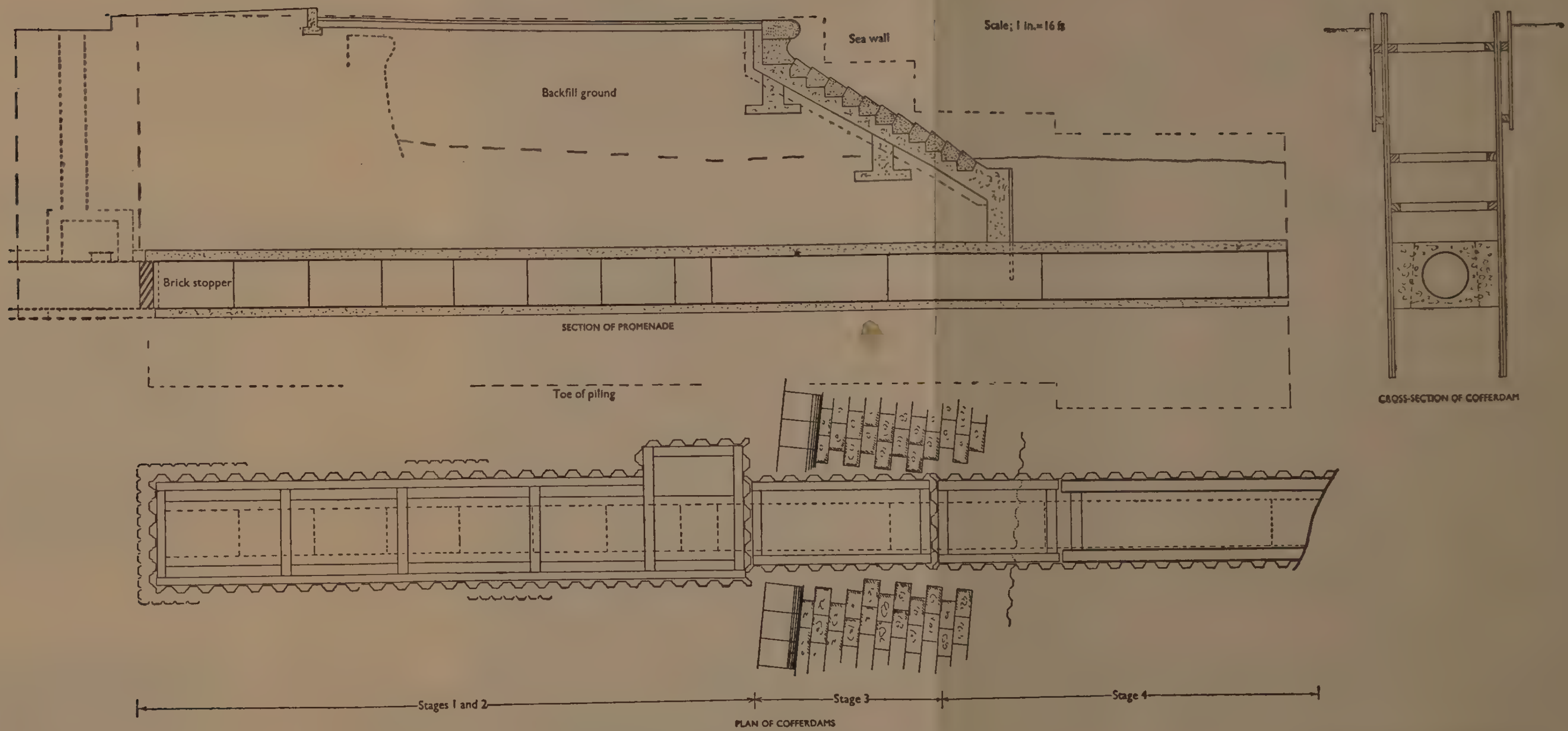

FIG. 2

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CONSTRUCTION OF 60-IN.-DIA. OUTFALL SEWER FOR MORECAMBE AND HEYSHAM CORPORATION

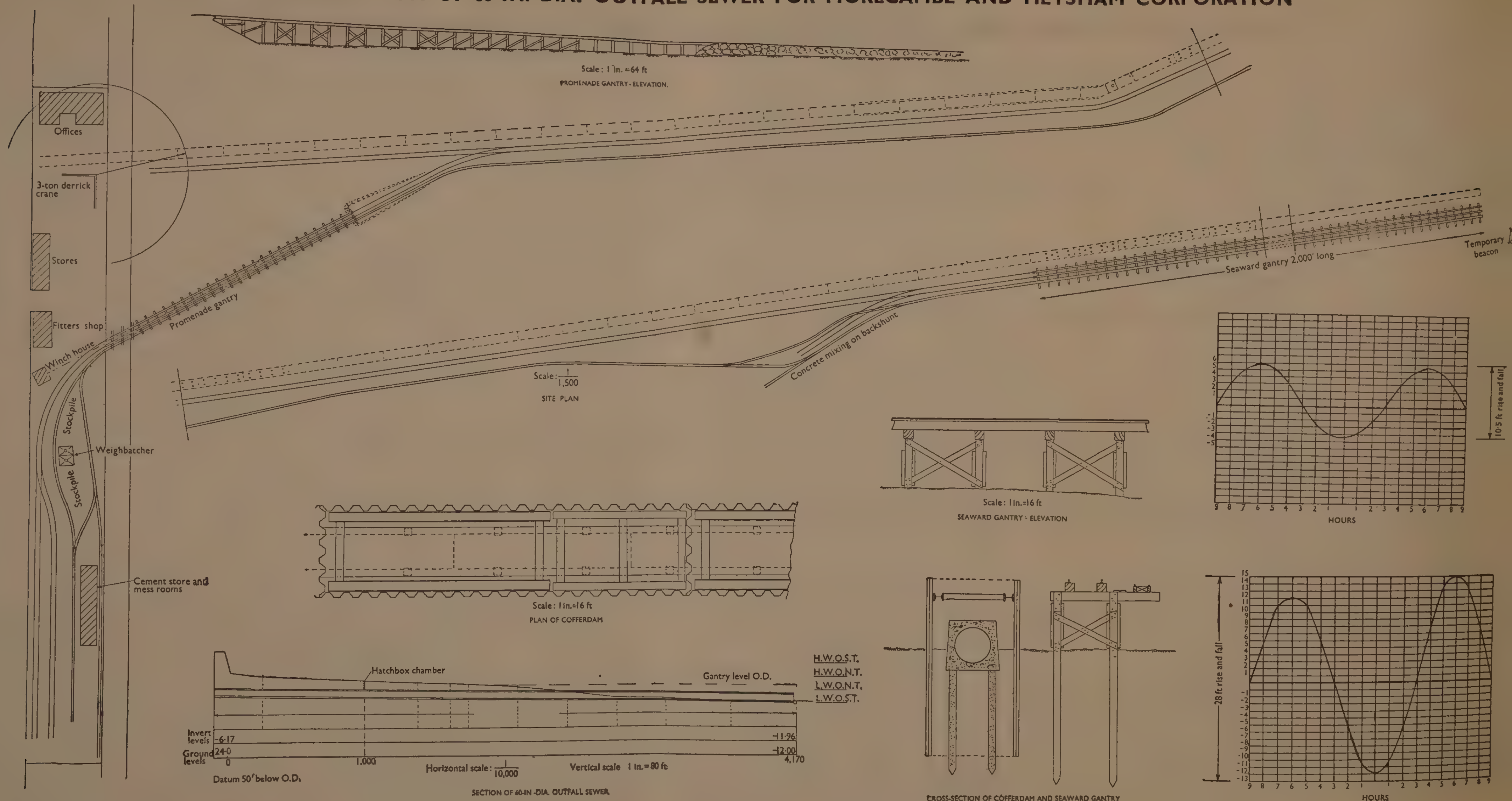

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FIG. 3

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progress, calling for either steam plant, compressors with their appropriate tools, umps, etc.

utputs

On the beach and gantry sections 164 pipes were laid in 129 working weeks. For ost of the time two units of two or three gangs each were employed. The pipe ying varied from two per unit week to one pipe in 4 weeks when conditions were bad.

The number of unit weeks was 224, giving an average of 0·73 pipes, or 18 ft 3 in. r week for each unit.

The total cost of the work cannot be closely assessed until after removal and sposal of surplus materials, but will probably be in the region of £375,000.

The pipes were supplied under a separate contract.

ACKNOWLEDGEMENTS

The Consulting Engineers responsible for the design and supervision of the work ere A. H. S. Waters and Partners, and the Authors would like to record their ppreciation of the readiness of the Engineers to consider possible amendments to e design to meet problems of construction, and of the helpful co-operation of the esident Engineer to the Corporation, Mr Andrew Nicol, M.I.C.E.

The Contractors for the outfall were Harbour and General Works Limited, acting der the control of the Managing Director, Mr W. R. Grigor Taylor, M.I.C.E.

The Authors would like particularly to acknowledge the work of the late Mr K. H. vans, M.I.C.E., who was a Director of Harbour and General Works Limited, and o did so much in drawing up the scheme of construction which was adopted.

The Employers were the Morecambe and Heysham Borough Corporation, the orough Engineer and Surveyor being Mr William Kilvington, A.M.I.C.E.

The Paper, which was received on the 18th October, 1955, is accompanied by nine otographs, and three sheets of drawings, from some of which the half-tone page ates, the Figure in the text and folding Plates 1 and 2, have been prepared.

Discussion

Mr W. Kilvington (Borough Engineer and Surveyor, Borough of Morecambe and ysham) said that on pp. 303–305 various methods of construction were outlined, all which had been considered. Method (e) had been put forward by other contractors ne years earlier, when Mr Kilvington knew less about such methods, but he had felt t it could not possibly succeed unless some evidence was put forward to the contrary. thought the method adopted was the only practicable one, bearing in mind the need economy. He had made trial holes on the shore, but he thought the Authors would ee that, though a valuable general indication, those which were nearest were frequently most dissimilar, so that it was impossible to attach the normal importance to them.

The mention of sand was specially interesting to them at Morecambe, apart from the stion of scour, because there was a tendency for a shortage of sand on their foreshore til about 10 years ago on that part of the beach and just to the east of it there had n huge accumulations of sand, but much of it had disappeared. He hoped that it now beginning to return; certainly sand was accreting, and he hoped it would conue to do so.

Finally, if the contractors contemplated a similar contract, would they, with the erience which they had now gained, adopt similar methods?

Mr W. Melville (Borough Engineer, Fleetwood) said that the Paper set out clearl considerations and problems of carrying out an important work on a foreshore where th distance from high- to low-water mark was almost a mile and where there was a tid range exceeding 30 ft at spring tides, particularly with a storm behind it.

His few comments were not particularly on the engineering side but rather on conditior generally in the Morecambe Bay area. On p. 308 the Authors had referred to subsoil cor ditions. Recently the Fleetwood Corporation had had reports on the ground condition relating to works of coast protection; evidence had been provided of frequent change and quite important variations in the ground conditions in the foreshore. On one pa of the beach was boulder clay at − 8·00 O.D., but that petered out and there wa nothing but wet alluvial silt down to − 40·00 O.D. farther north. It suggested a earlier delta formation of that part of Morecambe Bay and increased the hazards c underground works, whether on land or on the foreshore, in such tidal conditions.

On p. 309 the Authors had referred to the large volume of water which had come throug the bottom of the excavations in the gravel and silt, as of course was expected, and ha said that perforated steel subsidiary drains had been laid to deal with that inflow. Ha that water, as it was collected in the perforated drain, been pumped out from a sum in the cofferdam, and had any silting-up taken place in the subsidiary drain? Th subsidiary drain had been removed on completion of the work and was presumabl transferred to the next operation in the next cofferdam. That, of course, was the metho used for dewatering many years ago. Fifty years ago, in the main-drainage works in th town which Mr Melville served, there had been a very difficult operation to perform. I that case subsidiary drains had been placed at the bottom of the trench-work, had helpe to keep it dry while the permanent work was being constructed, and had then bee abandoned. Recently it had been necessary to make connexions to the original outfa sewer and the sub-drains had been a great embarrassment. They had been found to b under a considerable head of ground-water, which discharged very freely when th excavations were opened out.

Mr A. P. Lambert (of John Mowlem & Co. Ltd) said that work of the kind i question was not only difficult to do but difficult to estimate for, the reason being tha so much depended on the methods and scheme adopted. Having decided on the scheme which necessarily involved a great deal of temporary works, it was possible to calculat the cost of the temporary works with a fair degree of accuracy, so far as that was possibl in civil engineering; it was then necessary to weigh that capital expenditure against the fa less tangible expenses involved in on-costs, overheads, and plant charges. Those expense linked up with the estimated rate of progress. If the progress did not match the estimate the scheme which seemed to be right on the basis of the estimated progress might no prove so in fact. The Authors had given a great deal of thought to alternative method and he believed that they had arrived at the right basic method and had rightly con demned the idea of floating craft and the idea of the independent cofferdam leapfroggin out to sea; they had been right to base their scheme on using the beach to a certai point and then proceeding with a stage.

He was not certain, however, that they had been right in details. In that connexio he would put in a plea for scheme (c) at the bottom of p. 303, with the cofferdam at + 9·0 O.D. and the stage at + 9·00 O.D. He based his plea on the data given in the tide curve in Fig. 3, Plate 2. Tides were a mystery to him but that diagram was relatively simpl and seemed to show that with a cofferdam at O.D. there would be 6 hours out of 12 whe the piles were out of the water. That condition seemed to apply, oddly enough, to neap as well as to spring tides. Time had to be allowed for plant to run into position and t be withdrawn before the incoming tide, and the cofferdam had to be pumped out. H thought that only 4 hours out of 12 might remain to get on with the job. With a cofferdar at + 9·00 O.D., on the other hand, there appeared to be full protection at all stages of th neap tide and about 9 hours out of 12 with the spring tide. Deducting 2 hours as befor that gave 7 hours out of 12 on the spring tides and 12 out of 12 on the neaps—an averag

9½ hours. It seemed, therefore, that that scheme might have given at least twice the ective working time. If so, how would it have affected the period of the contract? , for instance, it would have shortened the contract by 9 months, how would that saving ve compared with the extra capital cost involved?

Another point not mentioned in the Paper which must have been given considerable ought was the advisability of providing separate access to the gantry section over the ach. Possibly the original programme had envisaged that the beach section would be ished before the gantry section began, so that the point would not arise. In reality at had not been achieved, and the Authors described the tremendous amount of work ich had gone on for about 18 months on the beach and the gantry.

He would like more information on the cofferdams. The job had really revolved ound the cofferdams. The piles were apparently 30 ft long. At the start, against the omenade, they had been driven to about 3 ft above beach level. The trench was given 18 ft, which meant that the piles there were driven to 9 ft below the trench bottom, ich seemed to be reasonable. What had been the Authors' limiting consideration in e depth of penetration below trench bottom? He understood that that depth was ft against the promenade. The beach fell considerably more rapidly than the invert the pipe, and the situation at 2,000 ft must roughly have been reversed if the piles re again driven to 3 ft above beach level as stated. The trench at that point was only ft deep, so that there would be 18 ft penetration below trench bottom, which seemed cessive. He imagined that pile-driving in that material was very troublesome, and in t the piles had suffered a good deal of damage. He wondered whether the Authors d driven those piles down to within 3 ft of the beach. If they had not done so, but d been satisfied with a penetration of 9 ft, they could have left most of the piles up, ile driving one or two down to ensure that the cofferdam did flood when it was designed flood. It was stated on p. 309 that considerable water came up through the bottom on the gravel and silt lengths, but that might not have come round the bottom of the es; it might have come through the interlocks.

With regard to the compound girder and timber frames, only one was shown in the otograph and diagram, but presumably two must have been necessary at the deepest rt of the trench. What was the maximum depth of trench at which the Authors felt ppy to use a single frame?

He sympathized with the Authors when they came across the large boulders. Mr lvington had referred to trial borings; did they disclose the area of large boulders? e only consolation was that it occurred on the beach; it would have been far more ublesome to come across such ground when out on the gantry.

The general arrangements for concreting had been extremely good and Mr Lambert l not think that the Authors could have done anything better, but they had been t with a run of 2,000 ft for which the mixed concrete had to be transported in skips. d there been any difficulty in discharging the skips at the end of that journey, or did Authors use any of the admixtures intended to prevent the segregation of concrete ich was apt to occur when it was taken for a long ride on railway track? He was a le amused to read on p. 309 that "The concrete mix was approximately 1 : 2 : 4 adjusted give maximum density and a low water/cement ratio" and then to read, two paragraphs ver down, that more water was put in to make it workable. That seemed an excellent angement on which it would be difficult to improve; the contractors got a concrete ich they could use, and the man in the office thought that he was getting the concrete ich he had specified.

Mr J. A. Lewis (Partner, Messrs Lewis and Duvivier, Consulting Engineers) said t, like Mr Lambert, he had wondered why a gantry at O.D. level had been chosen. m the tide levels given, the half-tide level was + 1·00 O.D., and therefore the gantry l been slightly below half-tide level. He had been engaged recently on estimating a posed outfall going considerably beyond low water, and after doing it on the basis of alf-tide gantry he had come to the conclusion that the time left for working, after getting

out on to the job, was so short that it would be essential to have the gantry level if not at high tide at least somewhere between half-tide and high-tide level.

On p. 307 there was a reference to the settlement of filling, and the Authors said that "sand and gravel filling was placed in layers, trimmed to slope, and blinded with 6-in. concrete. Following tides consolidated the filling." He could not understand how the consolidation had been done by the following tides after the concrete had been placed on top. He had had an unfortunate experience with the consolidation of gravel and sand filling behind a sea wall. After placing it in layers, running heavy plant about on it, and soaking it with water the conclusion had been reached that no further settlement could possibly take place, and a concrete decking had been cast on it. No further settlement had taken place for about 4 years, but then there had been a big storm and the ground must have been well shaken by the vibration of the waves; subsequently it was discovered that ½ to 1 in. of settlement had taken place. The consolidation of sand and gravel filling seemed extremely difficult; it did not roll well. It might be thought that if it was saturated with water maximum consolidation would be obtained, but it seemed that something could still happen if it was subsequently subjected to vibration. He wondered whether the only way to obtain complete compaction in a short time was to use vibrators on it as if it were concrete.

He was interested in the laying of the track in ballast. On the north coast of Kent, where the beaches were wide and flat below O.D., and the sea went out for a great distance, there were spits of shingle in the clay, presumed to be old river beds, which remained remarkably stable. It seemed, therefore, that where there was a silty foreshore, which must indicate that there was no great degree of disturbance, the laying of ballast would be quite satisfactory, and he had been surprised that it took so many men to maintain it. He wondered whether the Authors' reference to five or six men on maintenance meant that it was a continuous job or only periodical. In what way had it needed maintenance? Did the ballast tend to bunch up into ridges longitudinally or to be scoured out between the tracks and carried to one side?

The reference to the sand building up in 3 years was also of interest, but there was so little said about it that he wondered what relevant factors there were. Had the sand not been present at all before, or had there been some change in the vicinity which caused a fresh supply of sand to begin to move across the foreshore? That might be indicated by the experience with sand at the outer end of the pipeline. It was extremely interesting that severe scour had occurred when the cofferdam had been at a high level and that when it was cut down the sand began to fill in the scour.

On p. 308, with typical modesty, the Authors had dealt with the whole operation of driving the piles, laying the pipes on cradles, placing the concrete, and backfilling in a few lines. It could not have been so easy; hitches must have occurred, and it would be of interest to know more about the timing of the operation. Perhaps the Authors would say how long it took. He assumed that the pipes had been placed on the cradles and the concrete placed immediately afterwards on one tide, the backfilling done on the next, and so on.

On p. 309 the Authors referred to the concreting inside the cofferdams and said that "Portland cement was generally used but sometimes 417 cement when early flooding of the cofferdam could be expected." He thought that if the sea was allowed to cascade over the top of the cofferdam on to the pipe it would have damaged the concrete considerably, and he assumed that the water had been allowed to rise inside; but, if the water was allowed to rise inside, it was surprising that it had been necessary to use 417 cement. Possibly that was due to rough weather, when despite filling the trench with water there had been some wave action which made the Authors think it desirable to have the concrete set more rapidly. At the outer end, where they had been doing the concreting in two lifts, he could understand that it would be an advantage to use 417 cement to avoid damage.

Mr D. Lowe (Senior Assistant, Messrs D. Balfour & Sons, Consulting Engineers, London), who said that Mr Lambert had dealt admirably with several of the points he

.d intended to raise, observed that the principal feature of the work had been the iving of the piles, which had, in effect, determined the progress of the work in general. ke Mr Lambert, Mr Lowe had not been clear about the depth of the piles and the driving, .d he would like to know to what set the piles had been driven and if anything had en laid down in that respect. Trial bores had been mentioned, but had any test ling been carried out on the line of the outfall? That might—he emphasized "might"— .ve been helpful in determining whether to use Larssen No. 2 or No. 3 piles.

Turning to the concrete, what type of aggregate had been used? Had a whinstone en used, and of what size? Had cube tests been carried out, and if so, what strength .d been obtained? It appeared that only ordinary Portland cement and 417 had been ed. Naturally in tidal excavations there was bound to be a high proportion of lphates. Was there any reason for not using sulphate-resisting cement with, possibly, here quick setting had been required, a percentage of calcium chloride added?

For the last 300 ft of pipeline the method of concreting had been changed, one pour ing made up to half-pipe level and then the top surround being placed. Might it not .ve been better to adopt a different surround and to pour the whole concrete in shorter ctions, thereby avoiding the horizontal construction joint? He felt that in a few ars' time that might be rather a weak link. Was it essential that the last 300 ft should vibrated, considering the conditions, and if so did it prove of any real use?

Mention had already been made of the build-up of sand. It might be too early yet to y, but were horizontal sections taken at intervals of the beach and was it forming a od groyne, or was something happening on those lines from the point of view both of orecambe and possibly of the coast in the neighbourhood, since with littoral drift fects might be produced several miles from the actual groyne or pipeline?

Mr H. J. B. Harding (Director, John Mowlem & Co. Ltd) said that when working ith the sea then one's work must be contemplated as permanent or it would be ansitory. The sea was so powerful that Papers such as the present one were useful in owing what ought to be done to counter its effect and what could and could not be one with a certain chance of success.

One of the most interesting parts of the Paper was the Authors' discussion of the fferent methods considered; one of their first remarks was that they gave up the idea of ating plant because of the attendant complication of moorings. He thought they had en right; it was easy to float the plant, but the moorings could be a complicated oblem. Moorings could usefully serve as a subject of a separate Paper.

Dealing with the various other schemes considered, the Authors said that the sinking caissons did not appear feasible or justified. It was perfectly feasible because it had en done on a number of occasions, but certainly for a pipe of the size and length of at in question it would not have been justified. Another Institution publication[1] had scribed the outfall for cooling water at Pigeon House, Dublin, where twin 9-ft pipes d been sunk in compressed-air caissons and joined up under a wall. In that case the nction under the wall had been made by tunnelling in chemically consolidated sand d gravel. In the present case that had not been considered suitable because there ere layers of clay and silty material below the axis as well as sand and gravel, and a mplete seal could not be obtained. It would be inappropriate to spend time and oney and only partially complete it.

Another point was the size of the temporary works compared with the size of the pe. The same effort in the temporary works would have permitted pipes of considerably gger size to be laid. It was as well to remind consulting engineers and others that the ntractor often had to have considerable temporary works, out of proportion to the lue of the permanent work, and they should bear that in mind to see whether better use uld not be made of them. It might mean that it was better not to be cheeseparing in

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[1] H. J. B. Harding, "The Choice of Expedients in Civil Engineering Construction". ks Constn Div. Paper No. 6, Instn Civ. Engrs, 1946.

the design but to think of possible future developments. In saying that, he did not have in mind the case described by the Authors, but in some cases it would be better to look ahead by, for instance, laying a bigger pipe for future developments so as to make use of the expensive temporary works.

Licences for steel and timber had still been rampant at the start of the job, and the Paper would be an interesting reminder to their successors of the ridiculous way in which work had had to be carried out in those days. The Authors had started to design temporary works in steel because they could not get timber, and then half-way through they had to change to timber because they could not get steel.

The Authors' layout of plant had been very good, and they had used strong plant with 5-ton cranes, which added greatly to efficiency.

Mr Harding had thought that the Authors might have made the gantry at + 3·00 O.D. instead of at + 9·00 as Mr Lambert suggested. Two or three more feet would have added at least 2 hours to the working time. If they had gone to + 9·00 O.D. the gantries and piling would have been much heavier.

He was glad to see that the Authors had used powerful pumps. Work was often made costly by timidity in choice of pumps.

The Authors had not given much information about the rate of pile driving. Their choice of Larssen No. 3 piles ultimately instead of Larssen No. 2 had been very wise. If piles were to be used a number of times it was a mistake to economize on the original cost; it was necessary to have strong and healthy piles which could be used a number of times without damage. He thought that no civil engineering section should be considered complete unless it showed the strata outside the work as well as inside it. In Fig. 2, Plate 1, the Authors had given a cross-section of the cofferdam, but it would be useful to have a better picture of the ground the work was in and also to know how the penetration had been chosen to ensure against possible piping or blows in the bottom. The Authors did not seem to have suffered from that complaint, but did they have any which had not been mentioned in the Paper?

Mr S. R. H. Beard asked for more information about the diaphragm on the end of the pipe, and whether it would not have been more advantageous to put a valve at the end. Without a valve, every time a length of pipe had to be added all the water had to be emptied from the pipe. A valve would to some extent obviate that.

What comparison had been made of oil fuel costs against coal? The cost of fuel on the job must have been enormous, but apparently only coal was employed. No doubt the contractors had considered the question of using oil instead of coal. Mr Beard knew that it was perfectly feasible to alter a coal-burning crane to burn oil because recently in Trinidad contractors for the deep-water wharfage had altered all their coal-burning cranes to burn oil, with considerable economy.

Apparently the piles had been driven from leaders suspended from the ends of cranes. Had any difficulty been experienced in driving piles in that way? How many times could the steel piles be used again when extracted, and had there been any trouble in extracting them?

If cross-bracing was used between the two ends of the cofferdam in such a way as to carry the top stress down to the ground, could not lighter walings have been used?

Mr R. V. Hughes (Senior Engineering Assistant (Electrification), British Railways, Southern Region), commenting on the Authors' reference to the subsoil on p. 308, said that in his youth at Morecambe he had devoted much time to trying to trace the lines of the drumlins in the area, which appeared to have continued into the Bay at some far-distant date; he wondered to what extent they could be traced by the piles of stones to be found in various parts of the Bay, which were known locally as "skears." Had the Authors found in their excavations that there was any correlation between the boulders on the surface and the beds of harder material down below; in particular, near the outfall of their sewer had they encountered the continuation of the skear which existed to the south-west of their pipeline? It had seemed to him that the general run of the drumlins was approximately north to south.

Mr C. D. C. Braine (Partner, G. B. Kershaw & Kaufman, Consulting Engineers) said he wished to ask how had the prices been fixed? His feeling was that for the kind of work in question the target type of contract was by far the best, and it would be interesting to know whether the Authors agreed.

In the course of designing their falsework the Authors must have considered the maximum size of wave likely to be met. No mention had been made of that factor but it was very germane to the whole subject of falsework. For what did they have to cater? It was obvious that they had catered for it very successfully, though one of their photographs showed that they had not been successful all the time. He was interested in the height of waves and the load which such waves brought on work of the nature described in the Paper, and particularly in the tremendous forces which had to be resisted when only the bottom portion of an outfall was buried in the sea bed.

Mr M. W. Leonard (Soil Mechanics Ltd) said that there were three lines on p. 311 upon which he would like to comment, where the Authors had stated "The contract had called for some type of cast-in-situ piles, with a bulb base. However, it was not found possible to interest any of the firms specializing in this type of pile, and it was decided to use precast piles."

Mr Leonard felt that the decision to use driven precast concrete piles was sound for the conditions under which cast-in-situ piles would have been constructed on a gantry, under tidal conditions, and subjected to stormy weather, all of which would have resulted in a lack of continuity of work for the cast-in-situ piling specialists.

Either type of cast-in-situ pile—driven or bored—would require a temporary steel casing tube to be pitched and sunk to the required depth from off the gantry; once the concreting of the shaft of the pile had been started it would have to be completed in one operation, otherwise the pile would not be properly formed and the temporary casing tube would be trapped in the concrete before it was withdrawn.

Under such circumstances the concreting operations were not likely to be commenced until optimum tide and weather conditions prevailed, all of which would undoubtedly have hindered the progress of the contract.

The alternative of using a precast concrete pile made up on the shore, taken out to the piling positions, pitched, and driven there was, he felt sure, the best answer to that kind of piling problem and the sequence of operations could be stopped at any particular point without affecting the final result; for example, the pile could be partially driven and should a storm arise, suspending operations, the pile could afterwards be driven to its final set and penetration quite satisfactorily.

Mr Leonard also wished to comment on the specification which referred to the employment of cast-in-situ piles with a bulb base. He considered that it would be very difficult to form an enlargement at the base of such a pile in the soils which existed below the Morecambe beach, since they did not lend themselves to such a deformation by means of applying pressure to the concrete using a heavy punner or application of compressed air, as was the usual method for forming such bulb bases. If there was any enlargement at all it would be very slight, and he did not consider that it could be reliably constructed to enable the bearing capacity of the pile to be increased. It was essential to design on the minimum diameter of pile which could be positively formed.

Mr G. B. O'Rorke (Senior Engineering Assistant, Wilton & Bell) asked whether any useful information could be gathered from the set to which the piles had been driven and whether the pull recorded bore any relation to the minimum set and possibly to the type of soil encountered.

*** **Mr N. K. Rose** (Chief Engineer, Planning Department, George Wimpey and Co. Ltd) asked for information regarding the design, methods, and time taken for the

*** This contribution was submitted in writing after the closure of the oral discussion.—SEC.

construction of 24-in. outfall which had been replaced by the new 60-in. outfall. Presumably similarly difficult conditions had been encountered and overcome, although the choice of plant would have been rather restricted.

Mr Brooks, in reply, said that Mr Kilvington's question as to whether or not they would adopt similar methods on another occasion was to some extent linked with Mr Lambert's suggestion for having the gantry at + 9·00 O.D., and the suggestion made later by Mr Harding of + 3·00 O.D. Mr Brook's reply was that they would use similar methods again. He agreed with what Mr Lambert had said about working time, but the effect would not be as great as might be thought, in that over the middle part of the tide the rise and fall were very fast in tideways where there was a 30-ft range. Possibly Fig. 3, Plate 2, did not indicate that clearly enough. Had the cofferdams been even a few feet above the level that they were driven it was very doubtful whether any time which was worth while would have been gained, and certainly it would not have justified the additional expenditure involved in constructing a gantry three or four times the size of that actually built. It would have meant carrying a gantry from the sea wall to the full 4,000 ft plus.

It was good practice where possible to work with the sea rather than against it and another point was that below zero O.D. the wave action was very much less than when the sea was above O.D. It was during the top half of the tide that the damaging waves occurred. For the lower half of the tides the site was protected to some extent by sandbanks which extended far out into the bay, and vulnerable temporary works were kept within that protection.

That might help to answer the questions about the length of the piles and the number of frames used. Only one frame had been used; there had been no lower frame. On the beach section the piles had been driven to within 3 ft of beach level; there had been a good toe below formation and the one heavy frame at the top had proved sufficient. Upon going farther out to the gantry section the piles had been driven to O.D. They had as a matter of fact been reduced in length to 25 ft, but there the problem had not been so much one of supporting the weight outside as of holding them steady against wave action. Lighter walings, strutted and tied, had been adopted out on the gantry section, and the additional toe hold, had been an advantage.

The question of water/cement ratio was always interesting. They had been asked to use a water/cement ratio of about 0·52 and to increase it to 0·57 to allow the concrete to flow under the pipes. There had been trouble in getting the concrete out of the skips, and generally after a long run it had had to be vibrated out.

There had been some silting in the subsidiary drains, but in view of the fact that the drains had been taken out every pipe-length and cleaned out it had not been a serious matter; they had kept clear long enough for the purpose for which they were intended.

Mr Brooks did not think that the delays and troubles caused by lack of separate access had been sufficient to justify the cost of running a second line. Back shunts had been put in off the line so that plant could pass, and that meant a slight delay when a machine had to be taken off an operation and put in the back shunt to allow another to pass, but, owing to careful planning, that had not occurred very often. With regard to trying to seal the joints of the cofferdam, they had tried caulking the joints but had not had much success with it. Possibly some of the trouble was the result of pressure on the inside from a falling tide when the dam had not been pumped out.

Mr Lewis had asked about the settlement of filling when going through the sea wall. Mr Brooks could only say that the method adopted had been one which the same firm had used when constructing the Morecambe sea wall some years before and which had proved successful. The filling had been placed and blinded on the sloping face, but the top surface had been left clear to be flooded on a high tide, and good consolidation had been obtained. In fact the blinding would settle, and it was not until after it had settled that the blocks with their underlying concrete had been placed.

The question of scour and of the comparison between what might be called a high groyne

and a low groyne was interesting. The major scour on the south side of the cofferdam occurred as a result of the increased velocity round the end of the dam. As the dam proceeded forward the beach had made up, even before the dam had been removed. Under the gantry there had been considerable scour, again owing to the presence of the piles and the increased velocity of the water running under the gantry, and that did not fill up until the gantry had been removed.

The possibility of flooding the cofferdams and ruining the concrete had been taken care of by having a low pile in the seaward diaphragm. The water flooded through that into the 10-ft space used as a pump sump, so that the rise round the concrete was relatively gentle.

No test piles had been driven. A fair knowledge of the ground had been available from past experience, and the piling used had been dictated by judgement. There had been no foreknowledge of the very hard layer 500–600 ft out.

The Authors had looked into the caisson methods used at Dublin and in the Rotterdam tunnel. Mr Brooks had never done a job in that way, but he felt that it would be difficult with a foreshore which dried out as did that at Morecambe.

Mr Beard had asked about the diaphragm at the end of the pipe. There was a valve, and Mr Brooks was sorry that it had not been mentioned in the Paper.

Mr Hughes had asked if any relation had been found between the "skeers" and the hard layer which had been found underneath. Mr Brooks did not think that there was.

Mr Braine wanted to know whether the Authors thought that a target contract was a good idea for work of the kind in question. Mr Brooks believed that it was; it was a very sound way of doing a job which was so very difficult to estimate, particularly under the conditions prevailing at the time when the work at Morecambe had started.

The Authors fully agreed with Mr Leonard's remarks on in-situ piles. It had been their opinion from the start that they should use a precast pile, but the other type had been called for and they had had to ask the firms concerned to consider the matter. Most of them turned it down. One firm had offered to hire plant, but Mr Brooks did not know how they could have operated it.

Mr O'Rorke asked about the relations between the set of the piles and the resistance to extraction. There had been a relation, and it was as a result of that relation that it had been possible at times to go about 200 ft without a test pile and then, when it was found that the set for a certain depth was changing or lessening, to test. Mr Brooks could not give exact figures for the set and so on, but the resistance to extraction had varied with the set obtained when driving.

In reply to Mr Rose, the old outfall had been constructed about 60 years previously, when various other sewage works had been in progress. Apart from the fact that some contractors had got into financial difficulties, very little useful information was available.

Mr Brown, who also replied, dealt with the question of gantry construction at + 3·00 or + 9·00 O.D. and said that one of the factors which had had to be considered at the design stage had been the quantity of materials involved in the construction. It had been a period when both steel and timber had been difficult to obtain, and it had been necessary to take that factor into consideration in addition to the practical one of the amount of time likely to be available for working. In actual practice they had found that if the track and the gantry had been at a higher level they might have gained an advantage at neap tides by getting a little longer working period on the cofferdams, but during spring tides that would not be possible. There was a limit to the amount of time that men could work, and they had found, working on double shifts, that about three double shifts in any week was about the maximum that the men could stand. In many cases the men had been leaving home at 3 a.m. to start at 4 a.m. and finishing about 9 a.m., being away for six hours and finishing on the second shift between 9 and 10.30 p.m. If they had had to turn out day after day at that rate they would not have been able to stand the pace. It followed, therefore, that any advantage which might be secured on paper by working from a higher level and having a longer period of work would have been nullified by that factor of the time that men could actually spend on the job.

Mr Lewis had asked for more information on pipe-laying and concreting. The procedure was that having, on a previous tide, bottomed-up the cofferdam approximately to formation level, they had had sufficient time to finish the bottoming-up and the preparation of the drains to take the water, lay the pipe, and still allow 2½ hours for concreting. The laying of the pipe had not taken very long. The pipe had usually been taken the day before to the site and laid in between the piles of the preceding dam, which had not then been extracted, and had therefore been protected from tidal action. For the laying of the pipe and the fixing of the shutters they had allowed themselves 1 to 1½ hour, and 2½ hours for concreting. That gave them sufficient time to stop pumping and allow the water to rise, so preventing damage due to the tide pouring over the dam on to the concrete. They had not always been successful; sometimes they had had delays and in some cases the concrete had been disturbed and the following day it had been necessary, to repair it.

So far as the sheet piles were concerned, no set had been specified. In ordering piling in view of the difficulty about licences they had had to anticipate what would be required, and from their knowledge of the ground, and bearing in mind that the maximum area had to be obtained from the limited tonnage allotted, they had thought that Larssen section No. 2 would be sufficient. The length of pile had been chosen to suit the dams at the beach end of the work. Naturally as the work had proceeded and those piles were being used continually it had been necessary to trim them and they had been constantly shortened. There had been no unusual difficulty in extracting them, and they had been able to keep up with their pile driving. So far as progress of sheet-pile driving was concerned, they had driven the 50-ft cofferdam in roughly 2 weeks, and they had decided that that would have to be the basis of their programme in order to achieve their target of a pipe a week. They had not quite attained that, but they had come very close to it. Sheet-pile driving had certainly been a main factor in the progress of the work, and they felt that they had been successful in achieving a good output.

To the question of why sulphate-resisting cement had not been used the Authors could only reply that that had been a matter for the consulting engineers.

The type of aggregate used was crushed pit gravel, 1½ in. down, and washed sand. Test cubes were taken and generally broke at about 4,000 lb/sq. in. in 28 days.

The matter of converting coal-burning plant to oil had been investigated, and although showing certain advantages, it had not been proceeded with owing to delays and the time factor in changing over.

Maintenance of track was a continuous operation, the main trouble being scour under the 2-ft-gauge track, which had not been as heavily ballasted as the standard-gauge track.

Correspondence on this Paper is now closed.—Sec.

HYDRAULICS ENGINEERING DIVISION MEETING

7 February, 1956

Mr W. P. Shepherd-Barron, Past-President, Chairman of the Division, in the Chair

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Author.

Hydraulics Paper No. 9

SOME HYDRAULIC INVESTIGATIONS IN CONNEXION WITH THE WADI THARTHAR PROJECT, IRAQ

by

*** Anthony Rylands Thomas, O.B.E., B.Sc.(Eng.), M.I.C.E.**

SYNOPSIS

To protect Baghdad and the country to the south from high floods in the Tigris river, a barrage is being constructed at Samarra to divert a part of the flood discharge of the river into the Wadi Tharthar depression. Provision is also being made for future power generation and for an irrigation canal.

The Paper describes investigations into flood probabilities and river hydraulics before and after the construction of the barrage, also the layout of headworks and river-training measures proposed upstream, with particular reference to the safety of the works, operation for flood control, and exclusion of bed material from the turbine intakes and irrigation canal.

Experiments were carried out with a large-scale model of the river and headworks. Some of the results obtained are described.

INTRODUCTION

FROM the earliest times known to man the plains of the Tigris and Euphrates rivers, comprising the most fertile part of Iraq, have been subject to devastating floods. Both rivers have their sources in Turkey, but the Euphrates is longer and, because its active catchment is far upstream, it rises slowly in Iraq and remains at flood level for long periods. Its floods can also be predicted with fair accuracy many days in advance. This is not the case with the Tigris, which draws much of its flow from tributaries with catchments in the hills of Northern Iraq (see Fig. 1, Plate 1). The Tigris rises in flood rapidly and, although an efficient warning system is operated by the Irrigation Department, floods cannot be predicted more than 2 or 3 days prior to their arrival at Baghdad (see Table 3).

Some catastrophic and several high Tigris floods have been mentioned by Richards.[1] To these may be added the flood of February–April 1954, which was close to the

* The Author is a Consultant.

[1] The references are given on p. 351.

maximum on record (1941) in rate of discharge and was much more prolonged. The major part of Baghdad, though protected by a flood bank, was in grave danger for several days and was saved only by the calculated breaching of river banks upstream after extensive areas had been evacuated.

From time to time proposals have been made for the protection of Baghdad and the cultivated plains stretching south towards the Persian Gulf from the menace of Tigris floods, but until recently finance has been a stumbling block. Now, however, the Iraqi Government is allocating a large part of its oil revenues to the construction of a co-ordinated series of works intended to reduce floods and also to provide storage for irrigation and power generation.

Among these works is the Wadi Tharthar scheme, which appears to have been first considered for flood control by Sir William Willcocks about 40 years ago, though not investigated by him. The proposal comprised the construction of a barrage across the Tigris and a dike or earth bank about 60 km† long, forming a large canal or spillway, to divert part of the river flow during floods into the Wadi Tharthar depression—a large natural depression where water could accumulate and be reduced by evaporation and seepage (see Fig. 1, Plate 1). The principle of the scheme is of great interest and almost unique, the only other existing scheme of this kind known to the Author being that at Habbaniya on the Euphrates, to the south-west of Samarra, which has recently come into operation, though the same principle may have been used in works constructed on the Tigris and the Euphrates in ancient times.[1]

Since it was first suggested the Wadi Tharthar scheme has received consideration on various occasions and in 1949 the Irrigation Development Commission, under the Presidency of Mr F. F. Haigh, in a valuable report[2] collating hydrological data and making recommendations in regard to many projects, put forward concrete proposals for a flood control barrage at Samarra and a canal offtake for irrigation. Later, Messrs Coode and Partners, London, Consulting Engineers to the Iraqi Government, who had earlier reported on Tigris control, prepared designs for a barrage to be located at one of the Development Commission's alternative sites, 4 km upstream of Samarra town (see Fig. 3, Plate 1). Construction of the dike was started and a contract was let for construction of the barrage.

Early in 1953, changes of policy required that the site of the proposed barrage and canal offtake should be altered to the present site near Samarra town (see Fig. 3, Plate 1), where it could be combined with a road bridge over the river; it was also decided to make provision for the subsequent construction of a hydro-electric power station. The scheme was thus to provide fourfold benefit—flood control, a road bridge, an irrigation supply, and power generation, but flood control remained the primary object.

The headworks (see Fig. 3, Plate 1) were to comprise:—

(1) A barrage to raise the river level for occasional diversion of flood water into the spillway, for continuous generation of power and for supply of water to the proposed Ishaqi Canal.

(2) An escape regulator with gates which would normally be closed to maintain the high river level but would be opened as required to admit flood water into the spillway.

(3) Earth banks to retain the pond, or water impounded by the barrage.

(4) Guide banks and other protective works to safeguard the structures.

† A conversion Table of metric units is given on p. 351.

It is to be noted that no storage capacity was required at Samarra other than daily balancing storage for power generation.

The project was to be taken in two stages. In the first the barrage, regulator, spillway, and retaining banks were to be constructed to enable the works to operate for flood control; in the second stage, planned for several years later, the Ishaqi Canal and power station would come into operation.

At this juncture the Iraqi Government invited the Author to study, with the aid of models, the general behaviour of the river and the operation of the control to be provided by the barrage and escape regulator. The Author was asked to make recommendations in regard to hydraulic aspects of the headworks and in particular to determine a suitable layout of protective banks and the best position for the Ishaqi Canal offtake, also to provide designs, based on model tests, for the downstream protection of the barrage and regulator and to report on certain other matters.

The Paper presents results of investigations into some of the above matters, namely, flood discharge probabilities and river hydraulics, operation of the headworks, location of the canal offtake, and layout of the guide banks. A large-scale model was used to assist in the study of the problems and to test alternative layouts.

Flood discharges

In the study of the probable range of discharge conditions account had to be taken of the programme of construction of storage dams in the catchment. The first of these was the Dokan Dam, for which Messrs Binnie, Deacon and Gourley are Consultants. This is located on the Lesser Zab upstream of Altun Köprü (Fig. 1, Plate 1) and is designed to impound $6{\cdot}8 \times 10^9$ cu. m of water, a proportion of which will be available for flood control. It is expected to be completed by 1957. So far as the Author is aware no final decision has been taken in regard to the second upstream dam, but he was instructed to proceed on the assumption that a dam at Eski Mosul, on the Tigris upstream of Mosul, would be constructed and in commission before the power station at Samarra would come into operation.

The conditions to be considered were thus:—

(1) Existing conditions, with no upstream dams and no ponding at Samarra for power generation. This is termed phase 1.

(2) As above but modified by the construction of Dokan and Eski Mosul Dams.

(3) Conditions with Dokan and Eski Mosul Dams in commission and the river continuously ponded at Samarra for power generation and supply of Ishaqi Canal. This is termed phase 2.

The catchments of the river and its main tributaries are shown in Fig. 1, Plate 1, and their areas are given in Table 1. It will be seen that the river at Samarra draws supplies mostly from the Tigris above Mosul, the Greater Zab and the Lesser Zab, two of which are to be controlled by dams and the third, the Greater Zab, representing 23% of the total area, is to remain uncontrolled, at least in the earlier years of development. Flood levels at Baghdad and to the south are affected also by the Adhaim, a river rarely in high flood, which joins the Tigris below Samarra, and the Diyala which joins just downstream of Baghdad. Floods in the Diyala generally occur simultaneously with floods in the Tigris and thereby raise flood levels at Baghdad, but a project is being considered for the control of the Diyala by a storage dam some distance upstream of the confluence.

TABLE 1.—CATCHMENT OF THE TIGRIS AND TRIBUTARIES

(Approximate areas in sq. km)*

River	Mountainous	Foothills	Total
Above Samarra			
Tigris: above Mosul	23,435	31,469	54,898
Khazir: above Manquba	860	2,355	3,215
Greater Zab: above Eski Killik	18,554	1,909	20,463
,, ,, ,, confluence with Tigris	19,470	7,003	26,473
Lesser Zab: above Dokan	10,940	750	11,690
,, ,, ,, Altun Köprü . . .	11,670	3,952	15,622
,, ,, ,, confluence with Tigris .	11,670	10,580	22,250
Lesser Zab: below Dokan	730	9,830	10,560
Tigris: above Samarra	54,575	55,920	110,495†
,, ,, ,, (%)	49	51	100
,, ,, ,, excluding area above Eski Mosul and Dokan Dam sites . .	20,200	23,707	43,907†
Tigris: above Samarra excluding area above Eski Mosul and Dokan Dam sites (%)	46	54	100
Below Samarra			
Adhaim: above confluence with Tigris . .	—	9,815	9,815†
Diyala: above confluence with Tigris . .	19,810	9,868	29,678†

* Areas from Irrigation Development Commission Report, 1949.
† Omitting relatively small areas of plains.

Much of the Tigris catchment is at high altitude but about half the area above Samarra is classed as foothills. There is also a proportionately very small area of desert and plains, which can be ignored in considerations of flood run-off. The mountains and foothills carry vegetation in varying degree but a large proportion of the area could be described as thinly covered or barren.

Precipitation in the catchment is confined mainly to the season from October to May, the maximum occurring in December and January and consisting of snow in the high mountains and rain in the foothills. Fig. 2, Plate 1, shows typical isohyets.

The flood season generally extends from December to May. Rainfall in the foothills during December and January results in sharp rises in the river but the floods are of short duration. In February and March rainfall may be accompanied by the melting of snow and during April and May, though rainfall tends to decrease, the river discharge is swelled by continuously melting snow. From June the river gradually falls, reaching a minimum between October and the first rise of the new flood season.

The range of discharge* is considerable: whereas the maximum discharge at Samarra (see Table 2) during the past 30 (and probably 48) years was estimated to be 12,800 cumecs (450,000 cusecs) in 1941 and the mean annual maximum is about 6,000 cumecs (210,000 cusecs), the mean annual minimum is of the order of 300 cumecs (10,000 cusecs).

The Tigris is subject to rapid rise and fall of floods though to some extent moderated by the tendency for the peaks arriving from the main river, above Mosul, to

* In this Paper "discharge" is used to mean rate, not volume, of discharge.

TABLE 2

ANNUAL MAXIMUM GAUGE READINGS AND ESTIMATED DISCHARGES OF THE TIGRIS AT SAMARRA

Year	Date	Gauge (G.T.S.): m	Estimated discharge:	
			cumecs	cusecs
1930	February 17	57·80	1,600	56,000
1931	April 15	60·56	5,600	198,000
1932	February 26	59·10	3,000	106,000
1933	May 1	59·44	3,780	134,000
1934	April 9	59·12	3,600	127,000
1935	February 18	61·40	8,200	290,000
1936	May 17	60·58	5,900	208,000
1937	April 14	61·88	9,000	318,000
1938	May 3	60·46	5,500	194,000
1939	April 15	60·30	5,300	187,000
1940	April 21	61·08	7,800	275,000
1941	February 12	62·80	12,800	452,000
1942	March 25	61·58	8,000	282,000
1943	April 10	60·10	5,040	178,000
1944	May 9	60·60	5,880	208,000
1945	January 22	60·06	4,560	161,000
1946	March 16	61·50	7,650	270,000
1947	March 17	58·90	2,700	95,000
1948	May 3	61·30	7,200	254,000
1949	April 2	60·92	6,000	212,000
1950	March 10	61·90	8,000	282,000
1951	April 30	58·88	2,600	92,000
1952	February 8	61·94	7,600	268,000
1953	March 4	61·84	7,400	261,000
1954	March 26	63·12	11,200	396,000

llow the peaks from the two main tributaries above Samarra, the Greater Zab and e Lesser Zab. The approximate times of transit of flood crests are given in ıble 3. In the 1941 flood the crest of the flood wave from the Greater Zab reached ıarqat at about midnight on the 10th February while the first crest from Mosul ached the same point about 24 hours later and the second, higher, crest not until days after this.

The mean rate of rise at Samarra in the 1941 flood was approximately as given Table 4.

To estimate future flood risk two approaches were available; an analysis of ailable data in the catchments concerned, on a frequency basis, and the use of a neral formula derived from data geographically widely spread. The former method s the advantage of making allowance for all local characteristics of the catchments d prevailing meteorological conditions, which are implicit in the data, but its ıitation generally lies in the short period during which observations have been ıde and hence the relatively small "sample" of meteorological conditions en- ıntered. For example, the period for which observations have been made may ve been one of abnormally high or low precipitation and it may or may not have luded any rare combination of weather conditions which produce catastrophic ods.

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TABLE 3.—APPROXIMATE TIMES OF TRANSIT OF FLOOD PEAKS

Gauge	Hours from Samarra	Hours from Baghdad
Tigris		
Paish Khabur	70	90
Mosul	46	66
Sharqat	28	48
Baiji	16	36
Samarra	0	20
Greater Zab		
Eski Killik	44	64
Khazir		
Manquba	45	65
Lesser Zab		
Dokan	49	69
Altun Köprü	37	57
Adhaim		
Injana	—	24
Diyala		
Diyala Weir	—	24 from confluence

TABLE 4.—MEAN RATE OF RISE OF FLOOD AT SAMARRA (1941)

Range: cumecs	Rise: cumecs/hour	Range: cusecs	Rise: cusecs/hour
3,000–5,000	140	100,000–180,000	5,000
5,000–12,000	290	180,000–400,000	10,000

On the other hand a general formula derived from data collected in catchments in many parts of the world ensures the inclusion of a wide range of meteorologica conditions. Its use enables the designer to calculate the maximum flood whic would have occurred in a catchment of equal area under the worst conditions fo which discharges have been recorded, but the probability of such conditions occurrin in the catchment under consideration remains unknown and may in fact be quit negligible.

The choice between the two methods must depend on the circumstances of th case. For the Tigris it was considered that the statistical method, based on pas observations, would provide useful results but that it was desirable to examine the in the light of a very general formula. The existing flood data relating to th Tigris at Mosul and the tributaries Khazir, the Greater Zab and the Lesser Zab, an Adhaim were studied. Flood frequencies for these rivers were computed by a simplifie

Hazen method and are shown graphically in Figs 4 and 5. The curves were extrapolated to indicate the probable frequency of high floods.* [3]

For the second method, a formula was used which is derived from substantial and broadly based data. This is

$$Q = CA^{\frac{1}{2}}$$

where Q is the maximum probable discharge, C a coefficient, and A the catchment area. This form has been used by Myers, Jarvis,[4] and (effectively for large catchments) by Inglis and de Souza.[5] In ft-sec units (cusecs and sq. miles) the Jarvis value of C was 10,000, which was high enough to cover almost all available data including some relating to unusual meteorological conditions. The Inglis coefficient was 7,000 which, when applied to data of India and North America, appeared to cover maximum observed floods in all but a few abnormal cases.

TABLE 5

River	Catchment area: sq. miles	Number of years of record	Maximum flood recorded:		0·2% flood by Figs 4 and 5		C corresponding to 0·2% flood
			cumecs	cusecs	cumecs	cusecs	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Tigris at Mosul	21,300	36	6,200	218,000	9,500	330,000	2,300
Khazir at Manquba	1,240	13	5,000†	176,000†	4,700	170,000	4,710
Greater Zab at Eski Killik	7,900	30	8,500	300,000	11,400	400,000	4,530
Lesser Zab at Altun Köprü	6,040	30	3,700	130,000	5,900	210,000	2,680
Tigris at Samarra	42,700	25	12,800	450,000	19,000	670,000	3,240
Adhaim at Injana	3,720	21	2,900	102,000	3,800	130,000	2,200

† Rough estimate only.

In Table 5 (columns (6) and (7)) may be seen the results of extrapolation of the statistical data to determine flood discharges of 0·2% (i.e., 1 in 500) annual probability. In the same Table (column (8)) are given values of C obtained by substituting the discharges of column (7) in the general formula. It will be observed that they vary among themselves and are all appreciably less than the Inglis value of 7,000.

Some variation was to be expected because the catchments differ in shape and run-off characteristics as well as in respect of incidence of storms and melting snow. But the coefficients in column (8) indicate that, taking the Tigris catchment as a whole, the run-off has not during the period of record indicated a significant risk of

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* Since there was some latitude in the graphical fitting of the curves to the data, Gumbel's method of calculation was used in arriving at the extrapolation.

It should be mentioned that in most cases the annual maximum discharges were derived from gauge: discharge curves based on previous discharge gauging and extrapolated for the highest discharges. In the case of the Khazir (Fig. 4) the highest discharge (1941) was much in excess of the next highest. Since there was some doubt about this figure and in any case the flood was a very exceptional one, the extrapolation of the curve in Fig. 4 was computed both with and without the 1941 figure and a mean curve adopted as shown.

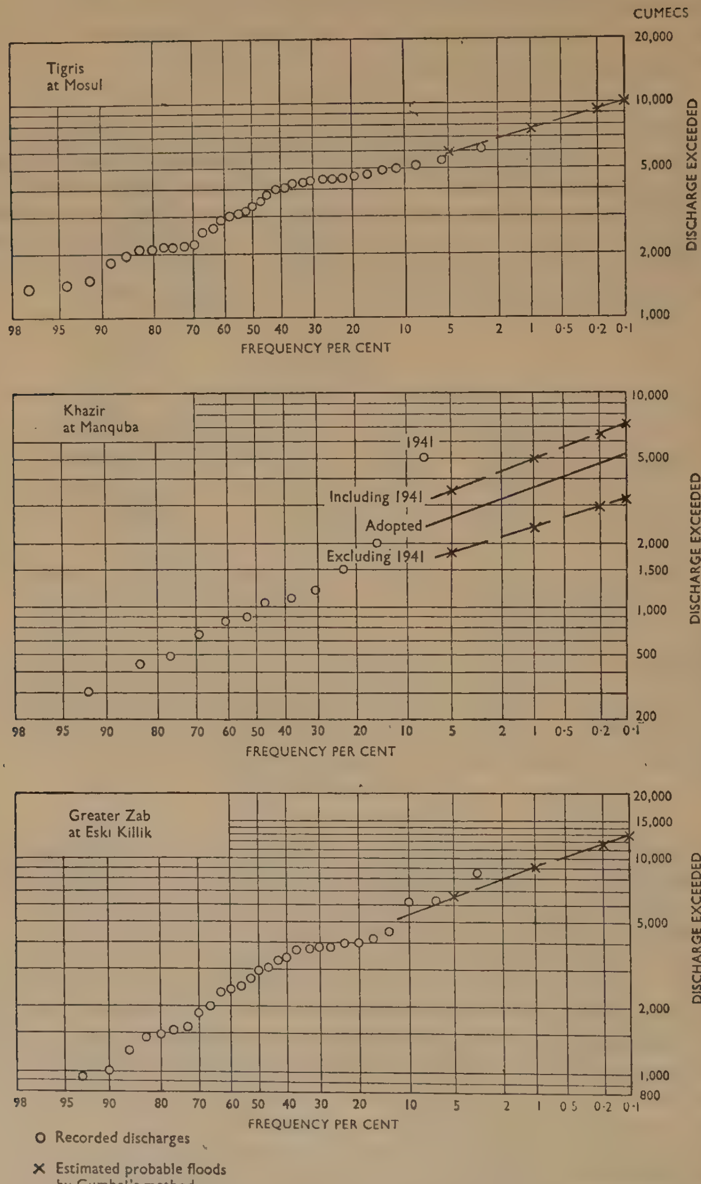

FIG. 4.—FLOOD PROBABILITIES

(Annual maximum discharges)

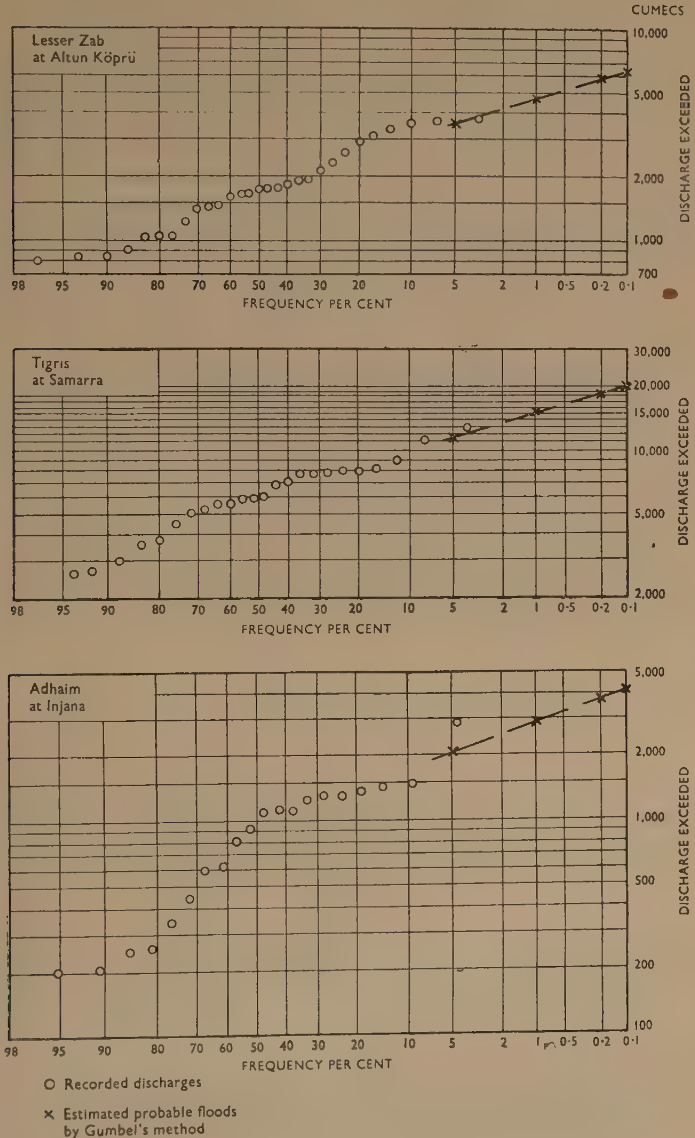

FIG. 5.—FLOOD PROBABILITIES
(Annual maximum discharges)

floods of extremely high intensity in a world context. Yet, in view of the evidence of catastrophic floods in the past, it would be unwise to ignore the possibility that the period of record was one of subnormal precipitation and the results shown must be subject to this reservation.

After the two proposed dams at Dokan and Eski Mosul are brought into commission, the distribution of river discharge throughout the year will be considerably modified; flood water will be absorbed in the storage reservoirs and discharged at moderate rates in the dry season. It is presumed that storage capacity will be reserved for flood control and (provisionally) that the outflow from the reservoirs arriving at the confluences will be capable of being reduced almost to nil at the times of peak floods in the main river. The effective catchment area for floods at Samarra may then be taken as the areas excluding the catchments above the dams—see Table 1, namely, 40% of the existing area. The proportions of mountains and foothills will remain almost the same but the Greater Zab will have a dominating influence. The conditions leading to high floods will be further modified by the greater relative length of effectively smaller catchment, the relatively greater valley storage, and the effect of timing of flood peaks at the confluences.

Insufficient data were available to attempt any detailed flood routing, though a study of volumes of water passing gauging stations during floods of recent years gave a quantitative indication of valley storage in the main river which was of value in the estimation of probable floods.

To obtain an indication of the future flood probability at Samarra, it may first be assumed that there will be no synchronizing at Samarra of any flood release from the dams at Dokan and Eski Mosul. A suitable value of the coefficient C may then be applied to the catchment excluding the areas above the dams. If the same value of C as obtained for the whole catchment were used, the 0·2%-flood discharges would be as shown in Table 6.

TABLE 6.—FLOOD PROBABILITIES AT SAMARRA

	Whole catchment	Excluding area above Dokan Dam	Excluding areas above Dokan and Eski Mosul Dams
(1)	(2)	(3)	(4)
Catchment area: sq. km	110,495	98,805	43,907
,, ,, : sq. m	42,700	38,100	16,900
% mountainous	49	44	46
Coefficient C ($= Q/A^{\frac{1}{2}}$)	3,240	3,240	3,240
0·2% flood: cusecs	670,000	630,000	420,000
,, ,, : cumecs	19,000	18,000	12,000

These results are, however, subject to some qualifications. The area above Dokan Dam is mostly mountainous and its removal may reduce the value of C; the discharge of 18,000 cumecs shown in column (3) of Table 6 may therefore be a slight overestimate though, as the relative catchment area is small, the difference could not be very appreciable. On the other hand the value of C for the catchment of the Tigris above Mosul, constituting more than half the total, is only 2,300 whereas the corresponding values for the Khazir and Greater Zab are 4,710 and 4,530 respectively (see Table 5), so without the areas above Dokan and Eski Mosul the

alue of C may well be higher than 3,240. Against this, the valley storage in the resent river between the Greater Zab confluence and Samarra will be greater lative to the flood volume than at present and any flood emanating from the hazir and Greater Zab would be considerably moderated when it reached Samarra. he estimate of 12,000 cumecs at Samarra may, therefore, subject to the limitations the recorded data, not be far wrong for the river in its present state and, although may be expected that in time valley storage will be reduced by siltation and the ood risk would then be somewhat higher, the value of C for the river at Samarra ill always be very considerably lower than the corresponding values for the Khazir d Greater Zab because of the length of the main river.

It is necessary to add a qualification on the assumption of complete cut-off at e dams. It is evident that flood storage is fully effective only within the range which discharge reaching the dam can be held until after the time corresponding the peak discharge in the main river, allowing for times of transit. Outside this nge, the greater the volume of flood water reaching the dam prior to the peak in e main river, the less effective is the control. The beneficial effect of flood storage the probability curve is therefore most marked in the lower range and, above a rtain critical point depending on the available storage capacity relative to the lume of discharge, it becomes less the greater the flood.

Flood storage which will be used up before the peak of a high flood is reached ay be worse than no storage, if it creates a false sense of security. It is evident at sufficient storage should be made available during the flood season to effect a eful reduction of the flood of probability equal to that for which the works are signed. For this purpose a deep-gated spillway at the dam is an obvious advantage d, as has been suggested by Mr Haigh, the maximum level to which water may be ored between floods could be progressively raised during the flood season as the obability of a high flood occurring in that season gradually diminishes. The nefit of flood protection can thus co-exist with the provision of a nearly full reservir for irrigation and power requirements at the beginning of a dry season, but ood *volume* probabilities should be carefully computed on a statistical basis and justed (where applicable) in the light of information gained from seasonal snow rveys.

For the most advantageous programme of gate operation in a flood an efficient stem of warning based on upstream rainfall and stream gauges is necessary. In ew of the complexity of river flood hydraulics an electronic computer might well of assistance in this connexion.

Bed material and transported load

The Tigris at Samarra flows in a bed of shingle and gravel in which particles up to cm dia. predominate and up to 5 cm are common. Occasional outcrops of hard nglomerate may be seen (Fig. 15, p. 344). Below Beled (see Fig. 1, Plate 1) e bed consists of fine sand.

A considerable load of fine sand and silt is carried in suspension in the flood ason. Samples were taken on a vertical above the deepest point below the ntoon bridge at Samarra, the sampling points being at 0·2, 0·3, and 0·8 depth. fair estimate of mean concentration on the vertical may be obtained by averaging e three samples and the results are shown graphically, plotted against discharge, Fig. 6. For comparison, a curve derived from samples taken at Baghdad in 1953 also shown and it will be seen that, despite considerable variation in the results each place, the concentration was of the same order at higher discharges whilst,

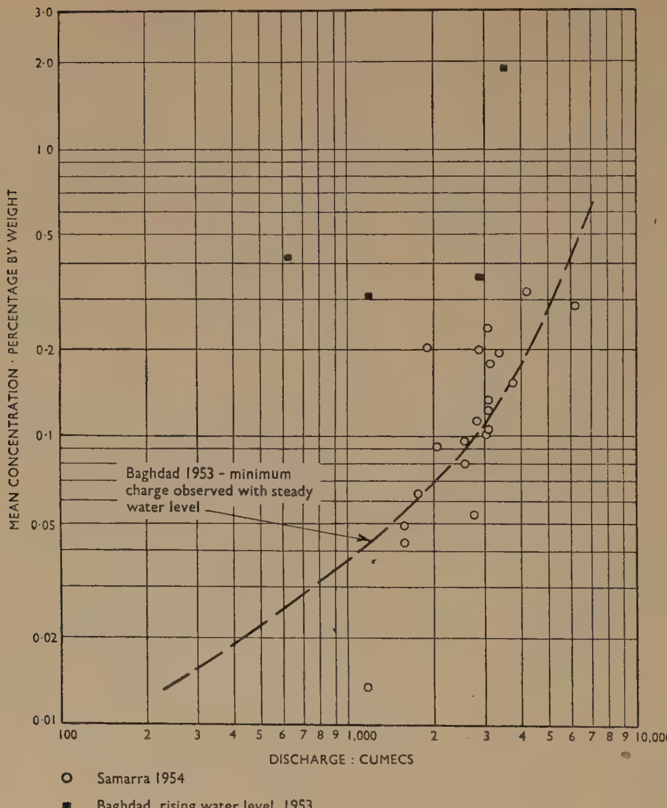

FIG. 6.—SUSPENDED LOAD IN TIGRIS RIVER AT BAGHDAD AND SAMARRA

as might be expected in view of the difference in bed material, it tended to be less at Samarra with lower discharges.

HYDRAULIC CHARACTERISTICS OF RIVER

The river approaches the barrage site in two channels (termed, in this Paper, the right and left main channels) which unite at K2, i.e., 2 km upstream of the barrage site (Fig. 3, Plate 1). The width of the single river channel averages about 600 m whilst the width of flood plain, or spill area, is approximately 4 km at Samarra, where the river is still incised, i.e., flowing at levels generally below the level of the surrounding country.

The average hydraulic gradient between Baiji and Samarra, covering 110 km of river, is approximately 42 cm/km, which is not very different from the local gradients indicated by the longitudinal section of the river shown in Fig. 7.

The mean annual maximum discharge at Samarra for the 29 years to 1953 was approximately 6,300 cumecs, or 220,000 cusecs. The model experiments showed that with this discharge the river is completely confined within its banks, the

"bank-full" discharge being about 10,000 cumecs, or 350,000 cusecs. Table 7 shows the hydraulic characteristics of the river obtained by averaging measurements on a number of cross-sections. It appears from the velocity/depth relation that the value of Lacey's f in the formula:

$$V = 1{\cdot}15\, f^{\frac{1}{2}}\, R^{\frac{1}{2}}$$

(in ft-sec units) is about 2·5, where V denotes the mean velocity and R the hydraulic mean depth.

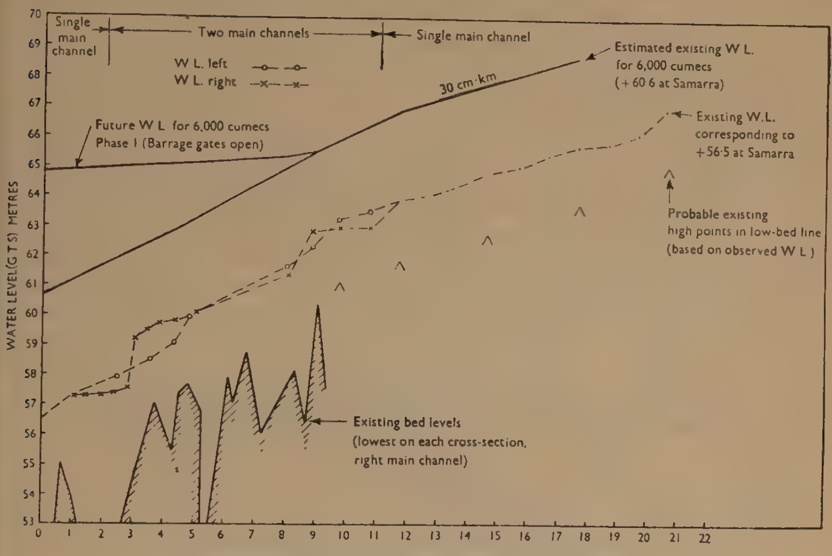

FIG. 7.—LONGITUDINAL SECTION OF TIGRIS RIVER ABOVE SAMARRA

TABLE 7.—HYDRAULIC CHARACTERISTICS OF RIVER TIGRIS AT SAMARRA FOR DISCHARGE OF 6,000 CUMECS (W.L. 61·0 AT SAMARRA)

Means of three cross-sections of single main channel and eleven cross-sections of left and right channels

	Single main channel	Mean of two main channels
Surface width (m)	600	400
Cross-sectional area (m²)	2,750	1,650
Mean depth (m)	4·6	4·1
Mean velocity (m/sec)	2·2	1·8
Hydraulic gradient	0·00042	

The lower curve shown in Fig. 8 represents the relation derived by the Irrigation Department by correlating known or estimated discharges at Baghdad and upstream stations with gauge readings at Samarra, prior to 1950. It is a good fit to the

earlier peak discharges but, as will be seen, subsequent estimated peak discharges have progressively moved away from it, indicating a trend towards higher specific levels. This was confirmed by a computation of cumulative volumes passing Samarra and Baghdad for the floods of February–March 1953, and February–April 1954, which indicated that the volume passing Samarra, computed by the earlier curve, was approximately 12% higher than the volume passing Baghdad, after deducting the Adhaim volume and eliminating differences in valley storage. Since

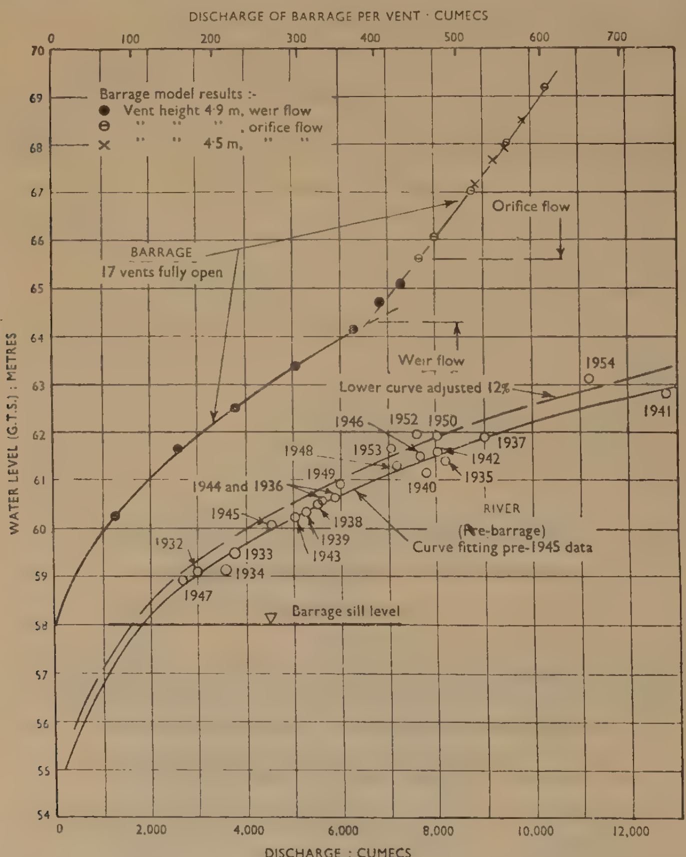

FIG. 8.—DISCHARGE : GAUGE CURVES FOR TIGRIS AT SAMARRA AND BARRAGE WITH BELLMOUTH SOFFITS
(Barrage calibrated in 3/100-scale model)

the low-water level appeared to be unchanged a uniform deduction of 12% was made and the resulting curve is shown in Fig. 8.

FUTURE RIVER LEVELS

It was necessary to consider at an early stage what effect the barrage would have on water levels in the river, first, because steps would have to be taken to evacuate

areas which would be flooded and to realign the main railway line and roads, and secondly, to provide data for the computation of pond capacity for balancing, making recommendations in regard to operation of gates, the design of protective works, and also for the construction of the model.

The changes in the river upstream of the barrage were considered to occur in two phases. Phase 1 represents the period before the power station comes into operation; the barrage gates will then normally be fully open but will be closed for short periods when it is required to reduce the discharge passing downstream by diverting water into the spillway. Phase 2 represents the period when the power station is in operation and a high pond level is maintained continuously.

Phase 1

The barrage is to have seventeen vents each 12 m wide with sill level 58·00 and gates to raise the pond to a maximum level of 69·00. Full pond level would thus be about 14 m above present low-water level.

In Fig. 8 is a curve showing the gauge : discharge relation for the barrage with gates fully open, as indicated by tests with the sectional model. This curve may be compared directly with the curve for the existing river shown in the same Figure. The amount of afflux due to the barrage varies between 2·5 and 3·5 m in the range up to 7,000 cumecs. Sand will deposit in the bed but, since the periods of high pond level will be short, siltation of the spill area is not likely to be appreciable during phase 1.

The calculated water surface for 6,000 cumecs is shown in Fig. 7. When the river is ponded to + 69 the surface would adopt a very flat slope meeting the existing 6,000-cumecs line at about 20 km upstream.

Phase 2.

In phase 2, however, siltation of the pond will be continuous in both channels and spill area. Fine sand and silt brought down by the flood in suspension will gradually raise the spill area to the controlled pond level, which, for reasons explained later, is assumed here to be + 67, i.e., 2 m below maximum water level in the flood season. The beds of the main channels will also be raised, but a comparison of future water level with existing channel bed and bank levels shows that the main river will tend to follow the existing channels except between the barrage and 4 km upstream, and between 6 and 8 km if the river-training works proposed are adopted. This will not preclude the formation of small channels meandering over the spill area near the left bank, but the presence of clay and gravel in this area makes it unlikely that a main channel will develop here in phase 2 (see Fig. 3, Plate 1).

Because of the reduction in flood discharge following the construction of the upstream dams, the occasions when gravel is moved in appreciable quantity in the river upstream of the pond will be very infrequent, and it is not anticipated that gravel will exert much influence on the river course through the pond for a very long time. For the same reason the channel bed in the pond will consist of newly deposited fine sand except where the existing gravel bed remains exposed.

The quantity of silt carried by the flood water is very considerable. A computation of volume, using the concentration curve for Baghdad shown in Fig. 6, showed that the silt which was carried past Samarra in the 10-year period 1944–53 probably represented about 500 million cu. m deposited volume. After the construction of the upstream dams, however, the volume carried will be much less—not because the dams will trap silt but because the discharge and duration of floods

will be less. Supposing, for example, all flood discharges were reduced by half, which is about what may be expected on the average, the concentration would also be reduced by about half and the volume carried would be reduced to 25% of the above amount. This reduced volume is comparable with the pond capacity corresponding to barrage W.L. of + 67 (after deducting the volume of the future channel), namely, 115 million cu. m.

Whilst a longer period than 10 years will be required before the whole spill area within the pond will silt to the level mentioned, because some of the silt will be carried downstream past the barrage, it may be expected that silt will deposit more rapidly along the margins of the river channel than at places farther removed from the flow. This will lead to berm formation, which in a few years will have the same effect on the channel flow as complete siltation of the spill area. These were the conditions reproduced in the river model for the purpose of testing various layouts of guide banks.

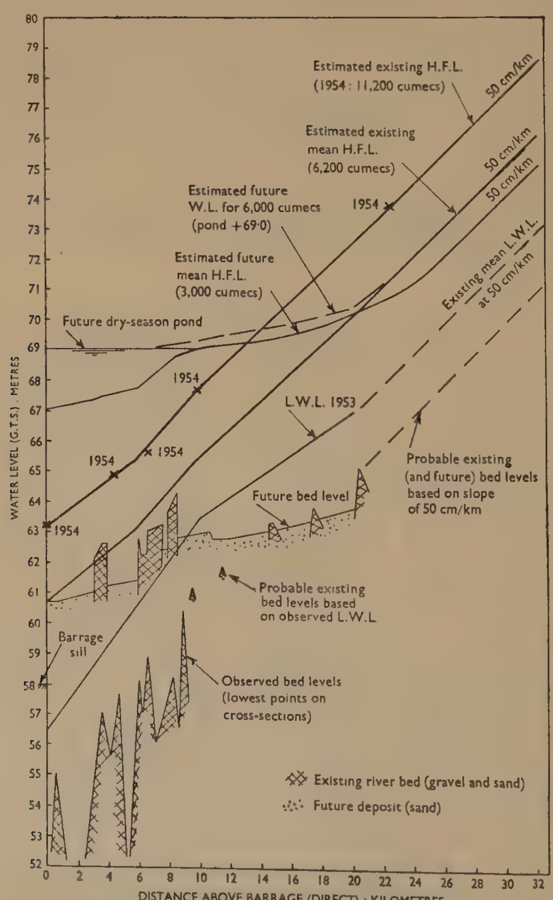

FIG. 9.—ESTIMATED FUTURE LEVELS UPSTREAM OF SAMARRA BARRAGE (PHASE 2)

The estimation of future river levels therefore presented some difficulty and had to be based on a number of assumptions. Discharge could be regarded on a statistical probability basis, but the next step was not merely a back-water calculation in the existing river course but involved the future silted cross-sections and determination of river course and hydraulic gradient through bed materials of various types.

The river course assumed was that shown in Fig. 10, Plate 2, as adopted in the model experiments described later in the Paper. It was a single channel—as may be ultimately expected when the bed material is alluvial sand—and no allowance was made for spill channels, all of which it was considered would eventually become insignificant.

The result of a step-by-step computation, based on data from the existing channels and using the Lacey formulae, may be seen in Fig. 9, in which the estimated future high flood and bed levels may be compared with those now existing. The presence of existing river bed (gravel and sand) at higher levels in the estimated future bed level is explained by the change in river course. Siltation will also occur on the spill area, which will eventually approach the future mean high-flood level.

Operating conditions

In the first phase of operation, the period when the only function of the barrage is to divert flood water into the spillway, operation should present few difficulties.

The maximum discharge to be allowed to pass through Samarra Barrage will depend on the flood conditions in the Adhaim and Diyala rivers which join the Tigris downstream and contribute to raising the flood levels at Baghdad and in the lower part of the river. The Haigh Commission proposed that the Samarra Barrage should as far as possible be operated so as to limit the combined discharge at the Diyala confluence to 3,500 cumecs. If the necessity arose, discharges in excess of this could be dealt with by breaching the flood banks at selected places in the lower river, as is done at present, but 7,000 cumecs is regarded as the limit beyond which extensive flood damage would be caused.

The peaks of the Adhaim and Diyala rivers generally precede the peak from Samarra, but the subsequent fall does not always occur in time to avoid a degree of synchronization, as illustrated by the recent flood of March 1954 when the Diyala was still passing a high discharge when the Tigris peak arrived at Baghdad.

Since the storms which have produced high floods have been accompanied by precipitation over the whole of the mountainous area in the catchment, it must be assumed that under those conditions which produce dangerous floods in the Tigris the Diyala flood would be both high enough and sufficiently sustained to synchronize with the Tigris peak flood.

To provide an example of the operation of the barrage in phase 1 the 1941 flood was taken, this being (prior to the 1954 flood) the highest combined flood recorded in 48 years. A step-by-step computation, allowing for pond capacity, showed that the combined flow of the Tigris below Samarra, Adhaim, and Diyala could have been controlled to 6,600 cumecs, compared with 14,400 without the barrage. The 1954 flood would, however, have presented a different picture because of the high and more sustained flood discharges, and (without detailed examination) it would appear that flood banks would have had to be breached even with Samarra Barrage in operation, although the situation would of course have been far less critical than it actually was.

The problem in operation will generally be to decide when to close the barrage gates and when to open them. This will depend largely on the estimation of flood

rises based on meteorological data and gauge readings at places upstream and on the Diyala. In the event of an extremely high flood, such as those for which the works must be designed, the barrage gates would almost certainly be closed during the rise of flood to divert the maximum flow into the spillway. The pond would therefore be effectively full before the peak is reached and no allowance can be made for reduction by absorption in the pond, but the diversion of flood water would result in increased valley storage available in the river downstream.

In the early years of phase 2 the safe limits will be the same as stated for phase 1, namely, the combined discharge at Diyala confluence should not normally exceed 3,500 cumecs, whilst 7,000 cumecs will be the safe limit, with breaches in the flood banks of the lower river.

After the construction of a storage dam on the Diyala river the risk of high flood at Baghdad will be greatly reduced because the floods from the lower part of the Tigris catchment and the Diyala will generally have passed Baghdad before the arrival of the peak from the Greater Zab.

A matter which should, however, not be overlooked in such schemes is the possibility of deterioration of discharge capacity of the downstream river by aggradation during a succession of years of low flood discharges. In the case now considered it is likely that, whilst the limit for normal floods may remain at 3,500 cumecs, the absolute safe limit may be gradually reduced owing to reduction of discharge capacity. The contraction of the river channel and steepening of the slope in the lower part of the river may indeed result in a gradual return to the conditions of dangerous floods, though of lower discharge than are now passed safely through Baghdad. This may occur in spite of the construction of upstream dams because the silt-carrying capacity reduces in greater ratio than the discharge (see Fig. 6). A remedy may then lie in increasing the effective slope by eliminating bends in the river downstream.

Control of pond level in flood season

When the power station comes into operation it will be necessary to maintain a high pond level continuously to create sufficient head through the turbines, and the higher the pond level which can be maintained the greater the design head and the less the cost of the turbines per kW.

In the dry season the water is effectively silt-free and the pond could be maintained at the maximum possible water level of + 69 during peak hours to reduce the flow required to be discharged from the storage reservoirs. The pond would provide balancing capacity for daily variations in demand.

In the flood season, when discharge is surplus to requirements for power generation, no balancing capacity would be called for but it will be essential to prevent siltation above, say, + 67 for two reasons:

(1) to maintain sufficient balancing capacity for dry season operation;
(2) to prevent the formation of a high silt bank across the entrance to the approach channel to the regulator and spillway.

This can be assured only by controlling the pond level at + 67 or below during the flood season when the concentration of silt in suspension is appreciable.

The control of siltation in the pond is indeed a matter of prime importance to the proper functioning of the scheme. As will be shown later, no layout of headworks could be devised which would meet all requirements and prevent the deposition of silt in the approach to the regulator when no discharge was being diverted to the

spillway, and an analysis of past discharge records shows that the number of days per year when the regulator gates will be opened may be very few. For example, in a period of low precipitation such as 1944–48 flood water might be diverted on an average of only 4 days per year. In the event of a sudden flood it may be required to divert water at 24 hours notice. It must therefore always be possible to raise the pond level above the level of the silt bank to provide head for scouring a channel, from which it follows that the silt bank must be kept below top water level. The limit of + 67 proposed would enable a scouring head of 2 m to be provided.

Discharge capacities of barrage and regulator

In view of the volume of impounded water the safety of the headworks should be on a lower risk level than the maximum flood assumed for operational design. The capacities provided in the earlier design were 7,000 cumecs each for barrage and regulator—a total of 14,000 cumecs (500,000 cusecs). It was, however, felt that this would not provide sufficient margin of safety in the period between the construction of the barrage and the completion of the upstream dams (phase 1). The matter is well illustrated by the statistical data (see Fig. 5 and Table 6), which indicate that the risk of a flood exceeding the safe limit for the headworks would have been at least 4% in the 2 years pending completion of Dokan Dam and more than 1% per annum after that until the second dam came into operation.

The designed capacity was therefore increased by widening the regulator to pass 9,000 cumecs and increasing the discharge capacity of the barrage by providing upstream bellmouths to the soffits of the vents. The discharge curve of the barrage with specially designed bellmouths based on experiments with a 3/100-scale sectional model is shown in Fig. 8, from which it will be seen that with this design a discharge in excess of 10,000 cumecs (350,000 cusecs) could be passed. The combined discharge capacity would then be 19,000 cumecs (670,000 cusecs), which should be ample for the protection of the headworks during the few years prior to the control to be provided by Dokan Dam.

In phase 2 the headworks would be capable of dealing with a Tigris flood of 12,000 cumecs (420,000 cusecs) while limiting the discharge in the river downstream to 3,000 cumecs, and in the event of a flood even higher than this the surplus capacity of the barrage would be of value in providing flexibility of control.

Layout of headworks and river training

At the time of this investigation the alignment of the barrage had already been determined, the barrage and power station had been sited for construction in the dry, and the contract for construction placed. The siting of the regulator was limited by the above considerations and by the alignment of the spillway, the dike for which was nearing completion under an earlier contract, and by the requirements of the future railway line to be carried across the regulator (see Fig. 3, Plate 1).

Other main requirements of the layout were that the works should be safe throughout the range of operational conditions, that flood water could be diverted into the spillway at short notice (say 24 hours), and would reach maximum designed discharge in minimum time, that excessive quantities of sand should not be carried into the spillway where it might deposit and reduce the discharge capacity, and that the quantity of sand drawn by the intakes of the turbines and Ishaqi Canal should be a minimum.

The layout considered to be most suitable to meet these requirements is shown in

Fig. 3, Plate 1, and Fig. 10, Plate 2. It was determined with the aid of experiments with a river model, described below, but since time was an important factor, construction having already commenced, this part of the investigation was carried out concurrently with the collection and analysis of flood data, and in consequence the discharge capacity of the regulator in the model was equivalent to 7,000 cumecs instead of 9,000 cumecs as adopted later. This did not, however, have any significant effect on the results.

The requirements for upstream guide banks in phase 2 differ from those in phase 1. In phase 1 the pond will not be raised to a high level except during a few days each year when flood water is to be diverted. Siltation will therefore be slight and during high floods when the pond level is raised the velocity of approach flow to the barrage and regulator will be low. Under such conditions short guide banks will suffice to protect the works.

In phase 2, under a continuously high water level, the pond will gradually silt with the formation of a channel leading to the barrage and the approach velocity will be appreciable. Long guide banks will therefore be required, as is general practice in the case of bridges and barrages across large rivers in alluvial terrain, to protect the works against oblique, concentrated, and sharply curved flow which might cause dangerous scour. The danger of scour is illustrated by Fig. 16 showing the regulator and right guide bank after an early experiment. If the guide bank had been shorter the deep scour arising from the curved flow would have endangered the regulator.

Except possibly in the bed of the future main channel the deposit would be composed of the fine sand and silt carried in suspension, and after the construction of the upstream dams it would be doubtful whether flood discharges would be high enough to carry and deposit within foreseeable time a sufficient quantity of gravel to cause a change in river course in the pond. It was therefore considered safe to assume that the velocities of flow would be comparable to those of rivers of this size in fine alluvium and would not be sufficient to scour existing deposits of gravel and shingle.

This assumption greatly facilitated the prediction of the future river course approaching the headworks, since the deposits forming the side spill area upstream of K3 (see Fig. 3, Plate 1) would tend to hold the river to its existing channels. Between K3 and the barrage, however, the river would have freedom to form main channels over the existing deposits.

It was also clear from the river survey (Fig. 3, Plate 1) that no satisfactory approach to the regulator or barrage could be obtained from the existing right main channel of the river—the oblique approach would be dangerous, as well as unsatisfactory from the point of view of sand being drawn into the spillway and into the turbine intakes. It was therefore considered necessary to close this channel and divert the whole (ponded) flow through the left main channel. The closure could be done during the first season of ponding for power generation and should be preceded by the erection of gravel banks to cause silt deposit on the island between the right and left main channels.

With these measures carried out and the pond maintained at + 67 during flood seasons, the river course indicated by the model was that shown in Fig. 11, Plate 2. To obtain the most direct approach to the regulator on the occasions when flood water is diverted, the regulator was sited as close to the barrage as other considerations would allow.

The positions of barrage, power station, and regulator having been determined

FIG. 15.—TIGRIS RIVER AT SITE OF BARRAGE

(Looking upstream from pontoon bridge at Samarra. Conglomerate cliff on right)

FIG. 16.—SCOUR AROUND SHORT RIGHT GUIDE BANK AFTER EXPERIMENT 10B (7,900 CUMECS)

For layout see Fig. 12

(Although there was deep scour at the head of the guide bank, this was some distance away from the regulator and there was no appreciable scour at the upstream face of the regulator)

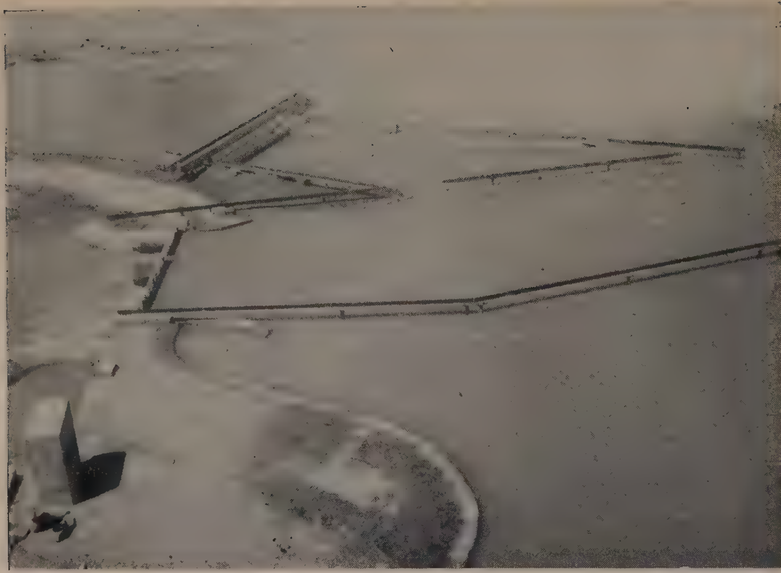

FIG. 17.—BARRAGE, REGULATOR, AND GUIDE BANKS AS TESTED IN FINAL LAYOUT. SEE FIG. 10

(View taken from direction of Samarra town. Closing bank may be seen in foreground. The rails were provided to facilitate observations)

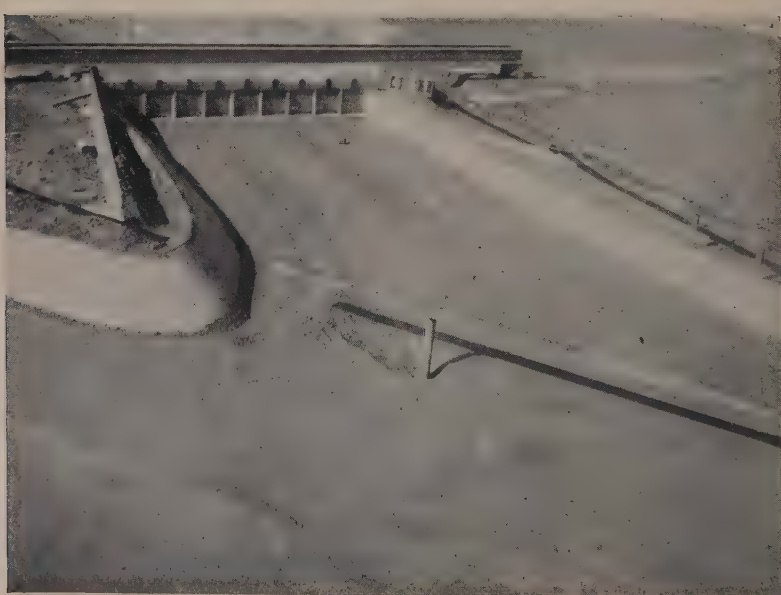

FIG. 18.—ESCAPE CHANNEL AND REGULATOR AFTER EXPERIMENT 26A (FINAL LAYOUT)

(Discharge through regulator had been 7,000 cumecs)

left guide bank was required to protect the barrage from oblique flow, particularly n the early years of phase 1 when the river would tend to follow its old course. A ight guide bank was required for the barrage which would ensure a direct approach o the power station and enable the latter to draw top water, rather than sand-laden ed water, so far as possible. Guide banks were also required to provide a direct pproach to the regulator. The most suitable layout for these guide-banks was etermined by experiments with the river model.

The intake to the Ishaqi Canal was located in the right abutment adjacent to the ower intakes (Fig. 10, Plate 2), so that measures for sand exclusion could serve both.

The river model

The model was constructed at a hydraulic research station established in the revious year at Hindiya Barrage, Iraq. The horizontal scale was 1/200, vertical cale 1/40, and discharge scale 1/50,000. The velocity scale was thus 1/6·25 which ffectively gave an equal Froude number ($V/\sqrt{gD}$, where V denotes the velocity, the gravitational constant, and D the depth) in model and prototype. Also, the orizontal scale was sufficiently close to the $\sqrt{Q}$-scale (where Q denotes discharge) s required by the Lacey-Inglis criterion for meander tendencies.[7]

The length of river reproduced in the model was approximately 13 km (shown in ig. 3, Plate 1). The model was thus about 200 ft in length and its greatest width vas 80 ft. A high-flood discharge of 10,000 cumecs (350,000 cusecs) was represented n the model by 7 cusecs.

The river-bed for the first series of experiments, in which the existing river conitions were reproduced with no barrage, was formed in sand of 1·5- to 6-mm grade, vhilst sand finer than 3 mm was used for the spill area. In subsequent experiments, n which the river was ponded by the barrage, fine sand of mean diameter about ·1 mm was used in raising the spill area to represent siltation and sand up to 3 mm n the channel beds. In certain experiments fine sand (0·1 mm) was injected to ndicate areas of deposition.

The modelling of the existing channels and spill area was based on a special survey f the river together with a survey carried out in 1948 by the Haigh Commission, n aerial photographic survey, and photographs taken during visits to Samarra.

The model could not reproduce the siltation of the pond and formation of new hannels by means of silt or sand carried in suspension because silt-carrying capacity relatively very much less in small channels than in rivers. It was therefore ecessary to form the river channels and spill areas at the higher levels which would ollow siltation. It was first necessary to estimate the probable levels and the ourses the river channels would follow. This was done, as described on p. 341, artly by calculation on the basis of a "dominant" discharge of 3,000 cumecs and artly by observing flow conditions in the model during a number of experiments etween which the siltation was advanced stage by stage. The river-training easures required to close the right main channel and spill area were reproduced the model and the predicted future river course was as shown in Fig. 11, Plate 2.

The guide banks were constructed in clay with cement plaster, to represent earth anks with rubble aprons (see Figs 17 and 18).

Experiments with layouts

Numerous experiments were carried out to test variations in the shape and length f guide banks but space does not permit a detailed account to be given here. Only

the final layout is presented (Fig. 10, Plate 2, and Fig. 17) and some of the results which led to its recommendation. Each layout was judged on the requirements mentioned earlier (p. 343), and it was necessary to consider only the conditions of phase 2 because the short guide banks required initially for phase 1 could easily be designed to suit.

A main requirement was that the discharge through the regulator into the spillway could be raised to the maximum within a short time. Long periods may be expected to occur when the river discharge is within the range from 1,500 to 3,000 cumecs, i.e., not sufficient for diversion into the spillway but carrying an appreciable concentration of silt in suspension. For this reason it was recommended that the pond level should normally be controlled at + 67 during the flood season, but even with this limit silt would deposit in the approach to the regulator and a channel must be scoured through the deposit before the designed discharge can be passed through the regulator.

This difficulty is illustrated in Fig. 12 which shows the flow lines under low-flood conditions when the regulator would be closed. It was seen that if, as in this layout, the right guide bank to the regulator was only of sufficient length to protect the work, extensive silt deposit would occur in the approach to the regulator. Scouring might then be a slow process.

The best solution was found to lie in the extension of left and right guide banks towards the main river channel, with the object of concentrating the discharge when the regulator is opened and thus enabling the silt deposit to be scoured more quickly than if the guide banks were the minimum consistent with safety and the discharge upstream was dispersed over a greater width.

With this arrangement of extended guide banks (see Fig. 13) it is considered that, provided flow through the regulator is completely stopped during these periods, the silt deposit will be limited in practice to a bank across the entrance to the escape channel, because silt will first deposit there and subsequent entry of silt-laden water will thereby be prevented. This assumption accords with the behaviour of an escape channel in rather similar conditions at Habbaniya and was borne out by observations on the model.

Experiments were also performed by raising the pond level to + 69 and opening the regulator gates to scour the silt bank shown in Fig. 13. Scour in the model was very rapid and, although a consolidated bank in the river would doubtless offer much greater resistance, it is reasonable to suppose that the gradient across the bank would be sufficient for the purpose. Since a given discharge has greater scouring power when more concentrated, the left and right guide banks forming the escape channel were aligned as nearly parallel as was consistent with other requirements to reduce the width at entry.

Considering the avoidance of excessive draw of silt and sand into the spillway, Fig. 14 shows the flow lines observed when the pond was raised, in the model, to + 69. It is inevitable that each time the regulator is opened the material forming the bank across the entrance will be carried down with the flow, in addition to the material comprising the transported sediment in the flood discharge diverted into the spillway. The main danger would appear to be a deposition of sand in the spillway bed, causing afflux and meandering of the deep channel, and there is no doubt that as the slope is only 1 : 10,000 the spillway capacity will in time be reduced by siltation. It will be desirable to avoid the diversion of small discharges. The material will, however, be mostly of a fine grade and there is a large area in which it can deposit. In view of the probable infrequency of use, siltation is not expected

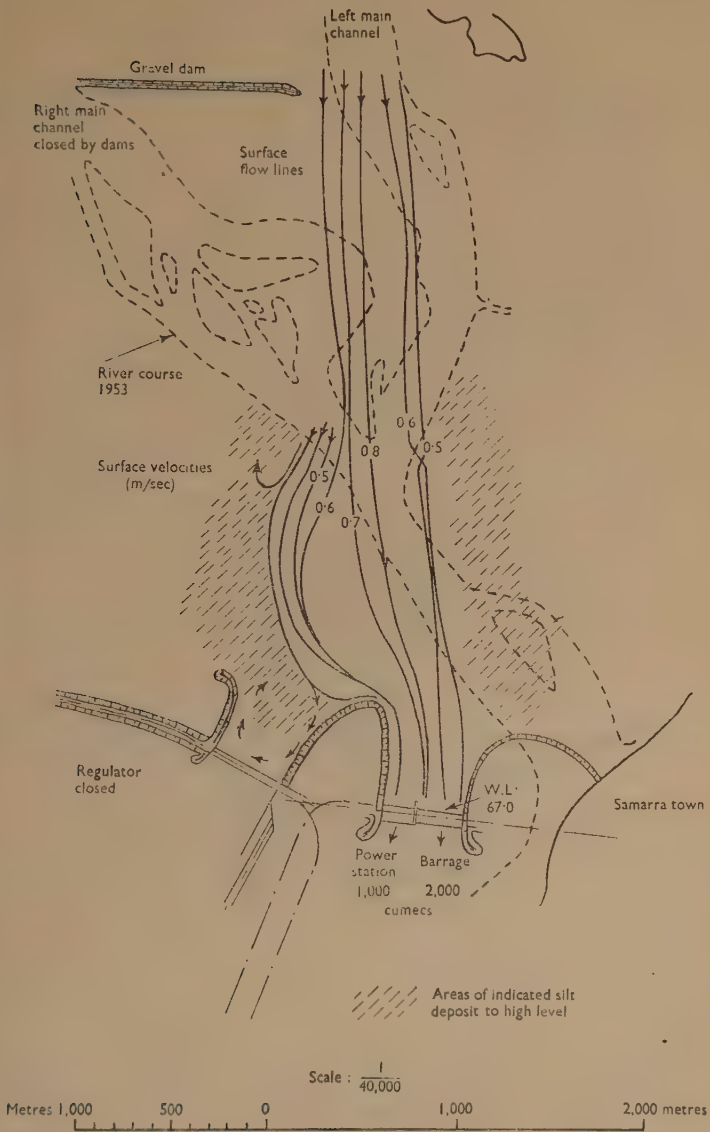

FIG. 12.—SHORT RIGHT GUIDE BANK

(Flow lines and velocities under low-flood conditions with regulator closed. Shows siltation of approach to regulator)

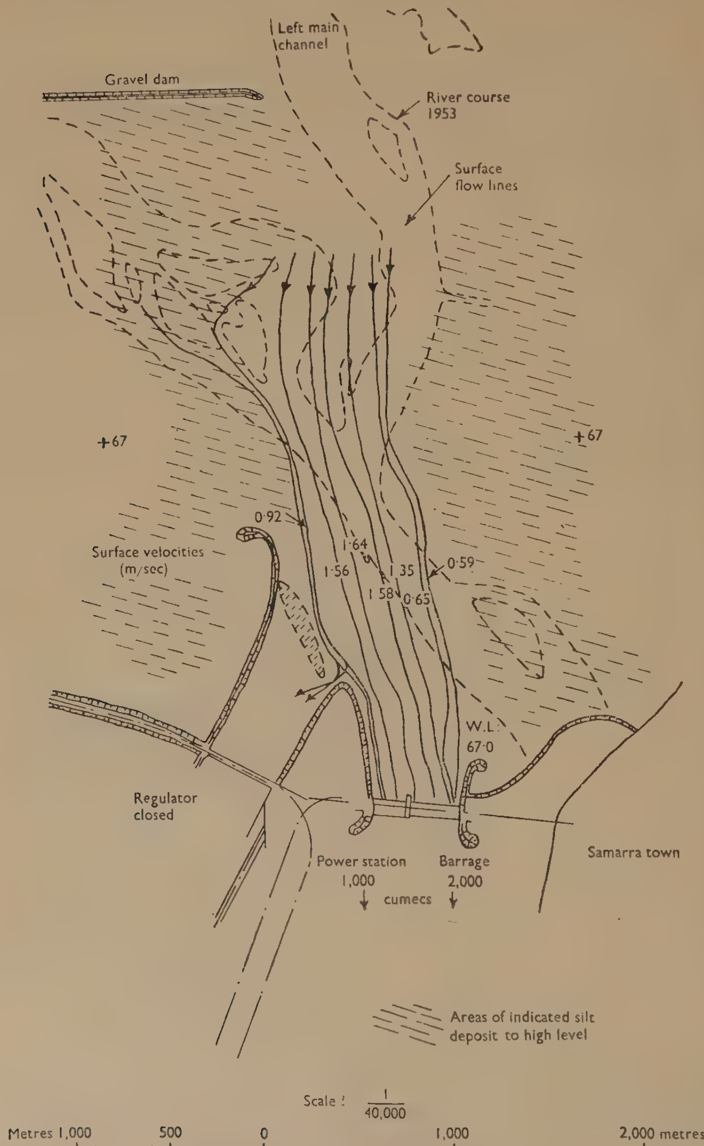

FIG. 13.—LONG RIGHT GUIDE BANK

(Flow lines and velocities under low-flood conditions with regulator closed. Shows formation of silt bank across entrance to escape channel)

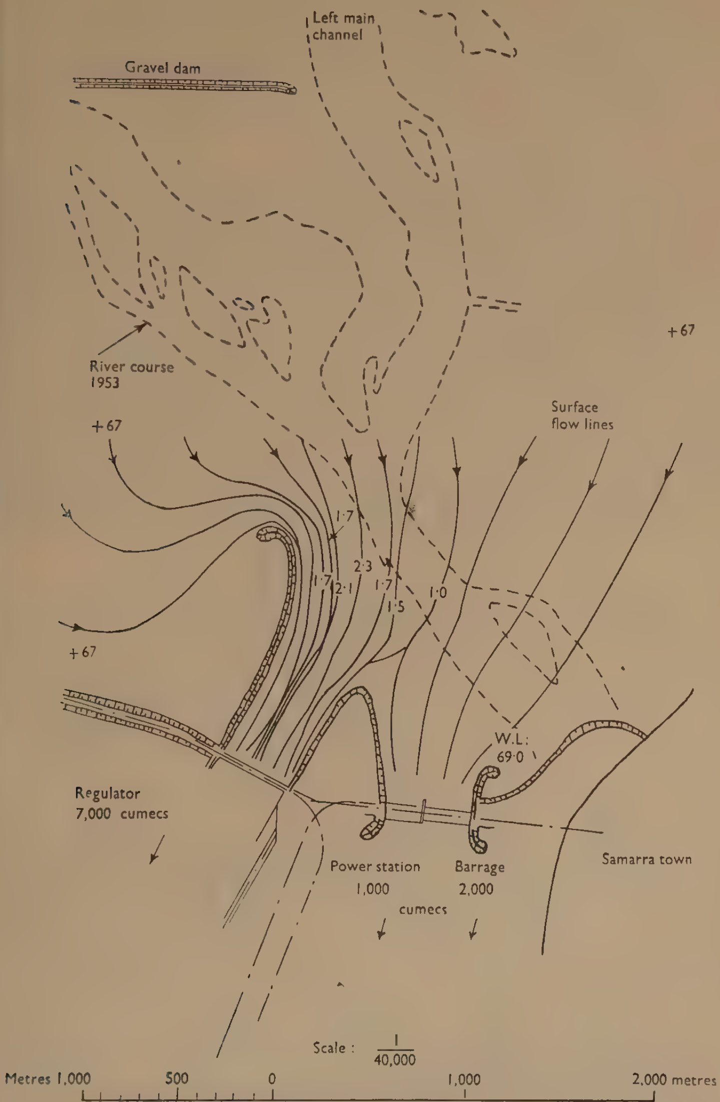

FIG. 14.—HIGH-FLOOD CONDITIONS. FLOW LINES AND VELOCITIES WITH HIGH POND LEVEL

(River discharge 10,000 cumecs, of which 7,000 cumecs passes through regulator)

to cause a serious reduction in discharge capacity before there has been ample opportunity for further upstream control projects to be brought into commission.

The need to exclude sand from the approach to the power station and canal intake presented a difficult problem. The conditions prevailing during the greater part of the flood season would be those shown in Fig. 13 or (more frequently) similar, but with the discharge through the barrage reduced to correspond with lower flow in the river.

The shape of the right guide bank to the barrage was found to have considerable influence. When the bank was at a right angle with the barrage line (as in Fig. 12) the flow tended to form a pocket of low velocity, or a slow return eddy, immediately upstream of the turbine intakes nearest the right abutment. This would lead to the formation of a silt bank at high level and unfavourable conditions for the turbine and canal intakes. The layout shown in Fig. 10, Plate 2, and Fig. 13, providing an oblique approach, gave better results; it was hoped that this angle of approach would, with suitable operation of the barrage gates, induce a convexity of flow past the turbine intakes. Whilst the undesirable eddy formation was eliminated, tests by the injection of dye in the surface and bed flow upstream did not show an appreciable tendency to the exclusion of bed sand. The provision of a divide wall between power station and barrage did not improve results since the flow pattern was a direct and inescapable result of the position of the main river channel and the proximity of the entrance to the escape channel. It would, however, be desirable in any case to safeguard the turbines and canal by providing excluder tunnels and if these are provided, as recommended, the layout shown would be satisfactory in this respect.

Finally, the guide banks were aligned so that their liability to damage in times of flood was reduced to the minimum. Their construction will probably be of local sand or soil encased in loose boulders and with boulder aprons. Such guide banks are most liable to damage by sharply curved flow at or near their heads and the need for a gentle curvature of the head of the right guide bank to the regulator, through a considerable arc, is evident from the flow lines shown in Fig. 14. In this case the pond level was at + 69, i.e., 2 m above the assumed level of silt deposit, so flow approached the entry to the escape channel over a wide area. Similar care was necessary at the point of divide between the approaches to regulator and barrage, but in this case a sharper curve is permissible.

For the protection of the guide banks and the upstream face of the regulator it was necessary to ensure a uniform distribution of flow across the escape channel. Not all the layouts tested were satisfactory in this respect under all conditions of operation, but the layout shown in Fig. 10 was adequate. Fig. 18 shows the scour occurring after a discharge of 7,000 cumecs had been passed into the spillway.

The experiments were all carried out with the river course shown in Fig. 11, Plate 2, derived as described earlier. Although this course is the one most likely to develop following the construction of the headworks and the river-training measures shown in Fig. 11, Plate 2, it may be mentioned that should the river follow a course more to the left, e.g., the existing course, from K4 (see Fig. 11, Plate 2) the conditions would be improved in all respects.

Fig. 10, Plate 2, shows the layout derived with the aid of model experiments for the conditions of phase 2, with regulator designed for 7,000 cumecs. The escape channel was then approximately 400 m wide. For the increased capacity of 9,000 cumecs the number of 12-m openings in the regulator would be increased from twenty-eight to thirty-six. The width of escape channel would then be approxi-

ıately 500 m and the length of right guide bank slightly greater, as shown in Fig, 3, 'late 1.

Fig. 10, Plate 2, also shows the short right guide bank considered adequate for he conditions of phase 1. The left guide bank to the barrage would be unchanged ut very short rounded banks would suffice for the right bank of the barrage and ower station and for the left bank of the regulator.

The river model experiments were commenced in November 1953 and completed ı April 1954.

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TABLE OF APPROXIMATE CONVERSION FACTORS

1 km	= 3,280·84 ft = 0·621 mile
1 sq. km	= 0·386 sq. mile
1 m	= 3·28 ft = 39·37 in.
1 sq. m	= 10·76 sq. ft
1 cu. m	= 35·31 cu. ft
1 cumec	= 35·31 cusecs

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The Paper, which was received on the 25th May, 1955, is accompanied by fourteen eets of drawings and four photographs, from which the half-tone page plates, lding Plates 1 and 2, and the Figures in the text have been prepared.

Discussion

Mr F. S. Hardy (Senior Engineer, Sir M. MacDonald & Partners, Consulting Engineers) asked if the Author could give references to "the evidence of catastrophic floods in the past" referred to on p. 334. Mr Hardy had been searching for factual evidence for many years but had not found any. Noah's flood was described in the Book of Genesis, but no real data were given except that it had rained for 40 days and 40 nights. It was also stated that the Ark had landed on Mount Ararat in Turkey. The prevailing wind in Iraq, however, was north-west, so that it was difficult to see how the Ark had reached Mount Ararat from Mesopotamia. By "catastrophic," had the Author meant catastrophic in its effects or catastrophic in respect of all previously experienced floods. There had been within living memory catastrophic floods in Iraq, in the sense that they had done a great deal of damage and could have done a great deal more, but it was unlikely that the Author would consider any of them as having been catastrophic in their volume by comparison with previous floods. In 1954 there had been a big flood, and Baghdad had only just been saved from a catastrophe by the excellent work of the Iraqi Army; but it had not been a particularly abnormal flood in peak discharge but only in its duration and the danger had arisen principally because, during the past 35 years, Baghdad had extended out into the natural spillway of the river. That sort of thing happened in many other countries, where towns had spread out into the natural spillway.

With regard to flood frequencies, the Author had rightly said that he used them with caution, and it should be emphasized that flood frequencies could be very misleading. The bottom chart in Fig. 5 showed the River Adhaim, and there was one point right out of line with all the rest at 3,000 cumecs. That particular flood had occurred on 26 December, 1952. If the flood frequencies had been calculated on 20 December of that year, it would have been found that the 100-year flood on the records up to that time was a little more than 2,000 cumecs, but calculating it again on the records 10 days later the 100-year flood was 3,000 cumecs. If one was brave enough to extrapolate to a 1,000-year flood, a calculation made on 20 December showed 2,500 cumecs whereas made on 30 December it showed 5,000 cumecs. Mr Hardy was not attempting to deprecate the frequency method; in view of the short period covered by the data and the lack of data the Author had had very little choice. The Author had said that he had had only river-gauging records and very few rainfall records, and that was correct. The rainfall records in Iraq, except in Baghdad, had been made only for a very short time, and the sites of the gauges frequently did not provide a representative picture of the rainfall. In the case of the great Adhaim flood in 1952, the rainfall records from the available stations in the catchment area had not shown any greater rainfall than in many other floods when the discharge on the Adhaim had been only half as much or even less.

Under the heading "Operating conditions" the Author had drawn attention to the possibility of the discharging capacity of the downstream river declining. Mr Hardy thought that that was a very great danger, and a danger which was always present with what the Americans called "off river" flood control schemes. The same effect would be obtained downstream as that to which the Author had referred upstream, that was, splitting the river, say half going downstream and half going to the Tharthar depression. The silt-carrying capacity of the downstream river discharge would be less but it would carry the same proportion of silt. Unless those who were operating the barrage and the scheme as a whole were extremely careful and kept the river discharge up to the maximum which it could carry in its present conditions, they were certainly going to get a very considerable decline in the river capacity, so that the last state of affairs with regard to flood relief might be worse than the present one.

Mr Gerald Lacey (Consultant, Drainage and Irrigation Adviser, Colonial Office) said that the Author had referred to the method of flood disposal which had been adopted as "almost unique" and attributed the suggestion that the Wadi Tharthar should be used for flood absorption to that truly great engineer Sir William Willcocks. Previous

SOME HYDRAULIC INVESTIGATIONS IN CONNEXION WITH THE WADI THARTHAR PROJECT, IRAQ

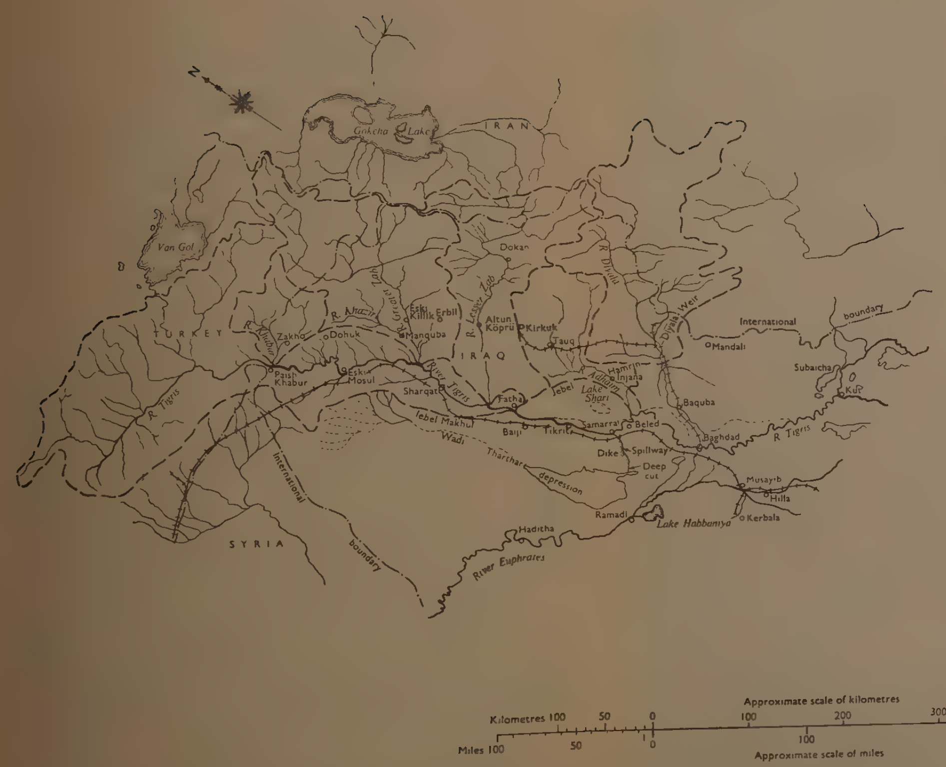

Fig. 1.—CATCHMENT AREAS OF RIVER TIGRIS AND TRIBUTARIES NORTH OF JEBEL HAMRIN

Fig. 2.—MAP SHOWING RAINFALL ISOHYETS (mm per annum)

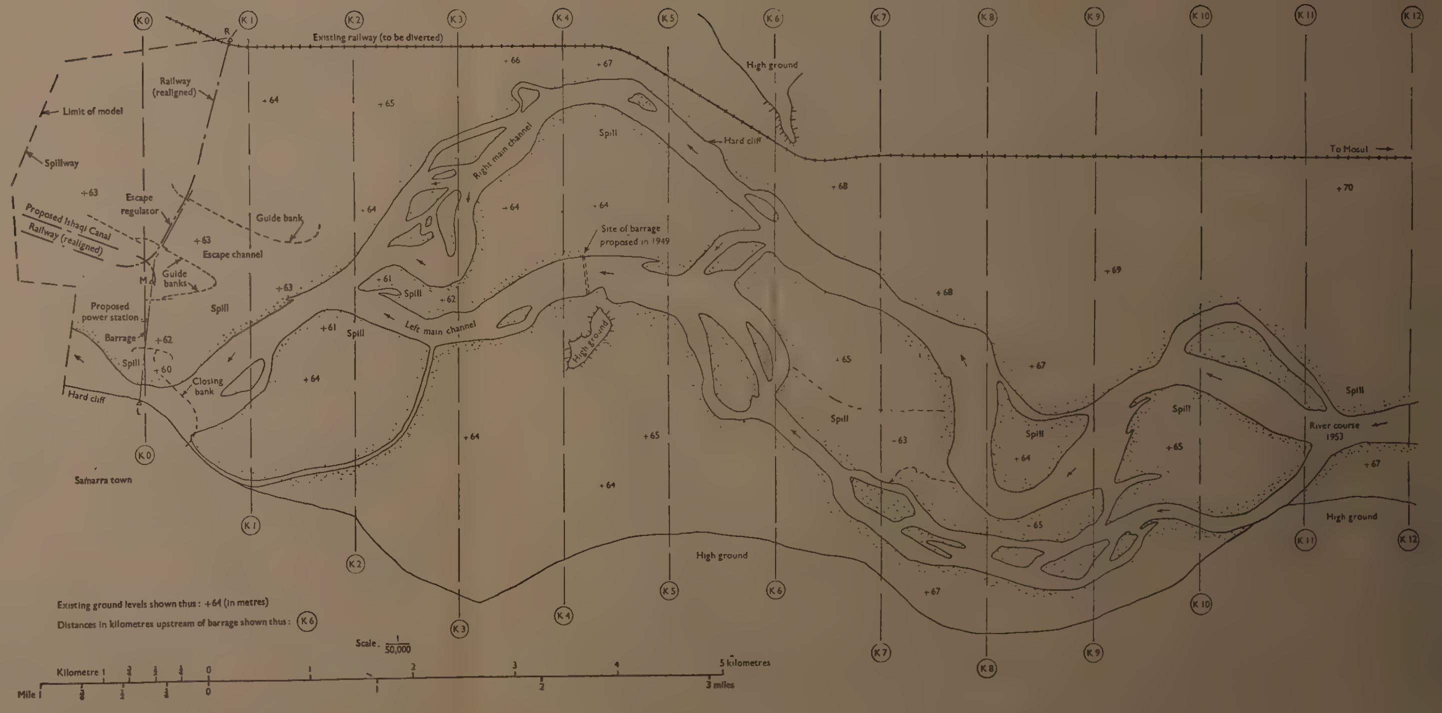

Fig. 3.—TIGRIS RIVER AT SAMARRA (SHOWING LAYOUT OF PROPOSED HEADWORKS)

A. R. THOMAS

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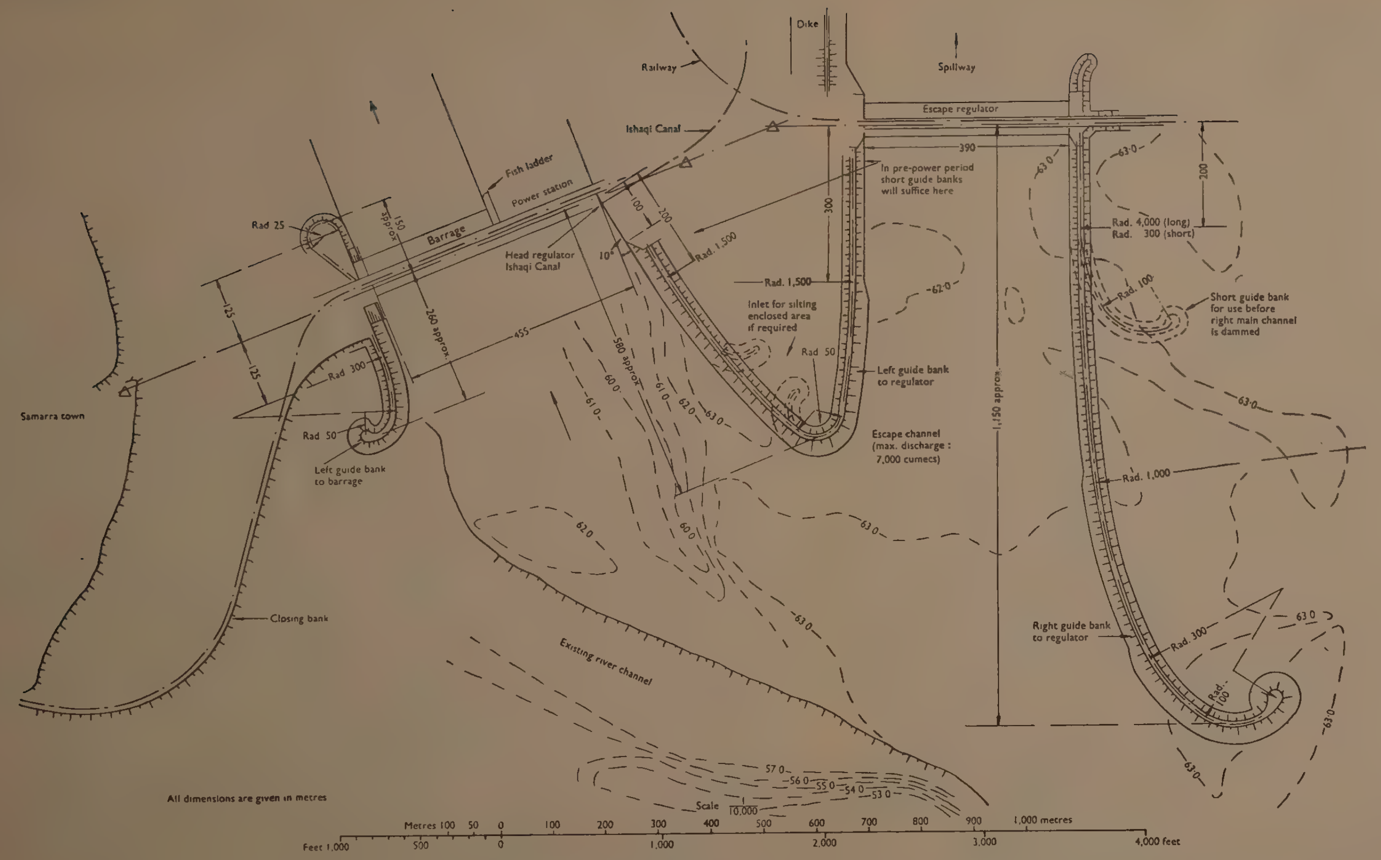

Fig. 10.—PROPOSED LAYOUT OF HEADWORKS—BASED ON MODEL EXPERIMENTS
(The escape regulator capacity in these experiments was 7,000 cumecs. For a capacity of 9,000 cumecs, as proposed later, the escape channel would be

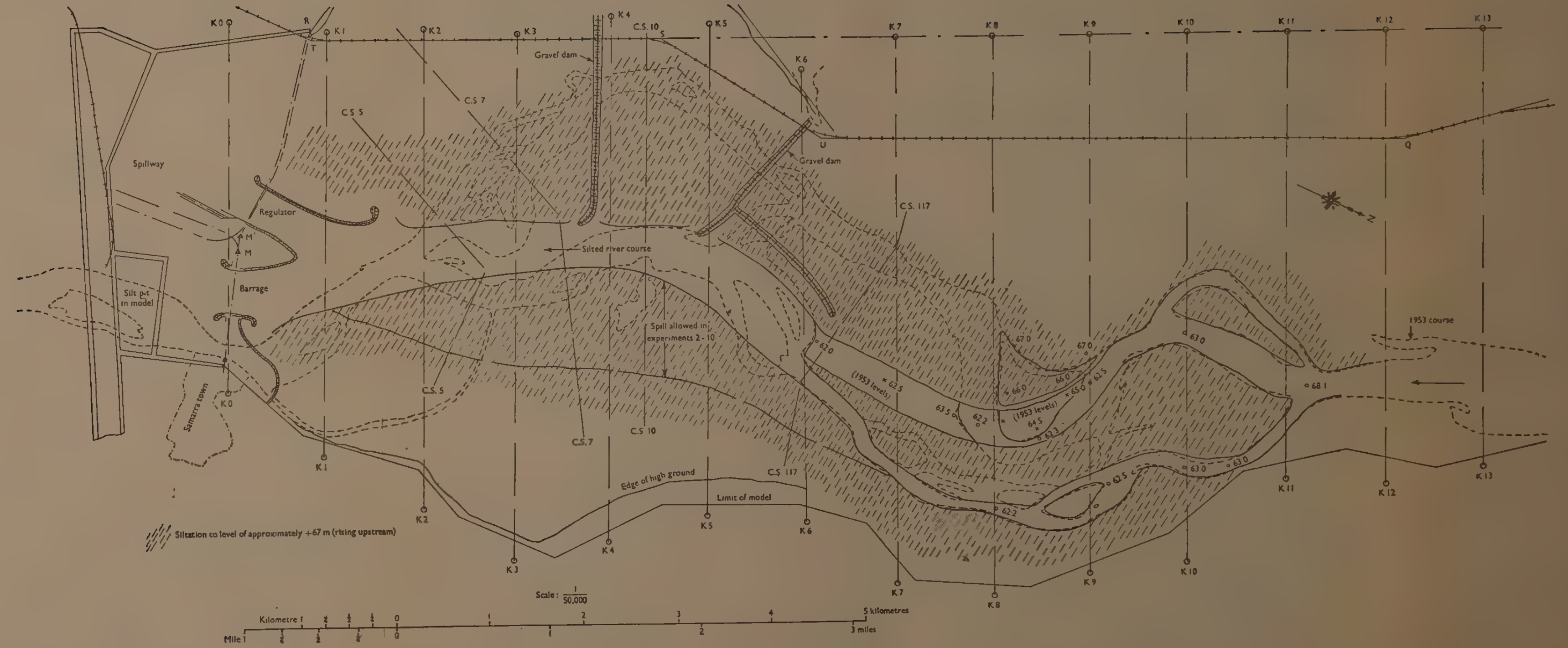

Fig. 11.—FUTURE RIVER COURSE THROUGH SILTED POND INDICATED BY MODEL EXPERIMENTS

o Sir William Willcocks's service under the Turkish Government he had been for many years in Egypt, where an almost identical proposal had been put forward that the Wadi Rayan, a very big depression, though not so large as the Wadi Tharthar, should be used for flood control on the Nile. Mr Lacey said that the capacity of the Wadi Rayan depression was between 25 and 30 $km.^3$

The Author, in estimating his flood discharges, had relied on two sources of information: records of existing floods, and empirical flood equations derived from large floods in other countries. He had truly said that to pay too much attention to those might mean legislating for a flood which was in fact an impossibility. Like Mr Hardy, Mr Lacey was somewhat suspicious of the diagrams for the smaller rivers in which the discharges were plotted on a probability basis. It was quite clear—no one was to blame for it—that the observations had been made over such a short period of time that it was impossible to assess with any attempt at accuracy what the maximum flood was likely to be. In particular, the most disturbing results had been produced in the case of the Adhaim. The Author had relied on certain equations such as that of Myers and Inglis for maximum floods. Presumably an equation of that type was a kind of "worst ever" equation and the locus of all the worst recorded floods which had occurred anywhere. It would be useful in an examination of the kind in question if the Author were to draw a diagram showing on it the Inglis equation and others for maximum floods, and were also to plot on that diagram the actual points on which the authors of those empirical equations had relied.

If an equation had been derived, for example, from American climatic conditions, it would have to be borne in mind that those were very different from those elsewhere. Recently, as was well known, owing to a hurricane proceeding up the east coast of the United States there had been at one place a catastrophic flood of a very unusual character. That had been due to a hurricane, and abnormal occurrences of that particular kind did not take place in Iraq. It was also well known that Sir Claude Inglis had produced his equation for fan-shaped catchments in India. The great majority if not all of them had no snow on them, so that that difficulty was disposed of, but there had been a very heavy Monsoon. When use was made of such an equation, therefore, it was not a case of comparing like with like. If dealing with catchments of the kind with which the Author was concerned, Mr Lacey would prefer to plot on a diagram observations which he personally knew something about or could identify, and where he knew the countries concerned, rather than rely too much on an equation. It was true, of course, that the equations might be on the safe side.

An attempt had been made to align maximum discharges as determined on a probability basis with empirical equations. For example, in the last column of Table 5 there was a coefficient corresponding to a 0·2% flood. Those coefficients were the signatures of the various rivers. It might happen with rivers such as the Lesser Zab, the Adhaim, and particularly the Khazir, that when that type of coefficient was applied with one type of equation they would be more or less in alignment, but with another they would disagree. Further, if a frequency of 1 in 50 or 1 in 100 instead of 1 in 500 were adopted, entirely different coefficients would be obtained.

Reverting to the question of hydrological observations, Mr Lacey thought it would be clear how very necessary it was that they should be systematically made and placed on record. Nowadays it was possible by means of models to produce results fairly quickly, particularly with tidal models, and years might be reproduced in a few weeks. Given the men, money, and materials it was also possible to carry out works far more quickly than one could have hoped to build them in the past. There was, however, no conceivable means of accelerating hydrological observations. If there were none, there was no quick way of rectifying that omission. He emphasized that because numbers of observations were now being made in the United Kingdom but, should there be an economy drive, it was just that kind of observation which might be stopped. That had happened in the past, and one of the saddest entries which an engineer could find in a record of rainfall or of river discharges was a whole decade during which observations had been

discontinued, though afterwards started again. That was a crime against hydraulics and hydrology, and he mentioned it because one never knew when and where the economy axe might fall.

Mr F. H. Allen (Assistant Director, Hydraulics Research Station) made a plea for more retrospective Papers describing the behaviour of schemes of the same type and showing in what way performance had matched prediction. That seemed to be a matter of tremendous importance and interest.

The model used by the Author had had a horizontal scale of 1/200 and a vertical scale of 1/40, which meant that the vertical exaggeration was 5. Those were large scales for a river model by any standards, and although a relatively short length of river had been reproduced the mobile bed model, built in the open air, was in fact 200 ft long and up to 80 ft in width. Need the scales have been quite so large? It was stated in the Paper that the model, for very good reasons, had not reproduced silting in the pond and had not been used for predicting the future course of the river or the future flood-water levels. In connexion with the entry to the regulator, experiments had been carried out to examine the scour which occurred at the entrance with various types of guide wall, but the Author had indicated that they were qualitative results. In addition to those qualitative experiments on scour at the entrance to the spillway channel, the model seemed to have been used primarily to examine the lines of flow at various critical points in the approaches to the proposed structures.

That was a very important function and one which he believed that only a model could perform satisfactorily, but it seemed possible that a somewhat smaller scale—perhaps 1/400, or half that actually used—might have been adequate for an investigation primarily concerned with lines of flow and qualitative degrees of scour. A scale of 1/400 would have meant that the area occupied by the model would be reduced four times, and presumably the cost of constructing the model would also have been substantially reduced.

It might be that labour in Iraq was very much cheaper than in Britain, and that would be a very good reason for constructing such a large-scale model—because he would be the first to agree that the larger the scale the better. What were the Author's motives in selecting such large scales and operating such a large model?

Mr P. O. Wolf (Lecturer in Fluid Mechanics and Hydraulic Engineering, Department of Engineering, Imperial College of Science and Technology), said that the Paper contained a painstaking analysis of limited data and showed the Author's far-sighted anticipation of the problems arising out of the partial control of the Tigris river system in future.

The limited hydrological data on which the Author had had to work had been brought to Mr Wolf's attention 4 or 5 years earlier when Mr Binnie had given him some of the flood records of the rivers which the Author had mentioned. Mr Wolf recalled having spent a great deal of time, particularly on the Adhaim record, which in his opinion defied precise treatment. In Fig. 5 the Author had made the best of what was obviously a bad job. It was unreasonable to expect an accurate forecast of floods on a river about which so few and so variable data had been collected and the method adopted in the Paper was not the only way of plotting the points which the Author had had at his disposal. Mr Wolf had tried about half-a-dozen, and none had been more successful than that used by the Author. It was therefore particularly important to heed the Author's warnings, not only of the possibility of error when the sample was too small but even more of the possibility of cycles of high flow and low flow: a sample which was fairly consistent in itself and gave a straight line on the Author's diagram might be completely different from another sample obtained 10–15 years later.

Mr Wolf particularly appreciated the qualitative discussion of the factors affecting forecasts of floods. There was now no doubt that further readings and observations would be collected, and a quantitative analysis was required at the earliest possible date. As soon as further surveys had been carried out, both of the river and of the meteorological and hydrological conditions, a better estimate of floods would be forthcoming. By then,

presumably, the large reservoirs, which would help to reduce flood peaks, would have come into operation. At the same time, of course, the total flood quantities would still occur, though their discharge would be spread over longer periods; their control might assist in dealing with the silting trouble, both in the river downstream of the barrage and in the flood spillway. A fully efficient electronic flow-computer, as mentioned by the Author, could not be devised until the results of the complete survey were available.

He had been puzzled to see on p. 335 that there was such a rapid change from 3–5-cm.-dia. gravel at Samarra to fine sand at Beled a short distance downstream. It was usual to associate the grading of the bed material with the hydraulic features in the river, and there did not seem to be any evidence of vast changes in the hydraulic conditions in the river which would justify such a change in bed material. He was particularly curious about it, because Fig. 7 showed a longitudinal section, though not extending far downstream, with a slope below the average, which did not indicate any particularly high velocities or scouring capacity at Samarra. The Author had mentioned Lacey's silt factor f, and Mr Wolf wondered whether or not the relation between f and the diameter of the predominant bed material was in fact satisfied by those very large pebbles: could Mr Thomas quote the numerical results of his analysis?

Had the model been constructed in accordance with the Author's assumption of silt banks forming in a certain way on both sides of the main channel? The Author controlled the flow on the right bank by his embankments, and silting would be expected to occur there; but had the somewhat speculative remark which the Author had made on p. 350 to the effect that different river channels would give better results "in all respects" with regard to silt behaviour, implied that the Author had in fact tried different river-flow patterns and different silting patterns in the pond upstream of the barrage?

Another point of interest was the possibility of scouring the channels downstream of the barrage. Presumably the volume of water in the pond formed by the barrage was much too small to send a flood wave down one channel or the other of sufficient magnitude and duration to make any difference to the carrying capacity: the silt would be picked up immediately downstream of the barrage and deposited a short distance further downstream; but would it be possible so to control the discharges from the upper reservoirs, say during the flood season, that the silt washed into the pond could be scoured right through by dropping the pond level for a short period. Could that artificially high discharge be expected to flush the river channel for many hundreds of miles? Naturally the performance of the whole scheme would depend very largely on the intelligent operation of the various barrage and regulator gates and power-station and canal take-offs. Had the Author an optimum operating cycle in mind to deal with the silt problem, which was a very serious one? As Mr Hardy had said, it could happen that silting would ultimately make the flood conditions worse than they had been before.

On questions of detailed design would the Author describe the silt-excluder tunnels he had mentioned? Presumably it was a question of tunnels constructed to intercept silt in front of the intake to the turbines and in front of the canal intake. With the layout shown in Fig. 10, the area marked A in Fig. 19 would be the one to be kept clear by the silt excluders, but if there was a tendency to build up silt across the intake to the regulator canal, would it not be equally possible to build a silt excluder at B? Another method which had occurred to Mr Wolf was a low-level scour culvert C–D from the front of the regulator back to the river. That would naturally require the distance between the regulator and the power station to be reduced as far as possible.

The Author had not stated to what extent he had been free to alter the whole layout on the basis of his model experiments. Mr Wolf, therefore, asked his indulgence in connexion with his final suggestion. A great deal of the silt trouble mentioned in the Paper was due, no doubt, to the flow approaching the works more or less in a straight line, as shown in Figs 13 and 14. Fig. 20 showed a revised layout based on the well-known fact that sediment tended to be moved from the outside towards the inside of a bend, and was consistent with the site conditions shown in Figs 3 and 11. The regulator R was brought right up to the main river channel, and its gates would be longer and shallower, so leaving

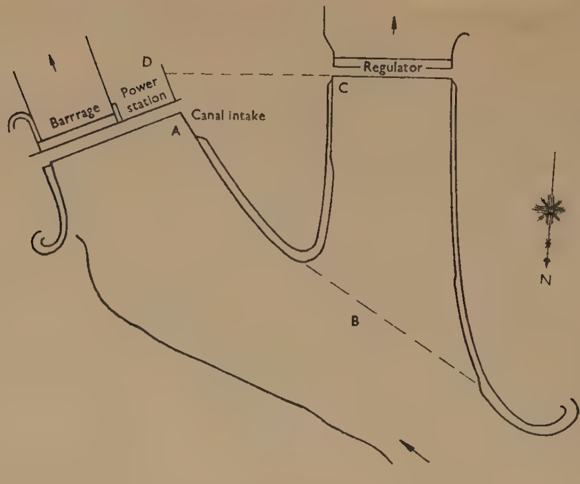
D
Power station
Barrage
Regulator
C
Canal intake
A
B
N

Fig. 19

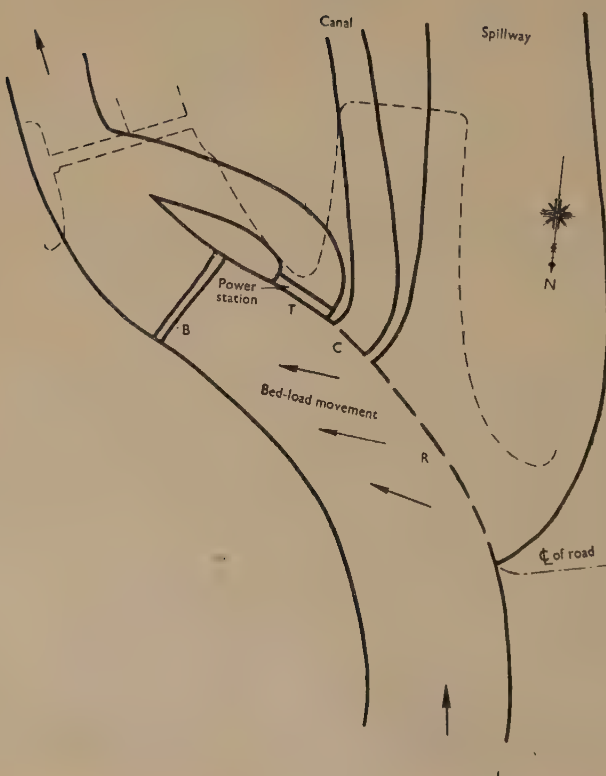
Canal
Spillway
N
Power station
T
B
C
Bed-load movement
R
℄ of road

Fig. 20

he sill to act as a silt-excluding step. There was no area of dead water in which silt ould settle when no water was drawn off to the spillway. The canal intake C and the urbine intakes T were also sited on the outside of the bend, whereas the deepest gates always open) of the barrage B were on the inside of the bend. Those deep gates would lso have the greatest draw on the silt-laden density layer on the floor of the pond. Naturally the road running across the works would require careful re-alignment.

Mr Herbert Addison said that the Author had pointed out that in the original lesign for the barrage normal sluiceways had been provided, but that when it had been ound that a greater discharge would be allowed for, bellmouth inlets had been added in he model, and Fig. 8 seemed to show that the rounded inlet had been quite effective, ecause, scaling off values from the discharge curve, the coefficient of discharge of the luiceways acting as orifices seemed to vary from 0·93 to 1·03. That variation might be ccounted for by the small scale of the diagram, but it suggested that the expedient in uestion had been quite effective.

Although the Paper was so comprehensive, it did not deal with the conditions on the lownstream side of the barrage. It would have been noted from some of the Author's hotographs that on the upstream side even the natural flow of the river had been suffiiently great to cause quite deep scour. If that were so on the upstream side, would not here be much more serious scour on the downstream side when the water issued from the ents under a head of 4 or 5 m? Although possibly the question of the downstream rotection of the barrage had not come within the Author's terms of reference, it would ave been interesting to hear what kind of downstream protection had in fact been proided to protect the river bed from the high-velocity jets issuing from the sluiceways.

*** **Mr B. D. Richards** (Consultant, Sir William Halcrow and Partners) was articularly interested in the Paper, because in 1948 his firm collaborated with Messrs oode, Vaughan-Lee and Gwyther in the preparation of a joint Report on control of the igris. That Report recommended the use of Wadi Tharthar depression as a floodtorage basin, the water being disposed of by evaporation and not used for irrigation.

The scheme described followed those general lines but with the addition of a future anal with intake at the barrage for limited irrigation. A future power plant was also rovided for as mentioned in the Report.

The hydrological basis of the 1948 Report was a continuous 5-day hydrograph of river low extending over a period of 17 years. From it a flow/duration curve was prepared nd the discharge in excess of 3,500 cumecs passed into the storage basin was found to be ·5% of the total flow in the period. As the water was subject to continuous evaporation, he amount accumulated in storage at the end of the period would be considerably less.

The Author had brought out clearly the difficulties in the design of the works arising rom siltation and erosion, and the value of the model experiments carried out. The hole basis of the scheme was, however, to provide for the predicted possible floods, and e would confine his remarks to that aspect.

The problems of control of a flood-storage basin would normally require the following ata:

1. The maximum intensity of flood to determine the gate capacity required to pass all floods in excess of a certain discharge.
2. The hydrograph of the flood to determine the total flow of water passing into the basin.

The Author had approached the problem of maximum flood intensity in two different ays. The first was by "flood frequency". That method had its limitations, paricularly if the records of flood discharge were not extensive. In the case of the Tigris here was a fairly long record including several major floods, and the frequency method hould give reasonably reliable results.

*** This and the following contributions were submitted in writing after the closure f the oral discussion.—Sec.

His other approach was the use of a formula $Q = CA^{\frac{1}{2}}$, in which there was only one variable factor—the area of the catchment A. All other variables such as rainfall intensity and distribution, and the shape and slope of the catchment were covered by the coefficient C. That coefficient would have a value peculiar to the particular catchment and values found for other countries and catchments would not be applicable. The determination of the correct value of C would require as much data as would the estimation of the maximum flood intensity.

Provided that sufficient rainfall records were obtainable, Mr Richards thought that the intensity might be determined by the method he had developed.[8] From the rainfall records, the coefficient of rainfall R could be assessed for various recorded major floods and for the maximum catastrophic flood. The other coefficients—shape of catchment and slope—being determined from the maps, there remained the coefficient of run-off K. Inserting the coefficients, assuming values of K and equating to a major recorded flood, the correct value of K could be found by interpolation and checked against a second recorded flood. Then using the value of K found and the maximum value of R, the maximum flood could be estimated. The hydrograph of the flood could also be drawn by the method shown and the actual flow passed into the flood storage basin determined.

The storage in the basin was cumulative, and at the same time it was being depleted by evaporation at a rate dependent on the varying level.

Given a long-term hydrograph of river flow, the water stored at any time in the basin could be determined by a step-by-step process, but since the capacity of the basin was very great, that was probably not significant.

Mr H. W. Stephenson (Messrs Maunsell, Posford & Pavry, Consulting Engineers) observed that the Author, in dealing with the problem of silting in the escape channel upstream of the regulator, stated that a bar might be expected to form across the mouth during the long periods when the escape channel was not in use. The question arose whether a bar would form in the manner predicted or whether silting would proceed throughout the whole of the inlet channel upstream of the regulator. The Author's conclusion rested on an analysis of flow lines and experience of the existing Habbaniya scheme, where a bar was stated to have formed across the mouth of the escape channel.

The Habbaniya scheme, as stated by the Author, was similar to the Wadi Tharthar project. A regulator and escape channel stood in the same relation to river and barrage, but the alignment of the escape channel was rather more favourable at Habbaniya because it was tangential to a bend in the river, offering a straight approach to the flood flow. The inlet regulator was completed in 1951 and thus offered evidence of the silting during the past 4 years.

An observation made in November 1955, possibly later than the Author's, showed that so far from forming a bar across the mouth silt had been deposited more or less uniformly over the whole of the area of the escape channel upstream of the regulator to a depth of between $1\frac{1}{2}$ and $2\frac{1}{2}$ m. The quantity of silt was about $\frac{1}{4}$ million cu. m; it was clear that it would throttle the discharge of the channel to a fraction of its design value. Even if natural scouring alone was relied upon to clear the deposit it was doubtful whether that it would take place, and the discharge capacity be fully restored, in time to intercept the main flood peak. Because scouring in the Samarra model under those circumstances was quick it could not be taken to demonstrate that it would also be quick in nature, since silting and erosion were not correctly reproduced in the model.

It appeared that if similar silting occurred at Samarra the deposit might throttle the escape channel to about 2,000 cumecs, against the (original) design of 7,000 cumecs.

He believed the problem was to be met at Habbaniya by removing the existing deposit first by mechanical plant, and in future by means of the newly completed barrage to raise the pond level and purge the escape channel at intervals. At Habbaniya there were arrangements for returning that water to the river downstream, but at Samarra the procedure would, in the months of low water, prove a loss of valuable irrigation water.

[8] References 8 *et seq.* are given on p. 365.

Mr F. P. Dath (Civil Engineer, Central Electricity Authority) commented on the ›vidence of catastrophic floods in the past". Sir Leonard Woolley, the eminent archae-ogist, writing on his excavations at Ur about 1930,* had stated that he had encountered layer of silt about 8–9 ft thick, which had been deposited by some abnormal flood.† nderneath that silt layer he had found further irrefutable evidence of a former civiliza-on, estimated to be about 3000 B.C.—the time of the biblical Great Flood.

In recent years the spade of the archaeologist had, on many occasions, revealed the ficacy of the ancient records contained in the Holy Writ. With reference to the remarks the Ark, the record clearly stated the measurements as being 300 cubits in length,) cubits in breadth, and 30 cubits in height (approximately 546 ft, 91 ft, and 55 ft spectively). Mr Dath added that sails were not mentioned in any form. Therefore he ıbmitted that a craft of that form would not be affected by a prevailing wind stated by r Hardy as coming from the north-west, but it would be affected more by the flow of ıe flood waters which was in the opposite direction. Mount Ararat was situated approxi-ately in that direction and therefore the Ark had eventually come to rest on the slopes that mountain as intended. In conclusion, he added, the record of the Flood stated ıat it "covered the world that then was".

Mr A. A. Middleton (Partner, Sir Murdoch MacDonald and Partners, Consulting ngineers) observed that the Author, in his introduction, had said that the principle the Wadi Tharthar scheme was of great interest and almost unique, and that the only ther existing scheme of the kind known to him was that at Habbaniya on the Euphrates, ıough the same principle might have been used in works constructed on the Tigris and ıe Euphrates in ancient times. Mr Lacey had said in the discussion that another scheme the same principle had been proposed for the Nile at Wadi Rayan, and thought that ıat scheme, like the Wadi Tharthar, had been first suggested by Sir William Willcocks. Sir William Willcocks had, however, himself recorded[9] that the Wadi Rayan Scheme ıd been suggested by an American gentleman, Mr Cope Whitehouse, in 1882, and it ight well have been first thought of at an even earlier date, for he had also recorded ıat the fame of the ancient Lake Moeris had made a profound impression on Mohammed li, who had urged on his Chief Engineer, Linant Pasha, the necessity of undertaking simi-r works on the Nile some time before 1870.

The ancient Lake Moeris (now the Fayoum) was supposed to have been used as a flood cape for the Nile in the time of the Pharoahs in the same way as was many centuries ter proposed for the adjoining depression, the Wadi Rayan. That may well have been ıe earliest scheme making use of the principle now adopted for the Wadi Tharthar.

The Author, in reply, thanked Messrs Lacey and Middleton for the information ey had provided in regard to the use of the Wadi Rayan depression and the ancient Lake oeris for flood control in Egypt, which was of considerable interest in view of the unusual ethod of disposal of flood water.

Mr Hardy had been searching for factual evidence of catastrophic floods. The Author

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* From 1922 to 1934 Sir Leonard Woolley conducted the excavations at Ur for the ustees of the British Museum and the Museum of the University of Pennsylvania.

† Quoting from Sir Leonard's writings:—"The bed of water-laid clay deposited against e sloping face of the mound, which extended from the town to the stream or canal at e north-east end, could only have been the result of a flood; no other agency could ssibly account for it. Inundations are of normal occurrence in Lower Mesopotamia, t no ordinary rising of the rivers would leave behind it anything approaching the bulk this clay bank: *eight feet of sediment imply a very great depth of water, and the flood which posited it must have been of a magnitude unparalled in local history.* That it was so is rther proved by the fact that the clay bank marks a definite break in the continuity of e local culture; a whole civilization which existed before it is lacking above it and seems have been submerged by the waters. . . ."

could not claim such extensive knowledge of Iraq but he thought that with its written history extending back about 5,000 years Iraq was in a unique position for the study of flood statistics. He agreed with Mr Hardy that a distinction should be made between floods which were so high that they were inevitably catastrophic and floods which were catastrophic only because of the lack of normal precautions. Where reliance had to be placed on historical records, however, the information available generally referred more to the results of the floods than to the rainfall and river levels giving rise to them, and the latter remained a matter of surmise. Even so, such information was of value and it would be a great service if someone would delve into history and throw some light on the frequency and (where possible) magnitude of high floods in the past.

The Great Flood of biblical times might or might not have occurred on the Tigris but it was very interesting to note that further evidence of such a flood had come to light in the last century as a result of Layard's discovery at Nineveh of clay tablets inscribed with a very similar story in some detail.[10] In that case, however, the storm had raged for only 6 days, the ship had been floated at Shuruppak in what was now lower Iraq and grounded on the mountain of Nizir, after which had followed the dynasty of Kish, which was a place near Babylon about 100 miles to the north-west of Shuruppak. It was generally southerly winds which brought rain in Iraq, but the conditions giving rise to such abnormal rainfall as caused the Great Flood might well produce winds different from those generally observed, so it was difficult to draw conclusions from the direction of travel of the ship.

Evidence of considerable floods had also been provided by excavations at Ur, Warka, Kish, and Farah, which revealed deposits of silt overlying remains of human occupation.[10] The Author was grateful to Mr Dath for the information he had contributed in that connexion. Whatever was thought about the biblical story it seemed likely that there had been at least one extremely high flood which had been catastrophic in its effect, probably over a large part of the country.

There were also the remains of breached dams in various parts of Iraq, though many of them may have been destroyed through neglect. In 1831 there had been a great disaster in Baghdad when the river had broken into the city and thousands of people had lost their lives. It was not known, however, what the river discharge had been and a contributory cause of the flooding might have been the neglect of the flood banks owing to a severe plague epidemic which had been raging in the city at the time.

It would naturally be expected that much higher floods had occurred in the past 5,000 years than in the past 50 years of records and the evidence supported that view. Though there might not be factual proof of them the Author considered that where the safety of populated areas was concerned it would be taking a grave risk to assume that such higher discharges could not occur.

When it came to estimating future maximum discharges, however, the designer had very inadequate data to work on. Doubts had been expressed on the value of the statistical method and they were justified to the extent that it was not a method to be used without consideration of other factors. Every method of flood prediction had its limitations; the occurrence of floods was in a large measure random and all that could be done was to estimate probabilities. There was no exact method of doing it and the best procedure generally lay not so much in the choice of one suitable method but in giving consideration to all data having a useful bearing on the question and drawing conclusions from the evidence provided. In that the statistical frequency method provided a unique line of approach; it took account of the local meteorological conditions and catchment characteristics without any assumption of run-off or other coefficients which might be a source of error, and the results were presented in the form of a probability curve which was a form eminently suitable for the estimation of a probability.

The limitations, however, should be recognized. It might be a reasonable assumption that over a long term of years the frequency distribution of rainfall and run-off in the future would approximate to that in the past, but the application of the method depended in most cases on a period of record which was short in the context of the risk to be estimated. Whilst, therefore, there was a probability that the records provided a sample of

ata sufficiently representative of the long-term pattern there was a small but significant hance that it might be quite unrepresentative and give false indications. The smaller he sample the greater was that chance and it appeared that in the case of rainfall and floods he chance was greater than would be the case in completely random events. The Author ad made the reservation that the period of record might be one of subnormal precipitation (it might also be one of above-normal precipitation) and Mr Wolf had referred to the ossibility of cycles of high and low flow. In another Paper presented in the current ession[11] Dr Hurst had shown that annual rainfall was not completely random in sequence nd that there was a tendency for high and low values to occur in groups. It did not follow hat the statistical method was unsuitable for the estimation of flood frequencies but ather that the probable errors of the mean and standard deviation calculated on the basis f small samples were greater than would be expected from the theory of random errors.

Mr Hardy had called attention to the anomaly of the flood data of the Adhaim (Fig. 5) n which the 1952 flood stood out much above the others and, when taken into account, esulted in a considerable change in the magnitude of extrapolated estimates. The Adhaim data were of concern in the investigations described in the Paper in so far as they rovided an analogy with the part of the Lesser Zab catchment below Dokan, but they rovided an example of an abnormal sample, as would be seen from the frequency distribution given in Table 8.

TABLE 8

Discharge: cumecs	0 to 300	301 to 600	601 to 900	901 to 1,200	1,201 to 1,500	1,501 to 1,800	1,801 to 2,100	2,101 to 2,400	2,401 to 2,700	2,701 to 3,000	Total
Number of years	5	3	2	4	6	0	0	0	0	1	21

Mean discharge: 917 cumecs.

n the statistical method too much importance should not be attached to the extreme alues in a series of data because the relative deviation from the mean trend was likely o be greater where the frequency of occurrence was less. It was always possible that a are flood might occur during a short period of record and the maximum might be a 20-ear, 100-year, or 500-year flood. The frequency diagram would, however, show whether flood was out of line with the pattern indicated by the other floods and allowance could e made by reducing the weight of a rare event, as had been done in the case of the Khazir Fig. 4). That provided the answer to Mr Hardy's criticism in respect of the effect of ne year's flood on the estimate. In the case of the Adhaim the Author preferred to make o adjustment in the weight given to the 1952 flood because the result indicated by the raph in Fig. 5 was consistent with other data of the Tigris catchment (see Table 5).

From what had been said, however, it was evident that the statistical method used alone as incapable of providing a firm estimate of flood risk within a given margin of error. t provided valuable information in terms of probabilities but should supplement or be upplemented by a different approach to reduce the chance of gross error. For that purose the run-off coefficient C for the estimated 0·2% floods had been calculated as shown Table 5, column 8. To facilitate a comparison with data of maximum discharges ecorded elsewhere, the estimated floods were shown in Fig. 21. The greatest observed ainfall intensities had invariably occurred under summer or tropical conditions, owing o doubt to the greater quantity of moisture which could be held by a column of warmer ir. The Tigris floods resulted only from precipitation in the winter and spring, and melt-g snow made a contribution to the run-off; no appreciable rainfall occurred in the catchent during the summer. Fig. 21 showed the North American winter and spring maxi-um flood discharges given by Creager, Justin, and Hinds[12] and it would be seen that the oints lay beneath an envelope curve representing a coefficient $C = 6{,}000$ in the formula

given on p. 331. The values of C for the estimated 0·2% Tigris floods given in Table 5, column 8, could be compared with that value, and to show those estimates against the general background they were plotted in Fig. 21.

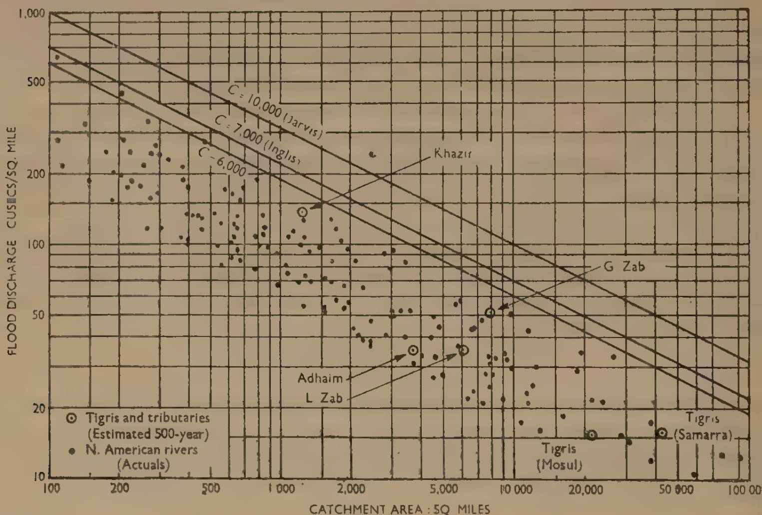

FIG. 21.—ESTIMATED 500-YEAR MAXIMUM FLOOD DISCHARGES OF THE TIGRIS AND TRIBUTARIES COMPARED WITH MAXIMUM OBSERVED WINTER AND SPRING DISCHARGES OF NORTH AMERICAN RIVERS AND GENERAL FORMULAE

The Author appreciated Mr Lacey's suggestion, which was partially met by Fig. 21, in which the curves for $C = 7{,}000$ (Inglis) and 10,000 (Jarvis) were shown. The data on which those formulae were based included many discharges of floods occurring under summer or tropical conditions and diagrams such as Mr Lacey had suggested were to be found in the original publications.[7, 12] As inferred by Messrs Lacey and Richards, those values of C represented envelope curves and did not apply in respect of a specified probability to all catchments. The pattern of plotted points, each representing an observed maximum flood discharge, reflected both the probability function and variation in the local characteristics of precipitation and catchment. The formulae and diagrams were, nevertheless, of practical value in providing a basis for comparison of results obtained by the statistical and other methods.

An approach on rational lines, such as by Mr B. D. Richards's method, would have had much to commend it; such methods were often the most reliable but they required the estimation of rainfall and snowfall intensities likely to give rise to extreme floods and the adoption of run-off coefficients. In the absence of data for the particular region those would have been a matter of some speculation and the values adopted would necessarily have been based on world data. In the case of the Tigris, not only were the rainfall records of short duration but they were too few to be representative of the catchments. The Author hoped that that deficiency would soon be remedied and he understood that steps were being taken to increase materially the amount of hydrological data collected in Iraq. Though long-term records would still be lacking it would then be possible to correlate flood discharges with precipitation and so predict flood hydrographs from hypothetical precipitations.

The Author fully agreed with Mr Allen's remarks that much could be learned from the

rformances of schemes which had been constructed in the past. It would be very in-ructive if the experience gained on schemes of the type dealt with in the present Paper ere published after a few years' operation. Mr Allen had queried the need for a large-ale model. It was true that the size of the model was large but that was because of the eat width of river valley included. The model discharges were not excessive—for ample, the discharge of 3,000 cumecs which had been passed through the barrage in veral experiments was represented by 0·06 cumec (2·1 cusecs). The experiments could doubt have been done with a smaller model but at some sacrifice in accuracy. In rticular, either the vertical exaggeration would have been excessive, resulting in a parture from similarity of flow pattern, or if the vertical scale were correspondingly duced the Reynolds Number and silt-carrying capacity of the flow would in places have en inadequate and surface velocities would have been more subject to error caused by nd. Contrary to Mr Allen's assumption the model had been used for predicting silt-posit tendencies and the future course of the river upstream.

Mr Hardy had referred to the dangers of decline in capacity of the downstream river by tation. The Author was glad that his reference to that question had received emphasis. dging by experience on Indian rivers there would first be a period of retrogression in vels while the greater part of the silt load was being deposited in the pond, but that ould be followed by a rise in levels, progressing to higher levels than now existing. xcessive silt load would be indicated by a wider shallower river with steepening slope. lt deposit in the pond could not be avoided and after siltation the pond would not con-in sufficient water for flushing the river channel. The release of discharge from upstream servoirs would be limited by gate capacity and operational factors and reliance would ve to be placed mainly on control at Samarra during floods. It seemed that the best urse would be to avoid splitting moderate flood discharges by delaying the diversion of flood until it became absolutely necessary, but the programme for operation to the best lvantage would no doubt have to be the subject of study based on experience and the scharge capacities of the river and spillway.

The change in bed material to which Mr Wolf had referred had been observed in other vers with boulder or gravel beds. It was associated with a flattening of the hydraulic adient and was known as the gravel "node". The bed material in most rivers became ner downstream but the interesting point with gravel nodes was that the change from avel to fine sand occurred within a very short distance.

With regard to the relation between the size of bed material and Lacey's f, it was fficult to determine with any accuracy the dominant size of gravel, or indeed the mean ameter, in view of the variation between samples. Table 9 gave extracts from the alyses of three samples of gravel from the river-bed.

TABLE 9

Sample No.	44	47	54
Distance upstream from barrage: km	2·6	4	5
Percentage by weight finer than			
0·15 mm	3	7	5
0·6 mm	4	15	13
2·3 mm	4	19	16
9·6 mm	8	40	40
25 mm	63	93	78
50 mm	93	100	92
Weighted mean dia: mm.	25	13	20

he value of f determined from the mean velocity V and hydraulic mean radius R (i.e., $_R=V^2/1{\cdot}3R$) in the case of rivers with boulder or gravel beds varied with the stage

and rate of bed movement. If the value was to be representative of the river the stage should be that of the mean channel-formative or "dominant" discharge, and that was often difficult to determine. Assuming that that condition prevailed in the data of Table 10 for the Indus and Miami rivers,[13] both with gravel beds, those provided a comparison with the Tigris (all in ft-sec units):—

TABLE 10

River	V	R	S	f_{VR}	n (Manning)	Bed material
Indus at Kalabagh (high flood)	13·1	28·8	0·00091	4·5	0·032	Shingle and coarse gravel
Tigris at Samarra (10,000 cumecs):						
Single channel	7·2	15·1	0·00042	2·6	0·026	Shingle and gravel (see analyses above)
Mean of two channels	5·9	13·4	0·00042	2·0	0·029	Shingle and gravel (see analyses above)
Miami at Dayton (near full stage)	6·0	12·1	0·00035	2·3	0·025	Clay, sand, and gravel

In reply to Mr Wolf's other questions, the whole of the flooded area in the model had been raised to represent general siltation. The positions of the banks and the channel course and shape had been evolved by a step-by-step method, noting where silt would deposit then building up by hand and allowing to scour. It was not considered necessary to experiment with the main approach from the left because that approach was not likely to occur and if it did it would clearly be more favourable.

Sand or gravel excluders had for some years been built in India at the headworks of canals. They had been developed in the Punjab and had in general worked very satisfactorily. They consisted of tunnels which drew off the bed flow in which the concentration of coarse sand was greatest. Similar tunnels had been used in other parts of the world, particularly in connexion with hydro-electric schemes. It would be possible, as Mr Wolf had suggested, to build excluders across the escape channel, but that would be rather an unwarranted expense, particularly since it was not expected to use the escape channel very often. Mr Wolf's proposal (Fig. 20) to increase the curvature of the river to create conditions favourable to the exclusion of sand from escape channel, canal, and turbine intakes were very interesting and appeared to embody the right principles. The layout illustrated, however, would not have been feasible for practical reasons; the barrage had already been sited and the position of the regulator was governed by considerations of railway alignment.

Mr Addison had referred to the bellmouth inlets of the barrage. The shape of the bellmouth had been determined in a model by replacing the diaphragm by a sharp-edged plate and tracing the shape of the jet springing from it. It was not possible to follow that curve all the way because of the space required by the gate mechanism and a minimum thickness of concrete, but provision of the bellmouth as proposed raised the coefficient of discharge from 0·79 to 0·93, representing an increase of 18% in discharge capacity. As suggested by Mr Addison, protection against scour had been required downstream of the barrage. That had been the subject of a lengthy series of model experiments but the inclusion of the results in the present Paper would have taken up too much space. In brief, the protection consisted of a pavement on which excess energy was dissipated in a hydraulic jump, which was stabilized by concrete blocks. Flow leaving the pavement would be sufficiently quiet not to cause deep scour.

In reply to Mr Stephenson, it was true that silt had deposited in the approach channel to the Habbaniya Inlet but the Author knew of no reason to suppose that that had occurred

hen the regulator was closed. He had not seen the latest survey but when he had in-
ected the channel in 1954 there had been no flow and the water in the approach channel
ıd been motionless and clear. The indications were that the silt had deposited when the
let was open. Somewhat similar conditions could be expected at Samarra, with the
rmation of a silt bar across the entrance when the regulator was closed and a tendency
r silt to deposit under certain conditions of flow. The greatest tendency to silt would
during periods of partial opening of the gates, and though such periods would necessarily
cur, for example when the gates were first opened and the spillway was filling up, it
ould clearly be desirable to reduce them to the minimum when the silt content of the
ater was appreciable. It was also probable that the greater width of channel required
r the increase in capacity to 9,000 cumecs would induce silt deposit near the right guide
ınk under certain operating conditions. Further tests with the model could throw
ght on that tendency and indicate the best method of dealing with it.

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The closing date for Correspondence on this Paper has now passed without the
eceipt of any communication.—Sec.

PUBLIC HEALTH ENGINEERING DIVISION MEETING

21 February, 1956

Mr C. A. Risbridger, Member, Chairman of the Division, in the Chair

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Author.

Public Health Paper No. 14

THE CAITHNESS REGIONAL WATER SUPPLY SCHEME

by

*** William Alexander Ronald Baker, B.Sc.(Eng.), M.I.C.E., William Mallinson Jollans, M.A., A.M.I.C.E., and John Neville Dale**

SYNOPSIS

The scheme for the supply of potable water throughout the rural areas of the county of Caithness is based on the exploitation of the rainfall on the catchment of Loch Calder, the water level of the loch being raised in order to provide the necessary initial storage. Only a small fraction of the water so found is eventually treated and pumped into supply, the greater part being employed for the creation of the necessary power for pumping the treated water and for the power, lighting, and heating of the treatment works.

The Paper describes the small dam constructed at the outlet of the loch, the raw-water pipeline to the treatment works, the treatment works and pumping machinery and the building containing them, the distribution network, and the sundry balancing reservoirs.

The reasons leading to the unusual expedient of turbines direct-coupled to pumps are explained, whilst Appendix II describes the resulting problems and their solution in more detail.

Sections dealing with the construction then follow, and the Paper concludes with some notes on the problems encountered in commissioning the scheme.

INTRODUCTION

THE PAPER describes the design and construction of the scheme for the supply of water to the rural districts of the county of Caithness. The population of the county—excluding the Burgh of Wick—is only about 15,500 persons and the scheme is therefore a small one when measured by that yardstick, but the unusual method of procurement and the efficient distribution of the water over considerable distances raised problems requiring considerable thought and of no little engineering interest.

The County Council had long had under consideration the question of improving the rural water supplies, which, except in the case of one or two villages, consisted of springs and shallow wells, the quality of the water from both sources being suspect. In 1943 the Secretary of State for Scotland instituted a complete survey of all water

* Mr Baker is responsible to his employers, Messrs Binnie, Deacon, & Gourley, for the design and subsequent administration of the works. Mr Jollans is the Resident Engineer on the works. Mr Dale is responsible for the design of the machinery for the manufacturers, Gilbert Gilkes & Gordon Ltd.

upplies then existing in Scotland, and in Caithness the survey was carried out by ne of the Engineering Inspectors of the Department of Health for Scotland, Mr eorge Smillie. Mr Smillie proposed, *inter alia*, a scheme for Caithness in which Loch alder was to be the source, and the Department supplied the County Council ith their findings in 1945. The Council then appointed Messrs Binnie, Deacon & ourley as Consulting Engineers to investigate the problem in detail and to prepare a cheme. The scheme so evolved is the subject of this Paper.

The consulting engineers, after a full investigation of possible alternatives, recom- ended a scheme based on the exploitation of Loch Calder but differing in some espects from Mr Smillie's proposals, and their recommendations were eventually ccepted. The adopted scheme consists of raising the top water level of Loch Calder o secure adequate storage, conveying the water through pressure pipelines to a pump- ng station near Hoy, on the Thurso River, and there treating the water and pump- ng it to supply. The bulk of the water is used for creating power to energize the umps and for generating electricity for the local needs in the station. From Hoy he water is pumped to two main reservoirs, whence it gravitates throughout the ounty in a reticulation of mains of various sizes.

The drafting of the Order authorizing the works was complicated by the existing ights of Thurso Burgh to the abstraction of the water of Loch Calder, the raising of he water level of the loch for the County scheme being such as would submerge the hurso headworks. Eventually it was agreed that the Burgh would relinquish their ights and obtain the necessary quantity of treated water from the County Council cheme. The Order was drafted accordingly; it includes provision for compensation ater in the Thurso River and in the Calder Burn (the natural outlet of Loch alder), which is a tributary thereof.

FUNDAMENTAL DATA

Fig. 1 shows the general layout of the scheme, which is to be developed in three hases; construction of only the first has yet been commenced. Phases I and III ill be supplied *via* Stemster hill reservoir, and phase II *via* a reservoir near the hill of noc Dubh.

The population of the county has shown a slight decline in recent years but, it eing anticipated that the decline will be checked if not reversed—the supply of ater, and of electric power from the North of Scotland Hydro-Electric Board will ontribute—it was decided to cater for a population of 30,000. On this basis, with ome allowance for livestock, the required supplies to the various reservoirs were as hown in Table 1.

TABLE 1

	Phase I: g.p.d.	Phase II: g.p.d.	Phase III: g.p.d.
rmlie reservoir (of Thurso Burgh)	350,000	—	—
temster reservoir	700,000	—	180,000
noc Dubh reservoir	—	320,000	—

The top water level of Loch Calder was previously 205·0 O.D., and this will be aised to 216·5 O.D. for the full operation of the scheme. Pending the ultimate

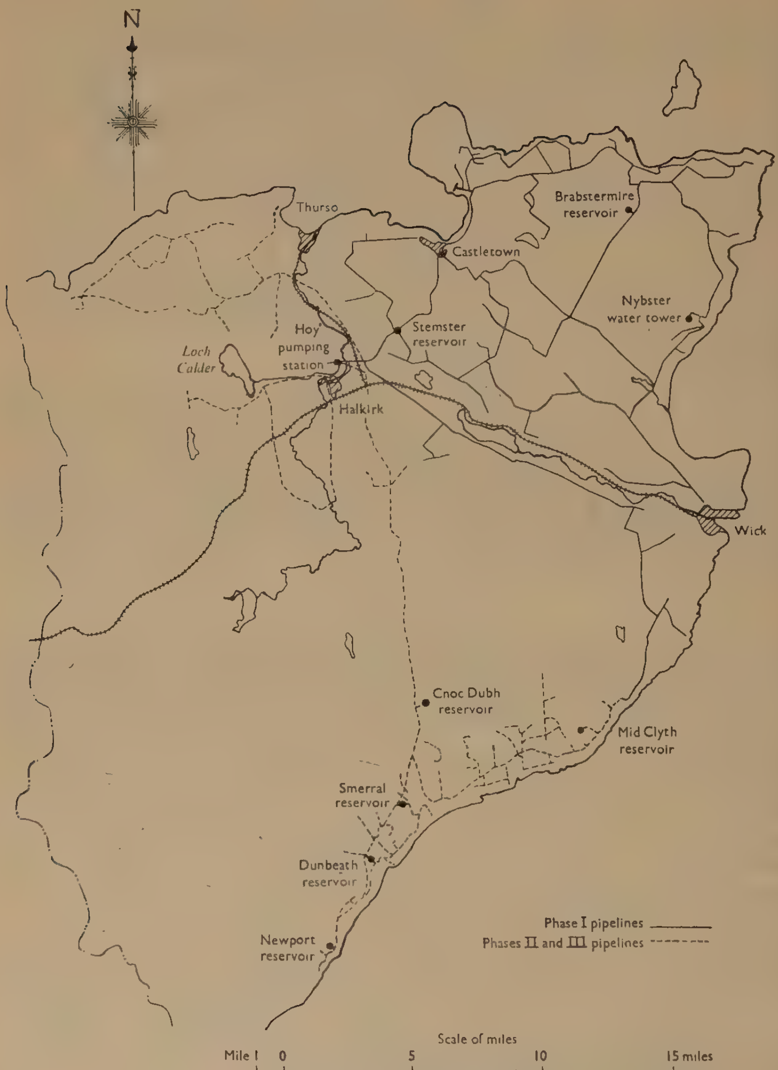

FIG. 1.—KEY PLAN

lemands on the scheme, the loch is being raised only to 211·5 O.D. to avoid inundating and at an unnecessarily early date, whilst also delaying part of the capital expendi-ure. The ground levels at the pumping station site were such as to make 75·0 O.D. in appropriate floor level, the level of the water of the Thurso River ranging from ibout 64·0 to 72·0 O.D.

The character of the Loch Calder water is such that initial sedimentation before reatment would have been desirable, but this would not have been economically ossible. If the sedimentation had been performed at Loch Calder it would have neant the creation of a separate operational establishment, and the water therefrom vould have required an independent pipeline to the pumping station. Had the treat-nent been carried out at the pumping station site, about 140 ft of head would have een lost to the treated water. It was therefore decided to dispense with sedimenta-ion and reduce the rate of filtration appropriately in pressure filters in the pumping tation building.

The common raw-water penstock from Loch Calder to Hoy is about 4 miles long and ith a static head difference of approximately 140 ft surge-pressure rises were bviously going to create an important problem. With a flow of about $5\frac{1}{2}$ million allons daily through one 24-in.-dia. pipeline for phase I, expedients for the reduction f surge pressures from operational interruptions of the flow were not considered racticable; consideration was therefore given to designing the machinery so that such ressures could not occur or, rather, would occur only to a greatly reduced extent. he governing of a hydro-electric generator is obviously the most dangerous cause of irge pressures and it was therefore decided to mount each pump on the same shaft s the turbine energizing it, the usual electrical link being omitted. Beyond this the iatter was left in abeyance until tenders for the machinery were received, so that the iatter could be gone into in detail with the manufacturers of the machinery. In the ise of the Thurso supply, the static level of Ormlie reservoir being lower than that of och Calder, a ball valve at the reservoir made the desirability of a centrifugal pump oubtful and hydrostats were specified. The anticipated power requirements were oout 10 to 12 kW for driving auxiliary equipment in connexion with the water eatment, about 5 kW for lighting, and about 15 kW for heating, and this power was be generated at the station by hydro-electric means. A simple mathematical check ould show that this power, with that for the pumping, cannot be met either by the ailable water from the loch or by the capacity of the pipeline, and it is therefore ressed that the heating is only to be provided electrically during the early years of e operation of the scheme, when the pumping demands are relatively low. But the ghtly increased cost of the generators is justified by the saving of other means of ating. Fig. 2 shows a diagrammatic layout.

All main pipelines were to be of spun iron, and concrete lined. For the raw-water nstock carrying peat-stained and aggressive water the lining is an obvious pre-ution but, as a safeguard against faulty treatment occurring accidentally, and ainst loss of future carrying capacity, it was decided that all should be so lined.

The reservoirs at Stemster and Cnoc Dubh were to contain 1,000,000 and 400,000 l respectively, which was rather more than one day's supply in each case. Other servoirs, for balancing local peak demands, were required (Fig. 1) at Brabstermire, bster, and Ulbster for phase I, and at Smerral, Dunbeath, Newport, and Mid Clyth phase II. An interesting point is that with the anticipated early commence-nt of phase II, with the supply from the higher initial head of Cnoc Dubh reser-ir piped northwards up the coast, Ulbster reservoir is not likely to be required, ce the demands on the phase I pipelines are unlikely to create, in the interim,

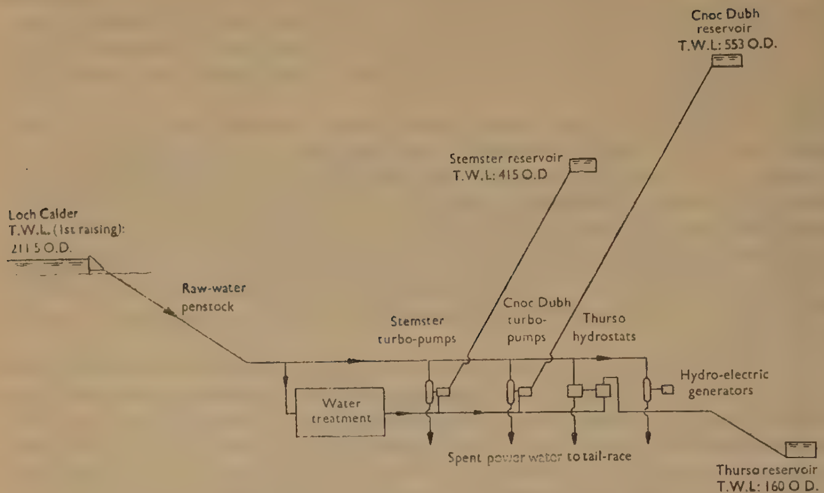

FIG. 2.—GENERAL ARRANGEMENT OF SCHEME—DIAGRAMMATIC

peak demands necessitating it. This is a matter which will have to be decided in due course as events develop.

THE RAISING OF LOCH CALDER

With a long-term average rainfall of 36·5 in. per annum, a catchment area of about 5,600 acres, loch surface area of about 840 acres at 205·0 O.D. increasing sharply with a raising of the water level, and assumed efficiencies for the machinery, it was expected that the figures given in Table 2 would be appropriate.

TABLE 2

	Phase I: g.p.d.	All phases: g.p.d.
Water supplied to Stemster	700,000	880,000
,, ,, Ormlie	350,000	350,000
,, ,, Cnoc Dubh	—	320,000
Total supply requirements	1,050,000	1,550,000
Compensation in Calder Burn	120,000	120,000
Water required for power	3,850,000	5,330,000
Yield needed from loch	5,020,000	7,000,000

The assumed pipeline sizes were 24 in. for the raw-water penstock, 12 in. for the rising mains to Stemster and Cnoc Dubh reservoirs, and 10 in. from the pumping station to the connecting point with the existing Thurso mains. For phases II and III the raw-water main and the rising main to Stemster were to be duplicated.

These assumptions necessitated raising the water level of the loch, as mentioned arlier, to 211·5 O.D. for phase I and to 216·5 O.D. for phase II or phase III. The vorks necessary for the raising were a main weir across the valley of the Calder Burn, road diversion at the south end of the loch, and, for the final raising, a small dam cross a saddle at the north end.

The road diversion calls for little comment except to mention that its construction nvolved the removal, and replacement by rock fill, of depths of peat up to 8 ft. No pecial bridging was involved. The existing road was at all points well above the evel of the first raising of the loch, but the culverts would have been permanently looded and impossible to maintain; the diversion was therefore carried out during the irst phase of construction.

The north end of the loch is interesting in that many maps show a watercourse from Loch Calder flowing into the Forss River. No sign of such a watercourse exists today, nd the reason for this cartographical error is obscure. However, the point of " outet " will require damming before the water level of 216·5 O.D. is created.

Achavarn weir

This structure, of traditional gravity design, is shown in Fig. 3. An extreme flood, f the order envisaged in the Interim Report of the Committee on Floods in relation o Reservoir Practice, would have involved a length of spillway far exceeding the vidth of the valley; but the lag effect of the loch, with its large surface area, peraitted a considerable reduction in the length of spillway and the weir now fits comactly into the valley.

The foundations were highly laminated beds of the Old-Red-Sandstone group and, onsequently, of high porosity. Pressure grouting was therefore necessary below the ut-off trench, but it was evident that there would be a risk of the grout lifting the verlying material and making matters worse. Consequently, pipes were fitted into he grout holes and were brought up through the cut-off trench and the superstrucure; the grouting was performed last of all, when the maximum weight on the foundtions had been created. This method made the normal water tests of the grout holes npossible and reliance had to be placed on judgement alone; however, should any eepage occur later, additional grouting can be performed by drilling down through he structure—not an impossible task in a dam of such moderate size.

The penstock chamber was duplicated, each chamber being protected by " rakeble " trash racks and by finer " lifting " screens. The provision of very fine screens vas considered impracticable and, with the large natural forebay which will permit ilt to settle, unnecessary. The actual intakes are, however, designed to draw water om near the surface, being of the " floating-arm " type.

Compensation water is supplied to the Calder Burn through a V-notch, the head bove it being kept constant by a ball valve in a small stilling pool.

The recorder house stands on a separate mass-concrete base adjoining the intake ructure. This recorder house contains water-level indicators of the loch level and f the level downstream of each screen to show any loss of head suggesting the necesty of cleaning screens; continuous recorders of the compensation water and of overow water over the spillway are also installed. The base of the recorder house ontains a dry sump, with a multiplicity of tubes and valves for the sundry instruients; these are for the purpose of checking and correcting the zero readings from me to time.

The raw-water penstock

The 24-in.-dia. concrete-lined spun-iron pipeline calls for little comment. The size selected is, if anything, on the small side, especially if deterioration in carrying capacity should be rapid; this apparently false economy was justified by the possibility of making the parallel future penstock slightly larger should experience then suggest its being desirable. The pipeline is provided at intervals with readily removable sections to facilitate brushing, when and if the residual head at the pumping station is found to have deteriorated beyond a tolerable extent. The pipeline is laid with a continuously falling gradient, but air valves and wash-outs are provided where judged expedient.

The treatment works

In the initial water samples satisfactory coagulation was obtained with moderate doses of alumina, but provision was made in the design for the addition also of sodium aluminate or soda ash. Chlorination both before and after filtration was considered as possibly desirable, and provision was made accordingly. Final pH correction, by the addition of lime, would be necessary.

Filtration is in 9-ft-dia. pressure filters, twelve being installed for phase I. This arrangement gives a rate of filtration of 68 gal/sq. ft/hour or, with two filters out of commission for washing, etc., 82 gal/sq. ft/hour. These rates were considered satisfactory in the unavoidable absence of initial sedimentation, and satisfactory results were later obtained in practice.

For subsequent phases of development of the scheme the number of filter shells will be increased to sixteen, but the chemical dosing apparatus installed initially is adequate for the ultimate requirements of the station.

Filter washing is by direct air scour and upwash, the wash-water being piped to settling tanks. After a period of settlement the supernatant water is pumped into the tail-race to the Thurso River, the sludge being pumped separately to lagoons. In spite of the very great dilution of the wash-water which will be effected by its addition to the spent-power water running to the river, the settling tanks were considered essential, the Thurso River being an important salmon river both from the sporting and the industrial angles. Figs 4 and 5 show diagrammatically the layout of the treatment works, and the pumping and treatment equipment.

The pumping and electrical generating plant

As mentioned earlier, the design of the machinery hinged largely round the question of surge pressures in the raw-water and rising mains. Surge towers were obviously impracticable and air vessels were also found to be inappropriate. Consideration was therefore given to the question of surge prevention rather than suppression, or in other words to designing the plant so that appreciable change in the velocity of the water in the penstock (or in the rising main) could occur only very slowly.

Stemster and Cnoc Dubh reservoirs are both to be supplied by multi-stage centrifugal pumps, direct-coupled to Francis turbines. The Stemster machinery (part of phase I) is installed and has been extensively tested, and the ensuing descriptions refer to this. The Cnoc Dubh equipment will be basically similar, but designed for reduced output at greater head. In the pumping station a constant indication of the depth of water in the receiving reservoir is available, and the same transmitter is used to close down the pumping set when the reservoir is full. The automatic devices are designed so that only small surge pressures can occur, and all manually

operated valves in connexion with the sets are equipped with worm reduction-gearing so that the operation is unavoidably protracted and therefore safe.

The Thurso supply is by hydrostat. There is actually a positive head of about 60 ft between Loch Calder and Ormlie reservoir at Thurso, but this is insufficient to pass the required supply through the filters and the pipelines. In point of fact the previous Thurso supply, direct to Ormlie reservoir, was boosted by a Diesel-driven pump near Loch Calder.

The operation of the hydrostats is controlled at Hoy to the required rate of delivery, which, in the initial stages of the scheme with small supplies and new pipes, will evidently be adjustable over a wide range. The closing of the ball valve at Ormlie will obviously bring the hydrostat discharge to zero with the filling of the reservoir and at a very slow rate.

Electrical generation is by 400/230-V alternators, three sets each rated at 16·5 kW being installed. All sets are capable of being worked in parallel, and the normal maximum requirements can be met from any two sets.

The instantaneous acceptance or shedding of an electrical load of these proportions would have created unacceptable surge pressures in the raw-water penstock, but all such possibility of surge has been eliminated by the use of impulse turbines, governed by jet deflexion. Conditions are not appropriate for the efficient use of Pelton wheels, but Gilbert Gilkes and Gordon Ltd, who were awarded the contract for the supply of the machinery, could commend their "Turgo" wheel for the duty. So here again surge problems are sidestepped; the main spear valve is first opened to an appropriate extent for the duty expected by means of worm reduction-gearing, and the set is then run up to speed. At this stage practically all water will be deflected from the wheel and, as load is taken up, the governor reduces the degree of deflexion appropriately. It is appreciated that this method involves some waste of water, especially should the handling of the plant be at any time otherwise than keen and efficient, but it is expected that the operators will soon discover their own private "calibration" of the spear valves, and will be able to set them to pass an amount of water appropriate to the maximum load to be carried in the near future.

The main switchboard

The main switchboard, which is about 20 ft long, carries indicators and recorders of the rate of flow to Stemster and Ormlie reservoirs and of the discharge down the tail-race; the latter is important since it forms the compensation water to the Thurso River. On the strictly electrical side the switchboard covers the usual requirements of alternators, with a swinging synchronizing panel at the left-hand end.

Further instruments will, of course, be required when the phase II machinery is installed, and blank panels have been provided for this purpose.

The pumping station building

The main hall, which is 110 ft × 60 ft, houses the filters and all main machinery. One wing of the building contains the ancillary machinery, etc., and the other the administration rooms.

On the east side of the main hall is the duct containing the incoming raw-water main. Branches therefrom lead to each turbine, from which the spent water falls to the tail-race underlying the machinery. All pumps are in duplicate, giving 100% standby.

A tee off the raw-water pipeline leads to another bus-pipe parallel to it and between the two rows of filters. This forms the supply to the treatment works. The filtered

water collecting pipe, which is laid in the same duct, discharges into the treated water main from which the pumps and hydrostats receive their supply.

Even the extremely compact layout thus created involved a building 60 ft wide, and the traditional travelling crane capable of lifting all items of equipment would have become a massive affair. This was therefore omitted, and a portable crane, on caster wheels, substituted.

Abandonment of the permanent crane made it possible to adopt the sweeping line of cross-section shown in Fig. 6. The effect is exceedingly pleasing externally, and perhaps more so internally. Mr W. Wilson, the County Architect of Caithness, co-operated enthusiastically with the consulting engineers in obtaining the best design. Adequate daylight is obtained by flat lights in the roof as well as by large windows at each end. Artificial lighting is by tubular fluorescent lights set in the angles between the roof and the stiffening ribs.

A shell-roof design is employed for the main hall and the wings are roofed with flat reinforced concrete slabs.

A large space such as the main hall is very difficult to heat, especially in this case where the roof is thin and the surface of the pressure filters absorbs heat rapidly. The general problem of condensation on internal surfaces is still under consideration, but surfaces which would contribute to the problem have been covered with insulating media, the roof being sprayed underneath with a vermiculite-cement mixture with an asbestos addition and the filter shells with the new proprietary material "Seculate."

The internal finishes are generally utilitarian, with granolithic flooring throughout except for a small area of wood-strip flooring in the administration wing.

Externally the whole building is clad in precast slabs, finished with exposed granite chips. This gives a pleasing though rather severe appearance, very suitable for the unpolluted atmosphere of Caithness. Two types of slab were employed, one forming permanent shuttering for cast-in-situ walls, and the other, $4\frac{1}{2}$ in. thick, forming the outer leaf of a cavity wall. The supplier made an excellent job of these slabs, the order containing a large variety of types and sizes. Actually no piece failed to fit and the carriage arrangements were so good that, even with the long journey to Caithness, there were few breakages.

The sludge settling tanks

These are formed with mass-concrete walls, between which span reinforced concrete floors designed to withstand the upward pressure of the external ground-water which can rise nearly to ground level. The walls had, therefore, to be able to resist the total uplift force, assisted only by the adjacent earth wedge, and therefore appear exceedingly massive for their apparent purpose.

Filter wash-water decanted into the tanks is allowed to remain there to permit the sludge to settle, when this is drawn off from the bottom and pumped to sludge lagoons. The supernatant water is drawn off through floating-arm outlets and pumped to the tail-race. The pumps are situated in an adjoining underground pump-house, and are thus permanently primed for either purpose.

The tail-race

The tail-race consists of a 48-in.-dia. concrete pipeline, leading to a simple outfall to the Thurso River. This type of construction was preferred to an open flume on account of economy of construction and also because its presence involved no potential harm to cattle. The construction calls for no special comment, but the hydraulics are of some interest.

Fig. 6.—Interior of Hoy pumping station

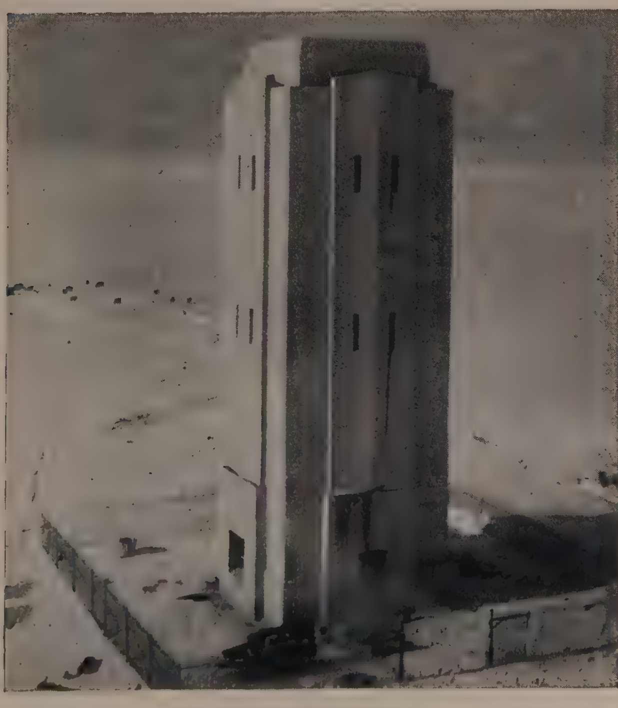

Fig. 7.—Aerial view of Nybster water tower

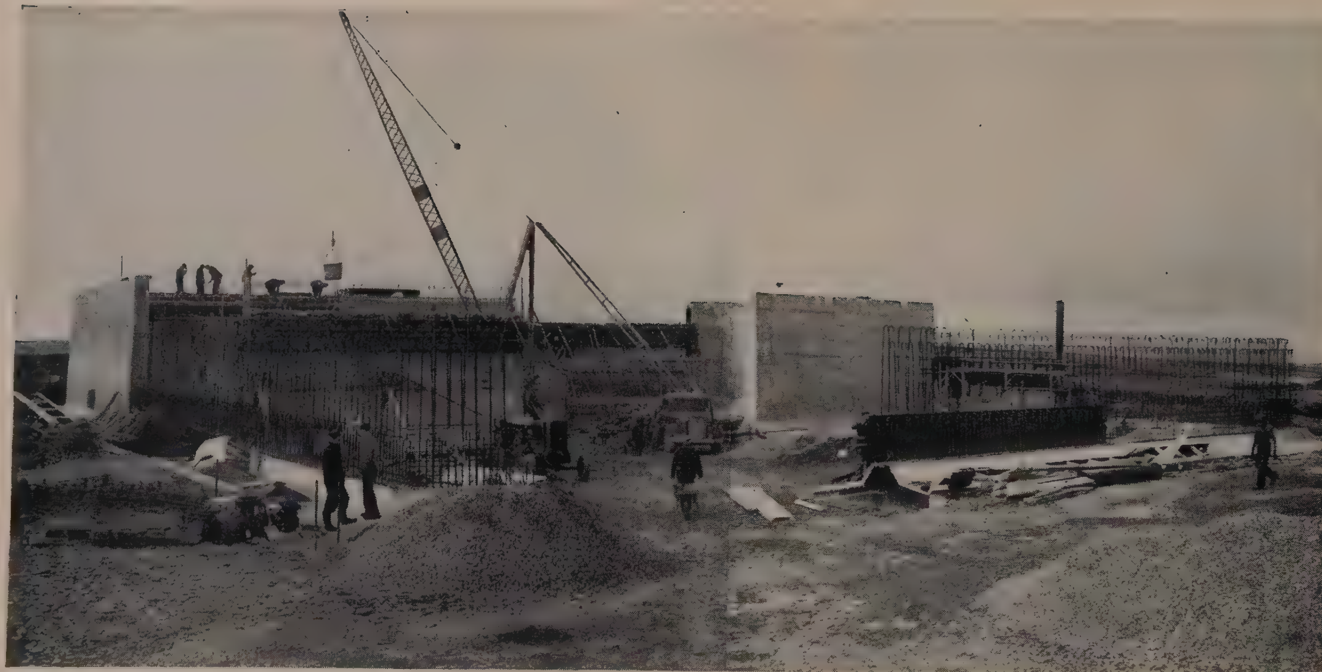

Fig. 8.—Stemster reservoir. View of site from south with water tank just visible above far wall
(From a composite photograph)

When the river water is high it can cause the pipeline to be filled at its lower end to a level above that corresponding to its maximum discharge. The resulting back-water curve becomes unstable. This is calculated to result, under extreme flood conditions, in the drowning of the standing-wave flume which measures and records the flow from the station. This drowning will occur at rates of flow depending on the river level; the anticipated incidence has been calculated and will be verified practically. It should be remembered that the resulting inaccuracy of the records is nothing more than an inconvenience, because the actual discharge can at all times be determined from the charts of the rates of flow entering the pumping station and being pumped therefrom; the difference is the water in the tail-race. The "drowning" trouble can, if later considered necessary, be minimized by removing the soffits of a few pipes at the downstream end of the tail-race.

The rising mains

The new main for the Thurso supply consisted of a 10-in.-dia. spun-iron pipeline laid only so far as to join the existing mains which had previously supplied the Burgh direct from Loch Calder,

To Stemster reservoir (phase I) the main is a single 12-in. pipeline, to be duplicated for phase III. For phase II a single 9-in. main is envisaged. Only the first Stemster main has so far been laid.

The point of interest in the pipelaying was the crossing of the Thurso River. This was performed in simple cofferdams—half the width of the river at a time—and the obvious expedient was adopted of laying immediately required and future pipelines in the same operation. Thus three water mains were laid, also a 4-in. threader pipe or the electrical signal cables.

A careful inspection of the river-bed suggested that it would have no tendency to move, and all pipes were therefore encased solid in concrete. The greatest care was necessary at this river crossing to avoid any interference with salmon; for instance, pumps were discharged well away from the river and the water allowed to spread over the land so that no sand or silt would cloud the river water.

Stemster reservoir

Generally, mass concrete was found to be preferable to reinforced, but in the case of Stemster reservoir, the fact that access by road was difficult made the use of reinforced concrete preferable, since the quantity of material to be transported to the site was less.

The design calls for little comment, being orthodox and in accordance with the Institution Code for Liquid Retaining Structures.

The reservoir contains one million gallons of water and is divided into two halves by a central division wall to permit the emptying and cleaning of either half.

Brabstermire reservoir

Brabstermire reservoir, which is a small balancing reservoir of 100,000 gal capacity, is of interest in that the use of grouted concrete was specified, the aggregates being pre-placed grouted with colloidal cement-sand grout mixed in a specially designed mixer.

This method of construction was adopted to permit the use of the local rock, the laminar structure of which made it useless for traditional concrete. But in the larger sizes that "post-grouted" concrete would permit, it was expected to be tolerable, and the subsequent remarks dealing with construction confirm this.

Nybster water tower

Nybster water tower is the only elevated balancing tank in the whole scheme; it consists of a sectional tank of 20,000 gal capacity supported on a lightly reinforced concrete structure. The tank is hidden from view by a thin concrete wall, the external lines of which form a continuation of the underlying structure. An aerial view of the finished structure appears in Fig. 7 and the intentional severity of the structure, architecturally, can be assessed.

Letting of contracts: considerations involved

So far the scheme has been dealt with from the designer's angle and it would therefore be logical to continue with a description of the constructional features, but in this scheme a most important administrative point was the rate of expenditure, which had to be controlled in accordance with the wishes of the Department of Health for Scotland who were granting financial assistance.

In general it was possible to control expenditure on pipes, etc., fairly well, by close co-operation with the manufacturers. Pipelaying contracts were intentionally kept small, so that expenditure could be controlled by the timely letting of contracts. But, for the headworks, this method had to be adjusted.

These works were thought to be rather too big for most contractors north of Aberdeen, say, and if let piecemeal would form contracts too small to permit contractors further afield to tender keenly. It was decided that Achavarn weir, the penstock, and the pumping station and ancillary works should form one contract. Nybster water tower, though small in value, was thought to be a type of structure suitable only for an experienced public works contractor and it was therefore decided, whilst keeping it the subject of a separate contract, to advertise that it would be awarded along with the headworks. It would appear that it might well have been incorporated in the same contract, but there were objections to this. It would have made accounting difficult and, moreover, the pipelines in the Nybster area were not scheduled for laying until long after the headworks were started. To construct the tower in advance of the connected pipe system would have been an unnecessarily early investment of capital and would have been politically unwise, suggesting to the public an uncertainty of intention as to the eventual availability of the water.

A further point for consideration in arranging the programme of construction, and therefore of capital investment, was the possibility of obtaining an early return for money spent. The scheme may be regarded as of three parts, Achavarn weir, the pumping station, and pipelines. Without any one of these " legs of a tripod " the scheme is inoperable.

While the headworks construction was being planned or implemented early contracts for pipelaying were let north from Wick as far as Keiss—a small section which could be supplied with water purchased in bulk from Wick—and in the extreme north of the county round Castletown. This latter and much larger area was supplied from the works installed by the Air Ministry during the war. Both these schemes have therefore been revenue earning for a considerable time; a small contribution, no doubt, but worthwhile.

A list of the principal contractors employed on the construction is given in Appendix I.

Details of construction

The scheme involved a considerable amount of concrete, of quality varying from

that suitable for bedding pipes to that for thin highly stressed reinforced sections. Perhaps the first major problem was, therefore, the question of concrete aggregate.

Apart from some local seashore deposits, practically the only possible concrete aggregate indigenous to Caithness is the highly laminar Caithness Flagstone of the Old Red Sandstone group. This material crushes to aggregates suitable only for the poorest qualities of concrete, the necessarily excessive water/cement ratio militating against both strength and weather resistance.

It was fortunate, therefore, that an excellent water-worn gravel and sand deposit existed near the mouth of the Strath Halladale River, in Sutherland. This is only about 20 miles from the headworks of the scheme and its use was therefore practicable.

The grading of the materials as won was often acceptable, but this had to be constantly watched. Rotor screens were installed at the quarry, but the intermediate grading was an important point. The sand was generally found to be acceptable from this angle. All aggregate was naturally clean and required no subsequent washing.

As mentioned earlier, the use of Caithness Flagstone aggregate, with grouted concrete, was permitted in the construction of Brabstermire reservoir. The large aggregate was pre-placed, generally by hand and with a minimum size of $1\frac{1}{2}$ in., care being taken that the flats were not against the shuttering, so that a good exposed surface was attainable. Generally it was intended to apply the grout from the top of the lift, but later it was found preferable to apply it through tubes leading to the bottom of the shutter. It was found that judgement had to be exercised, based on experience of this method of concreting, in deciding the mix and the consistency of grout for a particular purpose.

Excavation

The laminated sandstone persisted throughout, and occurred in pipe-trench excavation to a greater or lesser extent throughout a high percentage of the total length. Trench excavation was generally by back-acter with a small rock bucket. The rock encountered was a considerable hinderance to progress, its nature, and frequent clay seams, militating against efficient blasting.

A surface layer of peat was sometimes present, which made the movement of transport and machinery troublesome.

At Achavarn weir the excavation had to be very carefully executed, blasting being controlled so that the surrounding rock was not disturbed more than necessary. Inattention to this point would, of course, encourage leakage round the limits of the structure.

Achavarn weir

The organization of work on this site presented no major problem. Excavation was effected by dragline, feeding into dumpers; the same machine with an extended jib was later used for concreting in conjunction with jubilee stock.

Concrete was mixed in two orthodox mixers, proportioning being in a static weigh-batching plant. This method involved, on occasions, excessive numbers of handlings, but the advantage of a static mixing plant was there and the crane was a general asset when concreting was not in progress.

The contractor accepted some degree of risk in the arrangements for diverting the Achavarn Burn across the workings. The water was guided by stanks through a 16-in. steel pipe spanning across the excavation, and any inadequacy in the arrangements in time of heavy rainfall would have flooded the workings. Some degree of

flood control was possible, however, since the outflow of the loch was through sluices belonging to the Thurso Burgh in connexion with their water supply, and the capacity of the loch could therefore be used for balancing the outflow in the burn. Fortune was kind and no untoward incident occurred until that mentioned below. The apparently small yield from the loch caused pessimists to wonder whether the mysterious outlet at the northern end of the loch—mentioned earlier as being shown on maps—did in fact exist! But when the mass of the dam was well advanced the water did get out of hand; it was possible, however, to divert it across the partially completed structure through a gap left for the purpose, and by then all worry was at an end, both as to the disposal of flood water and as to the correctness of the calculated yield of the Achavarn Burn!

The concreting and subsequent under-grouting of the structure, and the construction of the various auxiliaries, proceeded to its conclusion without noteworthy incident. Impounding followed, and no signs of leakage have been observed.

Hoy pumping station

The construction of the shell roof was a new experience to the engineering staff and to the contractor, and therefore formed an interesting test of the suitability of this type of construction in general engineering. Timber formwork was used, the stiffening ribs being constructed first, followed by the edge beams and the shell. Some damage was caused to one of the stiffening ribs in the early stages by a very strong gale, with gusts as high as 100 m.p.h., but no further troubles were experienced. The accident to the stiffening rib could not, of course, be held to be the result of a risk inherent in the structural design. Construction was very carefully performed and the satisfactory result, in this remote part of the country, is an indication that this form of construction might be far more widely employed than it is.

The walls of the main hall are not load-bearing and were constructed up to the underside of the edge beams, and of the horizontal lintels of the end frames. Some trouble was caused by the inevitable shrinkage of the in-situ concrete away from the beams, and, where perfect watertightness is essential, this is a matter requiring careful thought.

The civil engineering work inside the building consisted of a multiplicity of ducts, foundations, and flooring, apart from the normal building operations, finishing, etc. The " ducts " varied in size from the tail-race to small channels for electric cables. The most interesting concrete work was that under the hydrostats, where pipework for raw power water, the spent power water, and the incoming and pumped treated water, had to be installed. The resulting three-dimensional puzzle required drawings comprehensible only to engineers, and a timber model, assembled in the order of the intended construction of the prototype, proved essential to the proper guidance of the tradesmen concerned, particularly of the joiners making the formwork. This is a method, it is thought, that might be more widely used for complicated structures.

Installation of equipment

No real difficulties were encountered in the installation of machinery and equipment, but a considerable number of minor problems inevitably arose. Three different firms were concerned; the filter plant manufacturers, the turbine and machinery makers, and the electricians who were sub-contracting to both the others.

With several hundred flanges on pipes, valves, and other equipment to be connected up, and only two flexible joints in the entire system, it was expected that some difficulty would be experienced in fitting everything together. The problem

was aggravated by the fact that some of the castings were not accurately made, but, after a good deal of trial and error, and a certain amount of ingenuity, the system was properly connected up.

Stemster reservoir

An excellent rock foundation was obtained for Stemster reservoir and since the angle of dip was very small it was found possible, with careful drilling and blasting, to take out the excavation to almost exact dimensions and levels.

The reservoir is on an exceedingly exposed site and more than usual care was necessary in the curing of the reinforced concrete walls, which are not of massive section. At construction joints, besides the usual precautions, copper strips were embedded to span the joints. Two slight vertical cracks were noticed in the walls just above pipes passing through. Conditions being so difficult, with practically continuous high winds, extra reinforcement was introduced at similar points thereafter.

The construction was let to a local firm, which, though of high repute, was not experienced in this class of work. It is interesting to record that this decision was well justified; Fig. 8 shows the excellence of the site layout and the perfection of the rock excavation.

The reservoir has been tested and only one or two minor defects have been disclosed, which have been easily remedied.

Pipelines

It has earlier been mentioned that the delivery of pipes, etc., was phased in order to be in accordance with the capital investment programme. From the site point of view more detailed phasing was needed to ensure that pipelaying contractors were not delayed for want of pipes, and, on the other hand, that pipes were not lying on site for too long a period in advance of requirements. The manufacturers co-operated admirably in this matter.

The major pipelines were of spun iron, lined with concrete, and coated externally with two coats of bitumen. In this connexion it was interesting to note the condition of old pipes which were cut for joining up or inserting valves. One particular pipeline—a 6-in. cast-iron main laid in 1876, which had been scraped internally on two or three occasions, was found to be very badly tuberculated and of much reduced carrying capacity. Other pipes had been eroded by the aggressive water they were carrying. But, except in the case of small-diameter galvanized-iron pipes, there has been little evidence of external corrosion.

The spun-iron pipes were shipped from the factory to Scrabster, and thence carried by road to the points of laying. Surprisingly little trouble was encountered through cracked pipes, such trouble as there was being almost entirely confined to the 24-in. ipes for the raw-water penstock. In many cases cracks which had escaped notice rior to laying were extended by the pressure tests which subsequently disclosed heir presence.

The pressure tests, it should be mentioned, were not easy to perform on pipelines preading over wild country where water was, in many cases, not available. Conractors exercised various forms of ingenuity in the matter. A further difficulty rose in cases where, a test having indicated leakage, the joint holes had filled with round-water and the small escape from the pipeline under test pressure was almost npossible to locate.

Pipelines were generally laid in the fields alongside the roads, the verges being too narrow for the purpose. For the smaller asbestos-cement distribution mains thrust-boring under carriageways to accommodate a copper connecting tube was found practicable in some cases and then held obvious advantages.

Wayleaves for laying the pipelines were nowhere necessary, the serving of notice being legally sufficient. To some extent for this very reason a special point was made, wherever possible, of visiting all owners and occupiers of land through which pipelines were to pass, a suitable interval before the work commenced. For the most part farmers were exceedingly co-operative and only on rare occasions was any ill feeling evinced. The restoration of land was carried out as thoroughly as possible, particular attention being paid to field drains since these can easily become a bone of contention if they are not, and occasionally even if they are, properly restored. The responsibility for unavoidable damage to crops within a defined width was intentionally shouldered by the County Council, instead of (as usual conditions of contract require) being the contractor's responsibility. There were two reasons for this. In the first place tenderers cannot tender scientifically for such a risk, since they are unable to foresee what crops may be planted before the start of the contract. Secondly, the County Council, being a permanent local body, can assess damage fairly without being suspected (generally unjustly) of trying to drive the best possible bargain in settling a claim. In laying more than 100 miles of pipelines very little discontent has been voiced.

Commissioning the works

Impounding in Loch Calder was commenced as soon as possible, and this coincided with the installation of the treatment-works equipment and the pumping machinery.

Water was admitted to the system and trials of the various parts of the installation commenced. At this stage one of the twin penstock chambers at Achavarn weir was incomplete and, with a great deal of floating debris in the loch from the newly flooded areas, proper alternate use of the screens was not possible so that much debris inevitably became admitted to the pipeline while the screens were out of use for cleaning. Thus various articles, ranging from pieces of wood to the battered corpses of salmon, eventually found their way to strategic points in the pumping station and their removal was a prerequisite to the continued operation of the apparatus involved. This trouble disappeared, of course, when duplicate penstock chambers were available.

The turbines, pumps, and generators had been tested at the manufacturers' works, therefore site trials of performances, etc., were not considered necessary. It was, however, necessary to check thoroughly the operation of all apparatus and to ensure that automatic devices worked as intended. Perhaps above all it was essential to set all controlling apparatus so that machines could be opened up or closed down only at speeds such as would cause no undue surge pressures in the connected pipelines. Such minor difficulties as arose were in due course overcome, all concerned co-operating enthusiastically.

Fortunately, or unfortunately according to the point of view, the Thurso River was in extreme flood soon after the trial operation of the station commenced. The tail-race measuring flume was duly flooded and recorder readings falsified so that some study of that problem was possible, but the true flow records would have been more useful at that time.

As always in the treatment of water from a new source a good deal of experiment

vas necessary to ascertain the most satisfactory chemical dosages. Until the plant ıas been in full-scale operation for a full year it will, of course, not be possible to say ıow the optimum dosages will be affected by seasonal changes in the raw water, but n the initial trials about 40 parts/million of aluminium sulphate and 5 parts/million of sodium aluminate has given good flocculation, whilst about 25 parts/million of lime ıas been needed for pH correction. Chlorine requirements have varied and it is, as vet, difficult to give an average figure. A residual of about 0·01 parts/million has een the aim, but, again, it is difficult to justify a particular figure, since the residual ıt the far end of the county must be considerably less than in the water supplied to onsumers geographically nearer Hoy. Appendix III gives examples of early analyses of the raw water of Loch Calder.

Conclusion and acknowledgements

It was a pleasure to all concerned that Her Majesty Queen Elizabeth The Queen Mother inaugurated the scheme on the 30th April, 1955.

The Authors wish to record their thanks to the Caithness County Council for per- nitting the presentation of the Paper, and to their respective employers for similar ermission and for encouragement and help.

The shell roof of the pumping-station building was designed by Mr H. G. Cousins, B.Sc.(Eng.), M.I.C.E., on behalf of Messrs Chisarc and Shell D, for the Consulting Engineers.

APPENDIX I

LIST OF PRINCIPAL CONTRACTORS

ohn McAdam and Sons Ltd, Aberdeen	Achavarn weir, raw-water penstock, and Hoy pumping station. Nybster water tower
eorge W. Bruce Ltd, Aberdeen	Facing slabs for Hoy pumping station
The Candy Filter Co. Ltd, Hanwell	Water-treatment equipment
ilbert Gilkes & Gordon Ltd, Kendal	Turbines, pumps, and generating machinery
ochranes Foundry, Middlesbrough	Spun-iron pipes and cast-iron specials
lenfield & Kennedy Ltd, Kilmarnock	Sluice valves, etc.
lexander Sutherland & Sons Ltd, Wick and Thurso	Stemster reservoir and pipelaying
. B. Russell & Sons Ltd, Aberdeen	Pipelaying
Villiam Tawse Ltd, Aberdeen	Brabstermire reservoir and road-diversion works
The Stanton Ironworks Co. Ltd, near Nottingham	Spun-iron pipes and cast specials for Wick to Keiss section
uest & Chrimes Ltd, Rotherham	Sluice valves for raw-water penstock

APPENDIX II

A considerable amount of time and thought had to be devoted to the selection of the vater-power units owing to the relatively high ultimate friction loss in the raw-water main. t was necessary to select units that would give a high efficiency under the varying con- litions of all the stages of the scheme. If during selection the capacity of one unit was ltered it affected the net head on the remaining units and the inlet pressure to the pumps, uite often making it necessary to change the characteristics of all the units.

It may not be immediately self-evident that, the pressure corresponding to the head of och Calder being retained through the system, the frictional loss in the raw-water pen- tock forms an addition to the pumping head. Thus, other things being equal, an addition o that frictional loss increases the pumping head and therefore the quantity of power vater required for the turbine driving the pump, and the increase of power water again

increases the friction in the penstock. This is elementary hydraulics, but it made it necessary to calculate conditions most carefully, since they were exceedingly sensitive to changes in basic assumptions.

Moreover, careful consideration had to be given to the control of the machines, since any rapid change of the rate of flow in the raw-water main, and to a lesser extent in the rising mains, would cause severe and possibly dangerous pressure surges.

The Stemster turbo-pumping sets

Each of the sets consists of a horizontal-shaft Francis-type turbine direct-coupled to a multi-stage centrifugal pump. In phase I the set is designed to deliver 700,000 g.p.d. and the turbine to develop a maximum of 58 b.h.p. when running at a speed of 1,320 r.p.m. In phase III the output becomes 880,000 g.p.d. and the maximum power of the turbine 70 b.h.p. Each set acts as a standby for the other unit since only one set is in use at any time.

The turbine is of the spiral-cased type and fitted with movable guide vanes working between renewable bronze facings. The runner is of bronze and mounted on a stainless-steel shaft carried on roller journal bearings. A ball thrust-bearing is provided to carry the axial thrust of the turbine runner. The drive to the pump is by means of a flexible coupling.

The power and consumption of water of the turbine is dictated by the opening of the guide vanes, whilst the speed of the set is fixed by that required by the pump for any given duty. The turbine guide vanes are arranged to be operated either manually or by electric control. The former is used only for emergency purposes and normally the guide vanes setting is controlled electrically, either by a manually operated switch on the control panel or automatically by remote water-level control from the reservoir at Stemster Hill.

The electric operating mechanism consists of a $\frac{1}{8}$-h.p. single-phase motor connected through suitable gearing to the guide-vane operating lever. A limit switch is fitted at the "fully closed" end of the travel of the mechanism to prevent over-running and there is a corresponding limit switch at the "full open" end of the travel. In addition, an adjustable limit switch is provided which may be set to any required position when it is desired to limit the opening of the guide vanes and turbine output to a valve less than the maximum. The arrangement of the mechanism is shown diagrammatically in Fig. 9.

An auxiliary limit switch is provided at the fully closed end and connected to a red indicator-lamp circuit. The purpose of the lamp is to indicate to the station attendant that both pumping sets are closed down.

When desired the electric control can be instantly disconnected by gently drawing the hand control wheel forward. This disengages the dog clutch connecting the control spindle to the electric-motor drive.

A hydraulically operated sluice valve is fitted at the inlet to each turbine. The opening of the sluice valve is carried out by a manually operated control valve but the closing of the sluice valve is controlled by a solenoid fitted to the hydraulic cylinder control valve. The solenoid is energized from the electric supply and if the supply is cut off or fails the sluice valve closes, thus shutting down the turbo-pump set. A solenoid switch is provided on the control panel for use as an emergency shut-down control and the solenoid also forms an emergency shut-down device for the failure of the electric supply to the whole station. It should be remembered that a failure of the electric supply would mean the cessation of chemical injection and the delivery to supply of improperly treated water.

The remote water-level control gear at Stemster reservoir consists of a float-operated electric transmitter connected by an underground cable to Hoy pumping station, where suitable receiving apparatus is installed. The remote-control gear operates two relay switches which start or stop the guide-vane operating motor at predetermined reservoir water-levels. Thus when the reservoir becomes full the set is automatically shut down. When the water level falls to a given level the turbine is started up and the rate of pumping depends upon the setting of the adjustable limit switch fitted to the turbine control mechanism. On the control panel selector switches are provided enabling either set to be put on manual electric control or remote water-level control. A water-level indicator and recorder are also fitted on the control panel.

The multi-stage centrifugal pump coupled to the turbine is of normal construction, provided with cast-iron casing, bronze impellers, and bronze diffusion vanes. The shaft is of nickel steel and runs in white-metal-lined oil-lubricated bearings. The axial thrust of the impellers is balanced hydraulically by a balance disk fitted at one end of the pump. The pump and turbine are mounted on a common cast-iron bedplate.

he future Cnoc Dubh turbo-pumping sets

The sets to be installed during phase II for supplying water to Cnoc Dubh will be con-tructed and controlled similarly to those for the Stemster supply.

The ultimate demand on these pumps is expected to be 320,000 g.p.d. but the pumps re not being designed on that basis. Cnoc Dubh reservoir being at a higher elevation han Stemster and more remote from Hoy, the ratio of power water to supply water is nuch increased. By designing the pumps for a delivery of only 270,000 g.p.d. the power vater available—increased actually by the reduced supply water—can create an increased ead at Hoy and this, coupled with the reduced delivery, permits a substantial reduction n the size of the pumping main. The design is therefore being evolved accordingly, rovision being made for boosting the delivery in the future if and when the demand xceeds 270,000 g.p.d.

he Thurso supply

The quantity of water to be supplied to Ormlie reservoir is 350,000 g.p.d. but the tota umping head is relatively small. It was decided that a hydrostat would be the most conomical and efficient type of unit to use. Two sets were therefore installed each apable of giving the required output. The hydrostats are of normal construction with he motive cylinder separate from the treated-water pumping cylinder. The isolating luice valves for the hydrostats are manually operated but are fitted with worm reduc-ion-gearing to ensure slow operation. The starting and stopping of the hydrostats is ictated by a ball valve fitted to the delivery pipe at Ormlie. The makers do not recom-nend that hydrostats be allowed to come completely to rest when the delivery valve is losed, so a spring-loaded by-pass valve is fitted between the pump inlet and outlet con-exions. This allows the hydrostat to creep when the demand for water ceases.

enerating sets

Electricity at the Hoy station is provided by three generating sets, one or two units in se and the third set acting as a standby. Each set is driven by a Turgo impulse wheel, hich is a high capacity type of impulse turbine. It has a maximum rating of 26·5 h.p. ut is a type of turbine that still operates with a high efficiency at small loads. The ırbine is driven by a single jet formed by an adjustable needle nozzle arranged for manual ontrol through worm gearing. The opening of the nozzle can be set to suit the maximum ad required at any time.

The speed of the turbine is controlled by a modern type of flyball governor, mounted on e end of the turbine shaft, which operates a deflector interposed between the jet and the ınner. Any surplus water not required for the load being carried is diverted from the heel. Such an arrangement gives close and rapid speed regulation without creating any ressure surges in the pipeline. Hand and electrically operated speed-adjusting mechanism provided for adjustment of the speed when two sets are running in parallel.

The alternator has a maximum capacity of 16·5 kW and runs at a normal speed of 750 p.m. It is direct-coupled to the turbine by means of a flexible coupling. The alter-ator is designed to give 400-V 3-phase 50-cycle current, but also provides 230-V single-hase current between any one phase and neutral. An automatic voltage regulator for ch machine is mounted on the main switchboard. The switchboard contains three enerator panels, a feeder panel, synchronizing gear, water-metering panels, and control quipment. Blank panels also exist, to be replaced in due course by metering and control anels for the Cnoc Dubh supply, and for reception of a supply of electricity from the public ipply should it ever be desirable to do so when the scheme is being worked to the limits of s capacity.

APPENDIX III

ANALYSES OF RAW WATER FROM LOCH CALDER

(1) *Sample taken* 10.6.48 (*Loch level normal following wet spell*)

Appearance	Coloured with small amount of suspended matter	
Colour	65	APHA scale
pH	8·05	

Free CO_2	1	p.p.m.
Alkalinity (as $CaCO_3$)	54	,,
Total hardness (as $CaCO_3$)	70	,,
Non-carbonate hardness (as $CaCO_3$)	16	,,
Calcium (as $CaCO_3$)	36	,,
Magnesium (as $CaCO_3$)	34	,,
pH 3 (equilibrium pH, 15°C)	8·75	

(2) *Sample taken* 20/12/49

Appearance	Yellowish, trace of organic sediment	
Reaction	Neutral (pH 7·0)	
Total alkalinity (as $CaCO_3$)	50	p.p.m.
Total solids in solution	134	,,
Chlorine in chlorides	36	,,
Sodium chloride	59·4	,,
Nitrogen in nitrates	nil	
Free or saline ammonia	0·036	p.p.m.
Albuminoid or organic ammonia	0·126	,,
Oxygen absorbed at 80°F in 4 hours	6·124	,,
Phosphates	nil	
Nitrites	nil	
Poisonous metals	nil	

The sample was yellowish in colour and gave a very small deposit of organic matter on standing. The proportion of free ammonia was fairly low whilst that of albuminoid ammonia was slightly above the usually accepted limit owing to the peaty nature of the water. This water is of a soft rather peaty nature which, from the chemical standpoint, is of satisfactory quality.

(3) *Bacteriological examination of sample received* 15/12/49

Presumptive coliform count	90 per 100 ml. of sample
Probable No. of faecal coli	90 ,, ,,

Plate counts

1 day at 37°C	20 colonies per ml.
2 days at 37°C	27 ,, ,,
3 days at 22°C	1,900 ,, ,,

The volume of water in the sample was inadequate for any examination for the presence of Cl. Welchii.

The presumptive coliform count and the probable number of faecal coli (90 per 100 ml.) indicate marked excretal pollution of recent mammalian origin. This water in its present state is bacteriologically unsatisfactory.

(4) *Sample received* 28/12/49

Direct and enrichment cultures made from this sample of water fail to reveal the presence of Cl. Welchii.

The Paper, which was received on the 28th March, 1955, is accompanied by three photographs and seven sheets of drawings, from which the half-tone page plates, folding Plates 1 and 2, and the Figures in the text have been prepared, and by the following three Appendices.

THE CAITHNESS REGIONAL WATER SUPPLY SCHEME

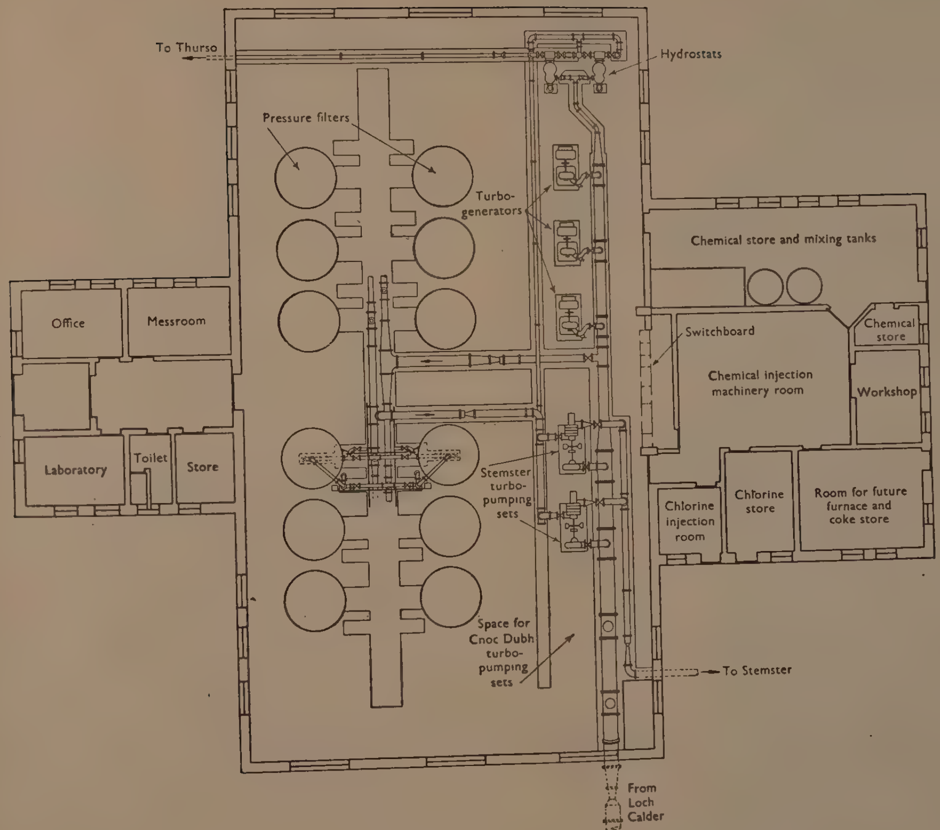

Fig. 5.—HOY PUMPING STATION—INTERNAL ARRANGEMENT

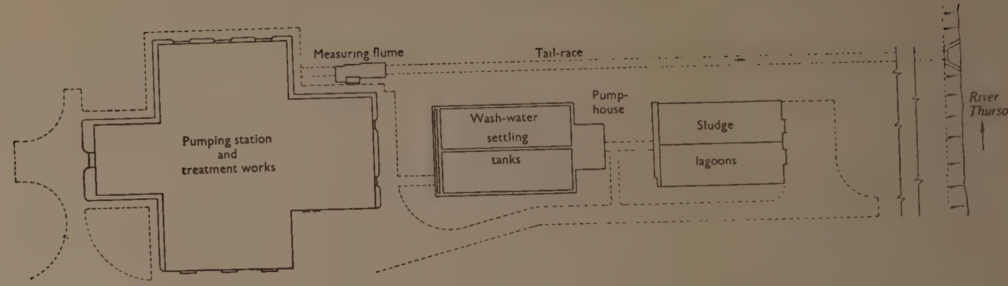

Fig. 4.—HOY WORKS—GENERAL LAYOUT

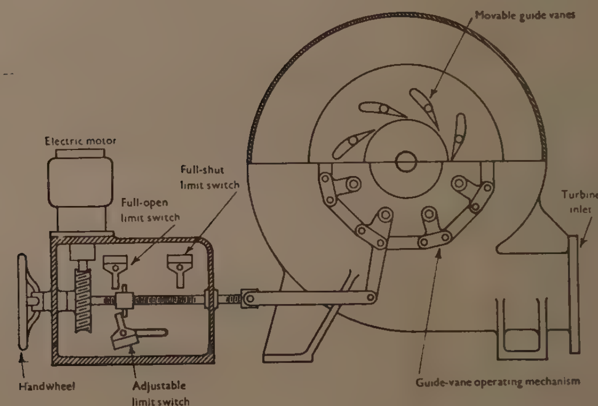

Fig. 9.—FRANCIS TURBINE CONTROL GEAR

William Clowes & Sons, Limited: London

THE CAITHNESS REGIONAL WATER SUPPLY SCHEME

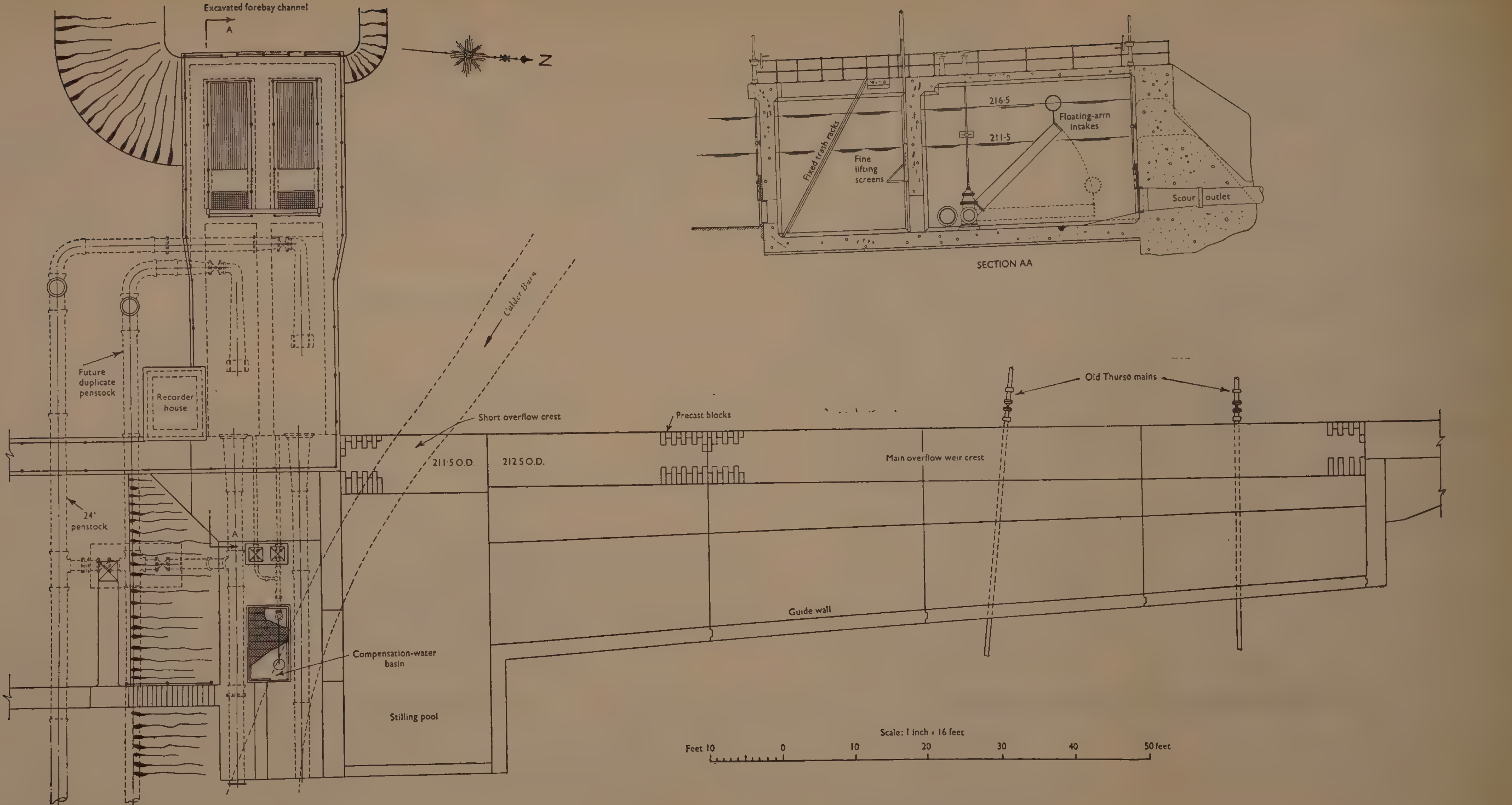

FIG. 3.—ACHAVARN WEIR

W. A. R. BAKER, W. M. JOLLANS, and J. N. DALE

WILLIAM CLOWES & SONS, LIMITED: LONDON

Discussion

The Authors, introducing the Paper, said that two problems of interest had been met during the first 18 months' operation of the scheme.

The first was icing of the screens at Achavarn. During the very severe winter of 1954/55 Loch Calder had frozen over and the intakes were hardly visible because of snow, but water had continued to pass through the screens below the ice and the head loss across them had never been more than a few inches. But trouble had been experienced on one occasion during the following winter when a severe and prolonged gale had swept across the loch blowing snow and ice towards the intake and choking the fine screens with a thick mat of honeycombed ice which had taken several hours to clear.

The other problem had been caused by eels getting in to the penstock at the dam and travelling down to the pumping station to get jammed into Venturi tubes or between turbine blades or in other equally inconvenient places. With the help of Dr Frost of the Freshwater Biological Association the eels had been identified as female silver eels whose migratory period occurred generally in the months of October and November. They were between 2 and 3 feet in length and entered the penstock by swimming under the fine screens when they were lifted for cleaning, so that the solution seemed to be to have double screens at the intake; that was being arranged.

Mr R. le G. Hetherington (Partner, Messrs Binnie, Deacon, and Gourley, Consulting Engineers) observed that the population served by the scheme was not very clearly stated. In fact, the scheme served the whole county, apart from Wick, which had an independent and satisfactory supply of its own from another loch. The present population in the area served by the scheme was thus about 20,000, and a future increase of 10% was allowed for, giving a future population of 22,000. The scheme was designed to give a supply of 1,550,000 g.p.d., which was therefore equivalent to an all-in consumption of about 70 gal/head/day. That might seem a very high figure for a rural area, but for some reason domestic consumption of water in Scotland was higher than it was in England, and in the present case 40 gal/head was allowed for domestic consumption and the remaining 30 gal was for agriculture and other metered supplies.

The long lengths of main involved in serving such a small population inevitably made the scheme an expensive one, and, apart from keeping the capital cost to the minimum, great care had had to be exercised to see that money was not expended unnecessarily soon and that, so far as practicable, each section of the scheme could be brought into use soon after it was completed. It was for such reasons of economy that the pumping rate in phase II, and consequently the size of the pumping main, was being reduced, as mentioned in the Paper.

In view of the necessity for economy the design of the Hoy pumping station and of Nybster water tower might be thought extravagant, but the whole county of Caithness was very open and barren, and those buildings were very prominent and could be seen from a great distance in all directions; therefore it had been thought necessary to make them look presentable. The Paper rightly gave prominence to the design of the pumping machinery. What were called the hydro-pumping sets had been the successful outcome of co-operation between the consulting engineers and the manufacturers.

There were references in the Paper to the fact that the supernatant water from the wash-water settling tank had to be pumped into the tail-race channel and also that the tail-race flume was occasionally drowned when the Thurso river was in flood. Both those disadvantages could, of course, have been overcome by raising the level of the pumping station, but it had not been economic to do so in view of the loss of head on the turbines that would have resulted.

Mr J. M. L. Bogle (Partner, Messrs Lemon & Blizard, Consulting Engineers) remarked that the Authors might not be aware that, by carrying out a water supply scheme in

Caithness, they were following in very distinguished footsteps. About 150 years ago Mr Thomas Telford, afterwards to become the first President of the Institution, had made a tour of the Highlands chiefly in connexion with roads and bridges, but he had found time to design a water supply for Wick. What had happened to that water supply, and was there any trace of it left at the present time?

Mr Bogle referred to the use of surplus water for power purposes. The idea of using water to pump water was a very alluring one to engineers; it was getting something for nothing, in a way, and it was therefore very attractive. In the present case no doubt there were good reasons for it. The only other case of which Mr Bogle was aware where that arrangement of using surplus water had been adopted was in the Tavy water scheme; in that scheme, described by Stuckey[1] in a Paper on the Plymouth Water Works, surplus head in the water main from Barrator had been used to pump up water from a low level in the River Tavy. That scheme had been possible only by a fortunate combination of circumstances, including one large 33-in. duplicate main which had a capacity almost entirely surplus to requirements. That had made it an economic proposition.

In the present case, it was stated at the foot of p. 370 that when the scheme was completed it was proposed to duplicate the 24-in. raw-water pipelines, which he imagined would cost about £168,000. No doubt there were good reasons for that heavy capital expenditure, but it might be worth while to work out what the comparative cost would be of pumping, with the capital cost taken into account, by the water turbine method and by diesels.

Mr Bogle showed a slide on which were given the approximate figures at which he had arrived. The Authors' scheme comprised two 24-in. mains to carry about 7,000,000 g.p.d., 7,000 yd long, costing £168,000. The additional cost of the turbine generators and pumps would be about £10,000; the loan repayments on the pipes for 35 years, and on the machinery for 15 years, came to £11,500 a year. The running costs, exclusive of maintenance, were nil, so that there was a total cost of £11,500 per annum.

In the alternative scheme, one 14-in. main costing £49,000 was ample for the 1,500,000 g.p.d. required for the supply, and diesel generators and pumps costing £13,000 would pump it up to the various reservoirs. The loan repayments, on the same basis, were £4,375 a year. The fuel oil would cost £3,000 per annum, making the total annual cost £7,375; there was therefore a big saving, if other reasons were disregarded.

Would the Authors say whether the use of diesel generators had been considered? In a similar scheme recently undertaken for the South Devon Water Board, on the other hand, it had proved economical to generate power for the filter house by providing a pipe 1 in. greater in diameter than otherwise necessary, which ensured 70-ft surplus head at all times. The calculations, which had been prepared by Mr Dale's firm, were complicated because of the varying factors to be taken into account—feed-water level, rate of flow, pipe friction, and output required from the generators. The water levels in the reservoir, which was about one mile away, would vary from a future top water level (after raising) of 1,131·5 and a present T.W.L. of 1,121·5 to normal low water level 1,085.

The rates of draw-off varied from the initial demand, 1,600,000 g.p.d., to the ultimate demand of 2,750,000 g.p.d. The basic load at the filter house for instruments, lighting, chemical gear, flocculating paddles, etc., was 17 kW. That was obtained from the generators at all times. If the wash-water pump were in use at the same time as the basic load it called for 44 kW. That could be obtained at all times when the reservoir was full, but when the reservoir was falling it was necessary to increase the flow down the pipeline to the maximum rate by temporarily running water to waste. The heating of the pumping station required 35 kW—much more, apparently, than in the case of the Authors' scheme, where the figure was 15 kW (p.369). That could be obtained only when the reservoir was full. That did not matter much, since heating would only be required in the winter, when the reservoir was nearly always full.

In order to lessen surge at the South Devon water turbine generator sets, very large

[1] P. J. Stuckey, "The River Tavy Scheme." J. Instn Wat. Engrs, vol. 8, 1954, p. 303.

flywheels had been recommended. Had anything of that kind been installed in connexion with the surge trouble anticipated at Hoy?

The Chairman said that he noticed that the Authors had taken the wise precaution of lining the mains with concrete, because the nature of the water in Caithness was not very different from that from the Elan valley. In both cases the water came from a rather peaty upland gathering ground. Had the concrete also been given two coats of bituminous paint? It had been found in the case of Elan water that unless that was done the water was slightly aggressive to the cement. To illustrate what took place, in connexion with one contract a 7-ft-dia. tunnel had been lined with concrete but had not been painted with bitumen. At the end of that tunnel, where there was a reduction to a pipeline, there was a steel special pipe lined with concrete and on the concrete was written with bituminous paint "Pattern No. Y". He had entered that tunnel about 20 years after its construction and those letters were standing out embossed about $\frac{1}{16}$ in. on the concrete. The rest of the surface of the concrete had eroded away, or the cement had been dissolved away, because it had not had the protection of the bituminous paint.

There was one other point he wished to mention in connexion with the pipeline. Very wisely, because of the economics of the problem, the Authors had not sited their treatment works just at the head of the raw-water pipeline, but he gathered that the water was of a peaty nature very similar to the Welsh water, and he wondered whether the Authors had been into that pipeline since it was laid to see whether peat was building up on the periphery of the main. At the Elan valley reservoirs, from the outlet to the roughing filters, which were situated about $\frac{1}{4}$ mile away, there was a tunnel about 7 ft $\times$ 7 ft, and once every year the aqueduct was emptied for the purpose of inspecting, cleaning, and painting, and every year they had to strip from $\frac{1}{2}$ to $\frac{3}{4}$ in. of peat from the walls. Below the roughing filter that deposition of peat had not occurred.

Mr Bernard Whitteron (Partner, Messrs Howard Humphreys & Sons, Consulting Engineers) asked for more information about the operating arrangements. Was only one shift worked at the filter house?

Mr Jollans had referred to ice forming at the head works—what was the arrangement for operation there? He inferred from Fig. 1 that the Stemster reservoir commanded the whole district, and therefore the supply to the farther service reservoirs was gravitational. If that was correct control arrangements would be needed only for Stemster. He raised that point because Mr Baker had mentioned the difficulty of controlling the reservoir in phase I.

It was stated in the Paper that various considerations made it desirable to use hydrostats for Thurso. What were those special considerations and why were hydrostats particularly applicable in that instance? So far as the demand was concerned, was the peak daily demand which had been expected in accordance with the usual 2 to $2\frac{1}{2}$ times the average, or did it approach some of the much higher figures which had been mentioned recently with regard to rural areas? The peak demand must necessarily vary with the size of the group of villages that was being supplied.

So far as the heating requirements were concerned, presumably the whole building was being warmed, although that involved heating a very large amount of space, usually just to keep a few instruments warm. The risk of freezing in the pipes might be rather unusual in the north of Scotland, but normally the water running through the pipes was unlikely to freeze unless the flow was checked, so it might be possible to save by merely restricting the heating to essential points.

What was the extra cost of casing the Nybster water tower compared with leaving it uncased or building a concrete water tower?

Mr F. H. Russell (Sir William Halcrow & Partners), referring to Mr Jollans's comments on the trouble that had occurred owing to the freezing of the fine screens, asked whether any thought had been given to the possibility of including an electrical circuit for the de-frosting of those screens, thus avoiding the rather arduous task of doing it manually.

It was mentioned in the Paper that in the construction joints at Stemster reservoir the usual precaution had been taken of using copper strips. He believed that nowadays the usual precaution was not copper—it was found more satisfactory and considerably cheaper to use either rubber or P.V.C., according to whether the movement was great or small.

Mr P. N. Wilson (Chairman, Gilbert Gilkes & Gordon Ltd) said he was interested in the question of the freezing up of the screens. With hydro-electric sets in northern Canada and the United States that was, of course, a major problem. The method of electrically or steam heating the grid bars was often adopted. In Britain it was a nuisance rather than a real problem, and it was a trouble that arose so seldom that it was a question of how much money should be spent on trying to avoid freezing up. Were the screens totally submerged or did the tops of the screens project above water level? It had been found that by totally submerging the screens the bars, or the grids, did not fall below the temperature of the water, and frozen ice was less likely to cling to them and block them. If the screens projected above the water level the cold air, which might be 10 or 20°F below freezing point, would chill the grids to that low temperature and the cold would be conducted to the bars below water level. That allowed the ice to stick to them.

Again, the freezing over of reservoirs was a considerable problem in northern Canada. Eighteen months ago he had been in the Yukon, where there were one or two small hydro-electric schemes. There, of course, everything froze during the winter, and any hydro-electric set could only draw from storage underneath the ice. Engineers there did not seem to worry at all if the head- and tail-races froze over; in fact, they encouraged it and endeavoured to establish a high level in the early part of the winter and then, if possible, drop the water level so as to get a layer of up to 1 ft of ice, with a small insulating air gap so that the water was able to flow underneath.

The problem of eels was an old trouble with small water turbines having very small water passages. Eels had always been a great nuisance. It might be imagined that any eel getting into a water turbine revolving at 1,000 or 1,500 r.p.m. would be properly "jellied" before it could do much damage, but that was not so. There had been a lot of trouble with small water turbines as a result of eels getting completely jammed in the runners. He therefore urged anyone designing a small hydro-electric scheme where the passages in the turbines, etc., were not sufficiently large to pass those pests to make certain that the screens would stop them getting there.

Mr H. G. Cousins (Consulting Engineer) commented on the Authors' statement in the Paper that the use of the shell roof was a new experience to them, but that the results suggested that it was a form which could be more widely used. The calculations for shell roofs always seemed rather frightening and the idea had spread that the construction of them was also very difficult. Mr Cousins suggested that the construction of a shell roof was probably simpler than the majority of concrete work, and certainly, in the present instance, it was far more simple than the work below ground, which called for such complicated shuttering that the drawings needed to be supplemented by a model. In construction, too, he had always found that the contractors who built roofs of that nature always approached them with trepidation, feeling that there was something very special about them. However, a shell-roof structure consisted mainly of very large plain surfaces, not cut up and not very difficult for the carpenter to deal with.

It had been suggested that the pumping-station structure was rather expensive. Mr Cousins did not consider that was true, because the quantities of materials used in the structure itself were very small. A concrete roof had many advantages in a district such as Caithness, where what was really needed was a permanent structure which could be erected by a local contractor using local materials. The first occurrence on the job had been the disappearance of the contractors' hut in a gale—that explained also the damage to one of the end-frames, as they were called, i.e., the two-pinned arches which supported the slab. In that gale one of the arches, which were about 10 in. thick, had been deflected sidewise about 2 or 3 in. causing a slight crack in the bottom. The crack, being at the

bottom of the column, was in the position where it would subsequently get most compression, and therefore it was not a serious piece of damage.

Mr F. E. Bruce (Reader in Public Health Engineering, Imperial College) asked whether the Authors could give a little more information on the hydrology of the scheme. He noticed that the ultimate yield for which the scheme had been designed, 7,000,000 g.p.d., coming from a catchment area of 5,600 acres, worked out at a usable yield of 20·1 in. of rainfall. The long-term average rainfall in that area was 36·5 in., so that the total allowed for losses, evaporation, and percolation, amounted to 16·4 in. per annum. When it was considered that the original surface area of the loch was 840 acres out of a total of 5,600 acres, and the Authors said that that increased considerably as the water level was raised, evaporation losses would be expected to be rather high. The rock at the site of the dam apparently consisted of highly laminated beds of Old Red Sandstone which were of high porosity. It would appear, then, that there might be rather high losses due to percolation as well as through evaporation.

Could the Authors give some idea of the assumptions which had been made in that respect, from which they had come to the conclusion that they could obtain a reliable yield of 7,000,000 g.p.d.? In the same connexion, on the information available at the moment it was not possible to calculate exactly the amount of storage that had been provided, but even ignoring the fact that the reservoir surface area increased considerably as the water level was raised, the amount of increased storage for phase I was very close to 1 year and for phases II and III would amount to considerably more than 1 year. Allowing for the increase in the water area, that storage might well amount to about 2 years, which seemed abnormally high. It was appreciated, of course, that with a relatively small structure it did not add a great deal to the cost to provide a large safety margin on the amount of storage, but perhaps the Authors could give some information about the variability of the rainfall which would explain the need for such a high allowance of storage.

*** **Mr R. W. S. Thompson** (Engineer, The Derwent Valley Water Board) observed that although the Paper described a relatively small scheme, it nevertheless contained many features of interest.

There were, very properly, frequent references to the economic and financial considerations, but no information as to the actual cost of any part of the work was disclosed. That was a rather unfortunate omission, since cost particulars of even a general character would have greatly added to the value of the Paper.

It was not apparent why a decision had been made to increase the size of the raw-water pipe and generate power for pumping. Without data on the cost of 4 miles of 24-in.-dia. pipe, or an alternative pipe of smaller size, and the cost of electricity in that locality, the reader was unable to judge whether the decision was a sound one. In view of the length of that pipe and the relatively small head available, it appeared to Mr Thompson to be very doubtful whether any economy had been effected. If the decision had been based on a desire to make the work self-contained with regard to power, that might have been stated.

The general use of concrete lining in cast-iron pipes was to be commended and was an example that should be more widely followed.

Mr R. E. Bartlett (recently with United Nations Technical Assistance Administration) observed that the Authors had adequately proved their point that on such schemes the population to be served was no yardstick of the engineering problems or interest.

The design of Hoy pumping station was obviously the result of considerable thought at the design stage; the shell roof and absence of crane rails in the pump and filter house would appear to be the ideal solution for such a building. The position of the switchgear

*** This and the following contributions were submitted in writing after the closure of the oral discussion.—Sec.

panel suggested that the design of the machinery had been well advanced before the building design was commenced; was the panel of the enclosed "unit" type (it would appear so from Fig. 5, Plate 2), and at what stage of the design had the switchgear manufacturers' details been completed? It would seem that a larger number of flexible joints in the pumping station pipework would have been an advantage—both from the point of view of installation and for future maintenance and repairs.

The Authors had stated that the population of the county was only about 15,500 and that it had shown a slight decline in recent years, yet the scheme had been designed for 30,000. That was equivalent to an annual increase in population of about 1·5% over the next 40 years. The Paper did not give any indication of the period that the design was intended to cover, but 30–40 years was generally considered to be sufficient; on the evidence put forward an average annual increase in population of 1·0% would appear to be adequate (the world increase at present is said to be only about 1·15% per year). At 1% for 30 years the design figure would be about 21,000 persons; for 40 years it would be about 23,000.

From Table 1 the total supply available at phase III appeared to be 1,550,000 g.p.d. to serve a population of 30,000; that was equivalent to more than 50 gal/head (including livestock allowance). Mr Bartlett thought that it would be generally agreed that a figure of 30 gal/head/day should be more than adequate for the domestic consumption of such a rural area, leaving an equivalent of 20 gal/head/day for livestock. What allowances had been made for livestock, and had they been based on any research in the area? It would seem that the figures to be used could be less than those usually taken (30 g.p.d. for a milking cow to cover drinking, milk cooling, and washing down, and 10 g.p.d. for a horse, etc.); the area would appear to be essentially rural and many streams would be available for stock watering.

On the construction side, had air testing of pipelines been considered as an alternative to water-pressure testing?

Mr I. R. White (Resident Engineer, Ampthill R.D.C., for Messrs Binnie, Deacon & Gourley) noted that the Authors had considered it worth mentioning the necessity of pipe-laying in fields, and that it had been found to result in smooth working of the procedures involving the serving of legal notices, the practical and tactful preliminary approaches to farmers, and the settlement of compensation claims. Along many trunk roads in rural areas there were pipelines that might eventually be covered by widened carriageways. The effect of that, both during the course of pipe-laying (even in verges) and during maintenance work after road widening, was bound to be an obstruction to the ever-increasing demand for fast traffic-flow. Being usually a matter of unforeseen expenditure by the government, road improvements were often in doubt. It might appear irrelevant, therefore, for an engineer investigating pipeline routes to concern himself with such matters or to recommend the siting of the pipeline in adjacent fields rather than in road verges on those grounds alone.

Had it been found, however, that the advantages during pipe-laying on the Caithness scheme and during maintenance (if any) of such pipelines extensively sited in fields outweighed the disadvantages, considered merely from the water authority's viewpoint?

Referring to the construction of Brabstermire reservoir, Mr White inferred that the "larger" sizes of the poor local aggregates referred to $1\frac{1}{2}$ in. upwards. Had the contractors taken any particular care in grading to reduce the voids in the grouted concrete caused by having to limit themselves to an apparently small range of aggregate sizes? It might otherwise appear to be an extravagant use of grouting constituents.

If the same method of construction had since been employed on other of their reservoirs, in both mass and reinforced work, would the Authors comment on its use where even good-quality aggregates were available in more accessible parts of the country?

Mr N. Sanyal (Reader in Civil Engineering, Government Engineering College, Jabalpur, India) asked the Authors to clarify some points.

First, the population of the county, now about 15,500, had shown a slight decline in

recent years and it was anticipated that the decline would be checked if not reversed. Why then was provision for 30,000 people being made?

Secondly, it was not clear why the storage of Loch Calder was not helpful in initial sedimentation. It appeared that sedimentation was to be dispensed with and reliance was to be placed upon a low rate of filtration; since it was known that filters worked efficiently when the turbidity in the raw water did not exceed 10 to 15 p.p.m., would it not be preferable to use (at least) micro-straining? It would certainly be interesting to know how the rate of filtration had been connected with the rate of removal of the suspended matter. Normally the rapid filter worked at about 80 gal/sq. ft/hour; how then could the rate of 82 gals/sq. ft/hour be considered satisfactory in the absence of sedimentation? Why was a pressure filter preferred to a rapid gravity filter?

The specified dosages of 40 p.p.m. aluminium sulphate, 5 p.p.m. sodium aluminate, and 25 p.p.m. of lime were very heavy and would result in costly maintenance. Since the raw water had a low pH value and required prechlorination, could not chlorinated copperas be tried with advantage?

A figure of 0·01 p.p.m. of residual chlorine had been aimed at.* It was not clear why that figure was so low, particularly when long distances were involved in the distribution of water. Why did not the Authors consider chlorination in stages compatible with the long distribution lines?

From Fig. 3, Plate 1, it appeared that the original concrete dam section had been designed for future raising. How was bonding between the old and new section to be achieved?

How would the cost of the extra storage needed to produce the power to energize the pumps compare with the purchased power?

Mr Baker, in reply, thanked Mr Hetherington for his clarification of the figures of population given in the Paper; the interpretation was quite correct.

Mr Bogle had presented figures showing that, with the long raw-water pipelines, the use of hydro-power for the scheme was extravagant vis-a-vis the use of Diesel engines. Mr Baker accepted Mr Bogle's estimates of the capital expenditure involved—in fact he considered that the difference Mr Bogle had indicated could justifiably have been increased somewhat for the incidentals. For instance, Mr Bogle had taken no account of the reduction in size of the headworks for the reduced yield which the abolition of hydro-power would permit. But Mr Baker did not agree with Mr Bogle's comparison of the annual charges involved. With Diesel engines, three attended shifts would be necessary, and that would involve at least £2,000 per annum for the salaries of the necessary staff for operation and maintenance. The cost of fuel oil was, Mr Baker thought, underestimated. The shaft horse-power of pumps for Stemster, Thurso, and Cnoc Dubh respectively, was approximately 70, 6, and 35; with local lighting and power the total requirements would amount to about 130 h.p. With reasonable allowance for efficiencies in the electrical generators and motors, the brake horse-power of the Diesel engines would have to be at least 150, and the annual cost of fuel about £4,000. So the additional annual cost of £4,125 (£11,500 less £7,375) mentioned by Mr Bogle should be reduced to about £1,000. But for that £1,000 per annum practically complete self-sufficiency was obtained, and that was very important in Caithness, where communications were liable to prolonged interruptions by snow; it should be remembered that practically the whole county was dependent on the scheme for its water supplies and reliability was a first essential. Moreover, any increase in the cost of Diesel oil would completely change the picture; Mr Baker, therefore, considered the decision to use water power was a wise one.

Several speakers had touched on the subject of the blocking of fine screens by freezing. Since those screens were normally submerged and protected by the trash racks, any such trouble was likely to be infrequent; Mr Jollans could better say what had in fact occurred.

Mr Whitteron had inquired why hydrostats were preferred for the Thurso supply. In

* A misprint in the advance copy of the Paper has since been corrected.—Sec.

the first place, owing to the relatively small capacity of those units, it had been found that the overall efficiency of a turbine-driven pumping set would be lower than that of a hydrostat. Again, in view of the delivery to Thurso being controlled by a ball valve at Ormlie Reservoir, it had been considered undesirable to permit a turbine-driven pump to continue to run against a closed valve. That would have involved waste of water and the risk of overheating of the pump, unless expensive control apparatus were also introduced.

The heating of the building and its apparatus presented a problem difficult of solution; at present a number of radiators were in use, but that aspect was being carefully watched.

It was difficult to answer Mr Whitteron's question as to how the cost of Nybster water tower compared with an uncased tank or an elevated reinforced concrete tank. A bare steel structure was not considered appropriate for two reasons; it would be undesirable aesthetically and, especially in the early stages of the scheme when demands were low, would be vulnerable to frost; it was, therefore, felt that a financial comparison in that case could not arise. With regard to a reinforced concrete tank, Mr Baker felt that there would not be much in it either way.

Mr Russell had mentioned that rubber or P.V.C. strips were more usual than, and were preferred to, copper sheets for the purpose of spanning construction joints in reservoir walls: Mr Baker felt that Mr Russell was probably referring to expansion or contraction joints. If so, that was not denied, but Mr Russell had, Mr Baker thought, misread the extract to which he referred. The word "usual" referred to precautions at construction joints and not to the use of the copper strips; the latter were inserted across horizontal *construction* joints as an additional precaution against seepage; expansion and contraction joints were separate matters.

Mr Bruce had raised questions concerning the hydrology of the scheme. The first thing to remember was that the scheme had had to be designed in the absence of adequate long-term records. The average rainfall had been assessed at 36·5 in/year, but its pattern was not certain and much less was known of the flow of the Calder Burn. The storage had been estimated from diagrams by Deacon and Lapworth which, of course, had been based on practical conditions. Those estimates had suggested that the ultimate design level was not extravagant. The Stage I level of 211·5 O.D. was somewhat liberal, but intentionally so in view of the paucity of information. For Stage II, should the available water have then proved to be somewhat less than anticipated, the second penstock could be of a larger size to reduce head losses and secure the same power from less water.

Mr Thompson, in his written communication, had asked for financial details. Mr Baker had, of course, touched on that aspect in his introduction. It was difficult to give a concise financial picture of such engineering works but the following figures might be of interest. Phase I would cost about £1,000,000 with loan charges of about £60,000 per annum; with the plant working to design capacity that would mean about 3/1d per thousand gallons of water; that was not too bad when the very extensive distribution network was considered. Phase I had been saddled with the cost of the headworks. Phase II was estimated to cost about £400,000; that included a very long pumping main to an area of small demand and the result was that the average cost of water would rise by about 4d per thousand gallons. The duplication of certain mains, mentioned in the Paper, would raise the cost a further 4d. Phase III would probably reduce average costs, but little was as yet known of that. None of the foregoing figures included local expenses, which amounted to about 2½d per thousand gallons. Mr Thompson would be interested to see Mr Bogle's remarks on the use of Diesel engines and Mr Baker's reply.

Mr Bartlett's and Mr Sanyal's points on population had been largely answered by Mr Hetherington's clarification of the figures. In general the scheme had been designed on the basis of 40 gal/head/day for domestic consumption, the corresponding figures for livestock being:—

Cows in milk	30 gal/head/day
Other horned stock . . .	10 ,, ,, ,,
Horses at work	10 ,, ,, ,,
Horses at pasture	6 ,, ,, ,,

Actually, streams for stock watering were not common in Caithness and piped supplies to troughs were extremely numerous.

Mr White had invited comments on the merits and demerits of laying pipelines in adjacent fields rather than verges or carriageways of roads; he had raised the question of possible widening taking place over the pipeline.

Mr Baker felt that the only disadvantages of laying pipelines in fields were the slightly more difficult access for construction and repairs, and possibly high compensation for land damage, etc.; if the carriageway were later extended over the pipeline the situation was no different to what it would have been had the pipeline been laid in the road, and in such a case one would have had the initial advantage of avoiding disturbing traffic and the trouble and expense of laying in the carriageway, coupled with the subsequent accessibility when the road was widened.

Regarding the Brabstermire reservoir the desirability of grading the aggregate of grouted concrete was, Mr Baker thought, a complicated question in that one had to balance the saving of grout against the cost of reducing voids in the placed aggregate; it was impossible to generalize on that matter, for circumstances varied so greatly.

Mr Baker felt that the grouting of pre-placed aggregate as a method of forming concrete, might always be kept in mind, but whether or not its use was appropriate for a particular application was a matter for individual judgement.

Mr Sanyal's points concerning sedimentation in Loch Calder was interesting, but unfortunately it was not effective; the type of initial sedimentation envisaged as desirable was, of course, accompanied by coagulation. That was not practicable at the pumping station unless all head were lost, or very expensive pressure sedimentation installed; if the treatment were at the loch it would, of course, mean two pipelines to the pumping station and the administrative disadvantage of a second establishment at the intake. Similar remarks applied to micro-straining and to the necessity of pressure filters vis-a-vis rapid gravity filters.

Mr Sanyal had evidently misread the quoted filtration rate; the design rate was 68 gal/sq. ft/hour, and 82 gal/sq. ft/hour only occurred when two filters were being washed simultaneously.

Concerning the raising of the dam, that could conveniently be done, and an effective key obtained, by removing the crest blocks on the water face; it was, of course, intended to raise the water level to 216·5 O.D.—as mentioned in the Paper under "Fundamental data"—from its present temporary level of 211·5 O.D.

Mr J. N. Dale replied to the question by Mr Bogle as to whether flywheels were fitted to the Caithness turbines similar to those for the turbines for the South Devon Water Board. In both installations the turbines were of the impulse type and the speed was controlled by a quick-acting governor operating a deflector fitted between the jet and the turbine runner. Changes of load caused the jet to be diverted to a greater or less extent from the runner, and that did not create any pressure surges in the pipeline.

The capacity of the turbine for the South Devon Water Board was small in relation to the power required by the wash-water pumps, and a flywheel was fitted to each turbine to deal with the momentary overload that occurred when the wash-water pump was being started up. Those conditions do not apply to the Caithness installation and therefore flywheels were not fitted.

The consumption of water of the South Devon Water Board turbines was regulated by means of a needle nozzle arranged for hand or electric operation. The flow of water through the Caithness turbines was regulated in a similar manner but the mechanism was arranged solely for hand operation. In both cases the mechanism was designed to ensure a relatively slow movement of the regulating gear, thus preventing the creation of any undue pressure surges in the supply pipes.

Mr W. M. Jollans, who also replied, said that he was very interested in the reference by Mr Bogle to Mr Thomas Telford who, as the engineer for the British Fisheries

Society of London had done a great deal for Wick and who was still commemorated in Telford Street. His work there was by no means confined to water supply but he seemed to have taken particular pride in the way in which he had overcome considerable technical difficulties to lead water from the Loch of Hempriggs into the Burgh by means of an open channel which closely followed the contours of the ground. The water was said to have been famous for its keeping qualities aboard ship and to have been the favourite of all mariners who frequented the port in the days of the Baltic Fleet windjammers. The system was to a limited extent still in operation, though parts of it had been allowed to fall into disuse.

With regard to the Chairman's remarks no evidence had come to light of peat building up inside the pipes to any noticeable degree. There was considerable evidence of the same peaty water having attacked a cast-iron pipe laid in 1876, for sections cut from that main were badly tuberculated but the concrete-lined pipes had not been in long enough for any definite conclusions to be drawn. No doubt a bituminous paint would give additional protection in certain cases though he could not say whether it would have been worth while in that particular instance. It was of interest to note that the pH-value of the raw water varied from 7 up to 8·05 and possibly that indicated that it was less aggressive than acid peaty water normally was.

Mr Whitteron had inquired about the operating arrangements at the pumping station. Only one shift was at present being worked though the hydrostats and occasionally the Stemster pumps were left running unattended overnight. It would probably be necessary initially to work two shifts when the Cnoc Dubh pumps were installed, and the demand throughout the county had somewhat increased. With regard to the ratio of peak daily demand to average he could not, unfortunately, give any really reliable figures. In the present stage of development pipe sizes and reservoir capacity were such that no operating worries were occasioned on that account.

The only trouble which had been experienced with freezing of pipes at the pumping station was in small-diameter pipes leading to gauges and Venturi meters and a low degree of local heating had quickly cured it. Elsewhere in the county frost had been responsible for the usual number of burst service pipes and in that connexion particular vigilance had to be exercised to ensure that bursts which occurred at field troughs in exposed and isolated places were detected and repaired expeditiously. All connexions to such troughs were fitted with frost cocks but short of metering them all individually it seemed impossible to avoid having some wastage there.

Mr Wilson was perfectly correct in stating that the freezing up of screens was a nuisance rather than a real problem, though one didn't make such a remark to the man who was working in a bitterly cold howling gale trying to clear them. The fine screens were in fact totally submerged during the winter but they were perhaps not deep enough to prevent pack ice from building up against them on occasion. Fortunately the trouble was of infrequent occurrence and seemed to be confined to thaw rather than to frost conditions. The suggestion that electrical defrosting of the screens might have been used was unfortunately not practicable, since power was not available at the site.

It was small comfort to learn that the eel problem was widespread, but now that duplicate fine screens were being fitted he hoped that no further difficulties would be experienced on their account.

Mr Bruce had raised the question of evaporation and other losses. Detailed rainfall and run-off figures were now being taken but unfortunately accurate run-off figures were only yet available for the 6 months October 1955 to March 1956; from them the evaporation and other losses amounted to 5·02 in. for the half-year. Rainfall readings had been taken regularly for about 4 years and no abnormal variability had been noticed.

Mr Bartlett had suggested that more flexible joints might have been included in the pumping-station pipework and there was no doubt that they would have been welcomed by those who occasionally had to dismantle sections for maintenance purposes. It should, however, be borne in mind that with so many bends in the pipework almost every joint was in tension and it would have been virtually impossible to have had anchor blocks or ties

ıt every bend. The bends themselves did of course allow short sections to be removed vithout great difficulty.

With regard to air testing of pipes there was of course a serious risk involved in the case f the larger diameters, particularly if the contractor allowed any appreciable pressure to levelop. Sometimes, however, when there was a slight leak in the pipeline and when every oint hole was full of water it was virtually impossible to locate the trouble by a water est and in such cases a low-pressure air test had been permitted prior to the specified vater test.

The chemical dosages quoted by Mr Sanyal certainly did seem on the high side. They vere estimated at a time when the plant was being commissioned and when numerous rials were being made. Subsequent experience over a longer period had shown that he dosages varied throughout the year as followed: aluminium sulphate 40–55 p.p.m., odium aluminate 2–4 p.p.m., lime 7–14 p.p.m., and chlorine 0·6–1·0 p.p.m. Prechlorination was not being used at present though the necessary equipment was installed. The otal cost of pumping and treating the water, including chemicals, labour, and supervision, but excluding capital charges, worked out at about 2¾d per thousand gallons, vhich did not seem unreasonable. Chlorinated copperas had in fact been tried experinentally on earlier water samples, but the method adopted had been preferred. Certain lifficulties had occurred in feeding it into the water and it was thought that the present ırrangements were simpler and quite satisfactory.

In commenting on the chlorine residual value Mr Sanyal had raised a matter which was ery much in the minds of those responsible for operating the plant. It was quite true hat chlorination in stages offered a theoretically better solution to the problem, but to ave adopted it would have involved greater capital and maintenance expenditure and hat had not been thought justifiable. The figure of 0·1 was arrived at empirically as the ighest value which could be used without receiving complaints from people living near he treatment works. There would be no hesitation about using a higher value if it were ound to be necessary. The figure of 0·01 mentioned by Mr Sanyal was a misprint which ıad now been corrected.

STRUCTURAL AND BUILDING ENGINEERING DIVISION

Tuesday, 28 February, 1956

Mr Ralph Freeman, Member, Chairman of the Division, in the Chair

The following two Papers were presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Authors.

Structural Paper No. 48

THE BASIS FOR DESIGN OF BEAMS AND PLATE GIRDERS IN THE REVISED BRITISH STANDARD 153

by

*** Oleg Alexander Kerensky, B.Sc., M.I.C.E., Anthony Ray Flint, B.Sc., Ph.D., and William Christopher Brown, B.Sc., Ph.D.**

SYNOPSIS

The Paper deals with the design of beams and plate girders with stocky and slender webs, and with equal, unequal, and curtailed flanges.

The problem of overall lateral instability of I-section members is stated.

Existing solutions for such girders subjected to uniform moments are extended to cover different types of loads (including loads on top flanges free to move laterally), intermediate and end restraints, and curtailments of flanges (applied to simply supported beams and cantilevers).

A single comprehensive formula for critical bending stress applicable to a practical range of girders (including T-sections) is evolved by the introduction of empirical constants, and by defining suitable effective lengths of compression flanges. The relations between critical stresses in an ideal and in an imperfect girder are derived and values of realistic critical stresses in compression flanges of practical girders obtained. Desirable and adopted load factors are discussed.

Recommended permissible stresses are presented in tabular form. The limitations and accuracy of the proposed method as compared with the exact rigorous analysis and results of special tests are discussed, and current standard methods of design are critically reviewed. Examples of allowable stresses in beams and plate girders calculated according to different specifications are given for a comprehensive range of girders.

The problem of lateral instability of webs and design of intermediate stiffeners is next discussed. The behaviour of slightly buckled webs is considered, and their ability to withstand loads greatly in excess of theoretical buckling values is emphasized. The action of web plates subjected to both shear and bending stresses is considered, and the load factor used with this combination is discussed. The minimum stiffener requirement based on theoretical and experimental results is included and the values of minimum stiffener inertia required are compared with the present standard methods of design.

Proposed design rules and several worked examples are given in the Appendices.

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* Mr Kerensky is a Partner of Freeman, Fox & Partners, Consulting Engineers; Dr Flint is a Lecturer in Civil Engineering, Imperial College; and Dr Brown is an Assistant Engineer, Freeman, Fox & Partners.

Introduction

MPROVEMENTS in the design of plate girders have become more desirable than ever vith the advent of welded construction. The stability of the compression flange nd of the thin web is assuming greater importance as considerations of economy ead to increases in the allowable stresses and reductions of thickness.

Whilst the behaviour of struts has been exhaustively investigated, the study of he analagous problems of compression flanges has lagged behind. It has long been ealized that a slender compression flange has a tendency to behave like a column. Reliance upon strut formulae is of little avail, however, since the end conditions, the ncremental application of load, the torsional rigidity of the girder, and the stabilizing ffect of the web and of the tension flange all affect the behaviour of the compression lange. There is, therefore, a great need to apply more rational theory to flange esign. Moreover, considerable advances have recently been made in the undertanding of the post-buckling behaviour of thin plates subjected to shear and bending. The application of this knowledge to the design of plate-girder webs greatly increases he range of their economic use.

Early in 1952, the British Standards Institution Committee revising the Specifiation for Girder Bridges, through its Chairman, Mr G. A. Gardner, O.B.E., pproached one of the Authors concerning the validity of existing formulae for ermissible flange stresses when applied to deep girders and half-through bridges. At about the same time, a panel of the Committee, under the Chairmanship of Dr Richard Weck, A.M.I.C.E., of Cambridge University, was set up to produce a set of ules for allowable shear stresses in girder webs, based on the work of Sparkes and Brown. From then onwards, for the next 3 years, theoretical analysis, evolution of implified formulae, and their experimental verification have proceeded.

The Paper sets out the results of this work. Data are given only for mild steel, lthough the high-tensile steels have also been considered and will be included in the Revised B.S. 153.

Permissible flange stresses

Review of existing British Standard design bases

The need for a comprehensive design formula applicable both to joists and plate irders, with equal and unequal flanges of uniform and variable cross-section has ong been felt, and has led to many researches into this field. The existing formulae iven in B.S. 153 (1937) and B.S. 449 (1948) are easy to use, but strictly speaking, are nly very approximately applicable to an average type of girder; when the range of pplication is extended to cover every variety, they can be over-conservative or angerously unsafe.

The straight-line formulae in B.S. 153 (1937) were developed in the days when the lastic theory had not yet been established and when really deep girders were proibited. It has now been shown experimentally that at failure, girders with ratios of $\frac{\text{depth}}{\text{web thickness}}$ and $\frac{\text{length}}{\text{radius of gyration about the } y\text{-axis}}$ not greater than about 90 evelop a plastic moment of almost maximum value, so that for these girders the llowable fibre stresses need not be sharply reduced from the maximum basic values iven by yield/factor of safety. On the other hand the compression flange of a

very deep girder will buckle laterally at a much lower stress than the flange of a rolled steel joist of the same width.

The equation $F_{bc} = 1{,}000K_1(r_y/l)$ given in B.S. 449 (1948) is based upon the solution for lateral elastic instability of girders subjected to constant bending moment and is derived by using average properties of rolled steel joists, omitting the term representing the influence of restraint of flange warping, and taking no account of the unavoidable physical imperfections. A factor of safety of 2 has been used in arriving at this simplified expression and it was thought that no transition curve from elastic to plastic failure conditions would be necessary. However, it can be proved theoretically and demonstrated experimentally that the ultimate-strength curve for any practical girder is, in fact, of the Perry–Robertson type. It is clear that the extension of the range of application of the simple expression into the transition zone of partially plastic behaviour or to deep plate girders, is quite inadmissible.

Other problems have for long been baffling the designer, or blissfully ignored by him, such as allowable stresses in girders with unequal flanges or in tees and angles, and the effect of curtailing flanges by varying either their widths or thicknesses. The solutions suggested herein apply to all these cases and make allowances for the imperfections of material and workmanship similar to those made for compression members. Unfortunately, problems of a flexural-torsional nature inherently produce mathematical solutions of greater complexity than those of bending or compression alone, and therefore the designer cannot expect to be provided with data which are simple yet comprehensive.

Proposed basis for estimating lateral buckling stresses

Members under uniform bending moment

The critical values of equal terminal couples, applied to a member in the plane of its maximum bending stiffness, and sufficient to cause overall lateral buckling have been derived by Timoshenko[1] and Chwalla.[2] The beam is assumed to be supported at its ends so that no restraint is afforded to bending actions, but with rotation of the end sections about its longitudinal axis rigidly prevented as in Fig. 1. The corresponding maximum fibre stress at the instant of buckling (the critical stress) may be expressed as:

$$f_{b\,.\,crit} = \frac{\pi}{Z_x L}\sqrt{\left[\frac{EI_yGK}{\gamma}\left\{1 + \frac{\pi^2 C}{GKL^2}\right\}\right]} \quad . \quad . \quad . \quad . \quad (1)$$

(Explanation of the notation used throughout the Paper is given on p. 460.) The beam is assumed to be of uniform cross-section, symmetrical about the x-axis, initially undeformed along its length, and of homogeneous material. It is implied that the section shape remains undistorted during buckling. For the particular case of the symmetrical I-section the above expression reduces to:

$$f_{b\,.\,crit} = \frac{\pi^2 EI_y h}{2Z_x L^2}\sqrt{\left[\frac{1}{\gamma}\left\{1 + \frac{4GKL^2}{\pi^2 EI_y h^2}\right\}\right]} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The second term inside the root represents the contribution of the torsional rigidity to the stability of the member. This is the governing factor for many shallow joists but is negligible for deep plate girders in which differential flange bending provides the major resistance to torque. When the web is sufficiently slender and unstiffened to allow sectional distortion of the types shown in Fig. 2, the influence of the torsional rigidity may be reduced. Nylander[3] has shown this effect to be of slight importance

[1, 2] The references are given on p. 443.

in beams of conventional proportions and it may be shown that the same is true for deep plate girders, even if unstiffened. Equation (2) has therefore been proposed as the basis for the clauses defining permissible bending stresses in the Draft B.S. 153.

It is seen that when the girder is very deep in relation to its span the critical stress becomes:

$$f_{b\,.\,crit} = \frac{\pi^2 EI_y h}{2Z_x L^2} \doteqdot \pi^2 E\left(\frac{r_c}{L}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Thus, in the limit, collapse of the compression flange occurs as an unrestrained strut under end axial load—a fact which justifies the application of many simple column formulae to the design of deep beams in the past.

These solutions require modifications when applied to members having unsym-

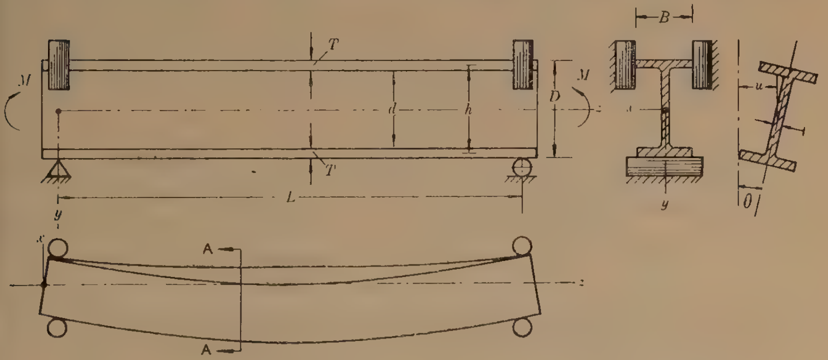


FIG. 1.—LATERALLY BUCKLED BEAM WITH THE TYPE OF END RESTRAINT ASSUMED IN DERIVING THE CRITICAL BENDING STRESS

metrical cross-sections. Where the section is symmetrical about the x-axis but not about the y-axis, as, for example, many rolled channels and zeds, equation (1) is still applicable provided that the relevant warping rigidity is introduced. For a conventional channel beam this term exceeds that for a similar I-beam and the critical stress is modified to:

$$f_{b\,.\,crit} = \frac{\pi}{Z_x L}\sqrt{\left[\frac{EI_y GK}{\gamma}\left\{1 + \frac{\pi^2 EI_c h^2}{2GKL^2}\left(1 + \frac{th^3}{4I_x}\right)\right\}\right]} \quad . \quad . \quad . \quad (4)$$

which for very deep members becomes:

$$f_{b\,.\,crit} \doteqdot \pi^2 E\left(\frac{r_c}{L}\right)^2 \sqrt{\left[\frac{I_y}{2I_c}\left\{1 + \frac{th^3}{4I_x}\right\}\right]} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Thus for similar flange and web dimensions, the channel section has always a higher buckling stress than a corresponding I-beam. By employing equation (2) as a basis, therefore, a conservative estimate will be obtained of the stability of members having webs offset from the flange centre-line.

If, on the other hand, symmetry exists only about the y-axis, as in Fig. 3, the shear centre and centre of twist no longer lie on the major flexural axis. As a result, the torsional effect of the applied moment on a slightly displaced beam will be increased when the shear centre is displaced towards the tension flange, and vice-versa. Moreover the warping rigidity must be modified and, in the limiting case of a tee section, disappears. Solutions to the critical conditions for members having such sections

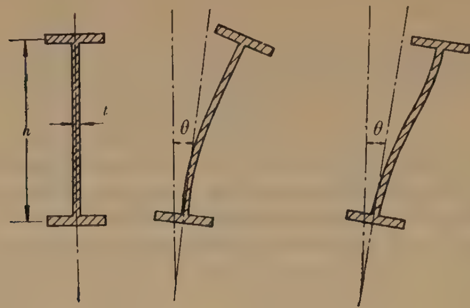


FIG. 2.—TYPICAL DISTORTIONS OF SLENDER UNSTIFFENED WEBS

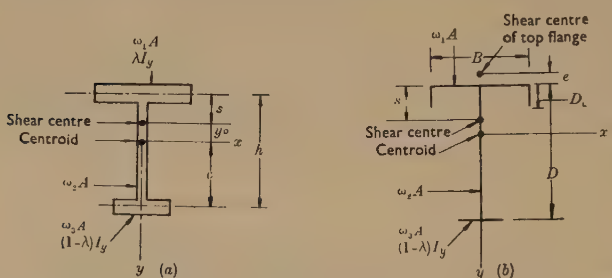


FIG. 3.—TYPICAL SECTIONS UNSYMMETRICAL ABOUT x-AXIS ONLY

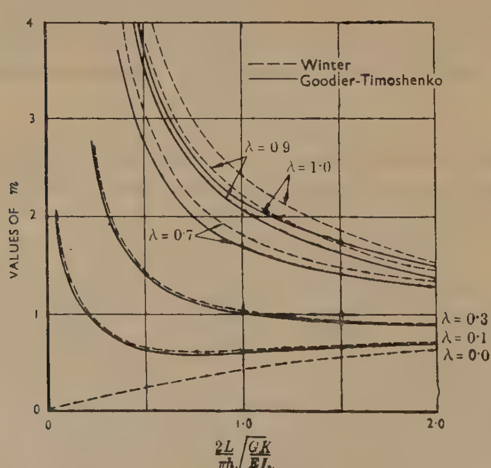


FIG. 4.—COMPARISON OF SOLUTIONS FOR MONOSYMMETRIC BEAMS UNDER UNIFORM BENDING MOMENT

have been derived by Winter,[4] Hill,[5] Goodier,[6] Timoshenko,[7] and Petterson.[8] Of these, the first solution, being of an explicit form based on simplified analysis, was adopted as providing a basis for design. The critical moment may be written as:

$$M_{crit} = \frac{\pi}{L}\sqrt{(EI_yGK)}\left\{\sqrt{\left[1 + \frac{\pi^2 EI_y h^2}{4GKL^2}\right]} + (2\lambda - 1)\frac{\pi h}{2L}\sqrt{\frac{EI_y}{GK}}\right\} \quad . \quad (6)$$

Owing to over-simplification of the expression for the work done by the end couples during buckling, Winter's solution[4] provides an optimistic estimate of the buckling moment in most instances, particularly where the larger flange is in compression. This may be seen by reference to Fig. 4 which shows comparisons between the solutions of equation (6) and those based on the Goodier-Timoshenko analysis for extreme values of the area ratios. It should be remarked that the latter solutions agree with those obtained by the direct analytical approach of Petterson. In its more accurate form,[28] the expression for the critical moment becomes:

$$M_{crit} = \frac{m}{L}\sqrt{\frac{EI_yGK}{\gamma}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where
$$m = \pi\left[\sqrt{\left\{1 + \frac{\tau_1 \pi^2 EI_y h^2}{4GKL^2}\right\}} + \tau_2 \frac{\pi h}{2L}\sqrt{\frac{EI_y}{GK}}\right]$$

The coefficients τ_1, τ_2 depend on the section proportions and are defined in Appendix III.

Cantilevers

The critical loading of cantilevers of symmetrical cross-section has been examined by Timoshenko.[1] Whilst no explicit solution may be obtained, the critical stress

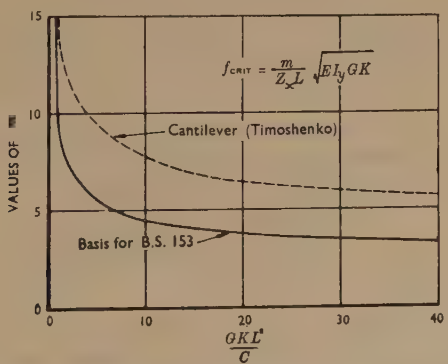


FIG. 5.—INFLUENCE OF WARPING RIGIDITY—CANTILEVER, POINT LOAD AT SHEAR CENTRE FREE END

for the case of point loading through the shear centre at the free end may be obtained from:

$$f_{b\,.\,crit} = \frac{m}{Z_x L}\sqrt{\frac{EI_yGK}{\gamma}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where m is plotted against the torsional parameter GKL^2/C in Fig. 5. This being the most serious loading for the cantilever it is evident, by comparison with the

values from equation (1) shown on the same diagram, that the adopted basis will provide a conservative estimate of the critical stress.

Derivation of the simplified formula

Taking equation (2) as a basis for members conforming to the assumed end conditions, the proposed formula for estimating critical (buckling) stresses of a perfect beam has been derived by the introduction of certain approximate geometric properties. These properties, evaluated for a wide range of symmetrical sections, provide a lower limit to the critical stress, and may be summarized as follows:—

$$I_x = 1{\cdot}1\frac{BTD^2}{2} \qquad \gamma = 1{\cdot}0$$

$$I_y = \frac{B^3T}{6} \qquad h = D \text{ (overall depth of girder)}$$

$$I_c = \frac{B^3T}{12} \qquad B = 4{\cdot}2r_y$$

$$K = 0{\cdot}9BT^3 \qquad E = 2{\cdot}5G = 13{,}000 \text{ tons/sq. in.}$$

Introducing these approximations into equation (2) the critical stress (in tons/sq. in.) is given by:

$$f_{b\,.\,crit} = \frac{170{,}000}{(l/r_y)^2}\sqrt{\left[1 + \frac{1}{20}\left(\frac{lT}{r_yD}\right)^2\right]} \quad . \quad . \quad . \quad . \quad (9)$$

It has been shown theoretically and by worked examples (see Table 1) that this formula gives nearly correct values for the critical stress for girders of the most unfavourable shape and a conservative estimate for the more normal types of girder. It is of interest to note that the numerical constants are almost identical with those proposed by Hussey in the discussion on a design formula derived by de Vries.[9] In assuming a torsion constant for a homogeneous section[10] and applying it to cover girders with riveted flange plates it is realized that some error does arise. It has been shown that up to a 20% reduction may occur, which in the worst instance would reduce the critical stress by 10%.[11] This contingency is covered by equation (9).

A simplified expression for girders with unequal flanges is obtained by modification of the above formula as indicated by equation (6). It is found that a lower limit to the critical stress in the broad flange is obtained from:

$$f_{b\,\cdot\,crit} = \frac{170{,}000}{(l/r_y)^2}\left\{\sqrt{\left[1 + \frac{1}{20}\left(\frac{lT}{r_yD}\right)^2\right]} + k_2\right\} \quad . \quad . \quad . \quad (10)$$

where k_2 is an empirical constant replacing $(2\lambda - 1)$ in Winter's solution and should vary from $+ 1{\cdot}0$ when $\lambda = 1$ to $- 1{\cdot}0$ when $\lambda = 0$. The inaccuracies introduced by the various approximations cause a wider scatter and this is allowed for by reducing the positive k_2-factor by 50%. The formula may now be used to treat both unequal flange I-beams and crane-gantry sections. Theoretical critical stresses for unsymmetrical sections have been evaluated and are compared with those derived from the proposed formula in Tables 2 and 3 for a range of sections.

All the above approximations are based on geometric properties of symmetrical I-sections and it may be observed that for these sections the simplified formula would be more correct if expressed in terms of l/B instead of l/r_y. This would also avoid the absurdity of apparently making the girders stronger by making their webs thinner. Substituting $B = 4{\cdot}2r_y$ gives:

$$f_{b\,.\,crit} = \frac{9{,}600}{(l/B)^2}\sqrt{\left[1 + 0{\cdot}89\left(\frac{lT}{BD}\right)^2\right]} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

TABLE 1.—THEORETICAL CRITICAL BENDING STRESSES OBTAINED BY "EXACT" AND SIMPLIFIED FORMULAE FOR SYMMETRICAL SECTIONS.

Section make-up			Type	$\frac{D}{T}$	$\frac{d}{t}$	$\frac{A\ web}{A\ flange}$	λ	k_2	$\frac{l}{r_y}$	Critical stress: ton/in.²		C		Critical / C	
Web	Top flange	Bottom flange								Uniform bending moment	Mid-span pt load on top flange	Normal	Top flange loading	Normal	Top flange loading
B.S.B. 114 6" × 6" I			I	12	19	·30	·50	0	200	20·4	—	17·0	—	1·20	—
B.S.B. 123 12" × 6" I			I	14	20	·48	·50	0	100	39·6	—	32·8	—	1·20	—
									200	18·2	—	14·6	—	1·25	—
B.S.B. 132 16" × 6" I			I	19	26	·66	·50	0	200	14·0	—	9·8	—	1·43	—
B.S.B. 129 15" × 5" I			I	23	33	·89	·50	0	200	11·8	—	9·4	—	1·26	—
B.S.B. 139 22" × 7" I			I	26	41	·87	·50	0	100	27·0	—	22·5	—	1·20	—
									200	10·4	—	8·5	—	1·22	—
12" × 3/4"	8" × 1½"	8" × 1½"	I	10	16	·38	·50	0	200	22·0	—	19·5	—	1·13	—
B.S.B. 139 22" × 7" I	12" × 1½"	12" × 1½"	I	13	26	·21	·50	0	100	43·6	—	35·3	—	1·24	—
									200	20·4	—	15·8	—	1·29	—
30" × 5/8" 4 flange ∠s 6" × 6" × 5/8"	18" × 2"	18" × 2"	I	14	34	·39	·50	0	200	16·8	—	14·5	—	1·16	—
33" × ½"	10" × 1"	10" × 1"	I	35	66	·83	·50	0	100	23·2	19·2	20·2	14·9	1·15	1·29
									200	8·0	7·3	6·9	5·4	1·16	1·35
92" × 3/4"	36" × 2"	36" × 2"	I	48	123	·48	·50	0	100	19·2	—	18·8	—	1·02	—
									200	6·0	—	5·8	—	1·03	—
58" × 3/8"	20" × 1"	20" × 1"	I	60	93	·91	·50	0	100	—	15·4	—	13·2	—	1·16
									200	—	4·9	—	4·1	—	1·19
128" × 3/4"	24" × 2"	24" × 2"	I	66	171	1·00	·50	0	100	20·6	—	18·2	—	1·13	—
									200	5·5	—	5·4	—	1·02	—
68" × 3/4"	24" × 1"	24" × 1"	I	70	91	1·06	·50	0	100	22·0	—	18·1	—	1·21	—
									200	6·2	—	5·3	—	1·17	—
94" × 3/4"	24" × 1"	24" × 1"	I	96	125	1·47	·50	0	100	21·6	—	17·4	—	1·24	—
									200	5·8	—	4·7	—	1·23	—
150" × ½"	30" × 1½"	30" × 1½"	I	100	300	·83	·50	0	100	19·6	15·0	17·4	12·2	1·13	1·23
									200	5·2	4·0	4·7	3·4	1·11	1·18
42" × 3/8"	4 ∠s 3½" × 3½" × 3/8"		I	112	101	1·58	·50	0	100	24·0	—	17·4	—	1·38	—
									200	6·4	—	4·7	—	1·36	—
15" × 3/16"	3½" × 1"	3½" × 1"	I	17	80	·40	·50	0	100	30·2	29·2	28·5	22·4	1·06	1·31
									200	13·2	14·4	12·1	10·0	1·09	1·44
37½" × 3/16"	5" × 3/4"	5" × 3/4"	I	52	200	·94	·50	0	100	21·6	17·3	18·6	13·4	1·16	1·29
									200	6·4	5·4	5·7	4·3	1·12	1·25
56¼" × 3/16"	8" × 5/8"	8" × 5/8"	I	92	300	1·05	·50	0	100	20·8	16·7	17·7	12·4	1·18	1·35
									200	5·6	4·4	4·9	3·6	1·14	1·22
75" × 3/16"	10" × 3/4"	10" × 3/4"	I	102	400	·94	·50	0	100	20·2	15·8	17·4	12·2	1·16	1·30
									200	5·3	4·2	4·7	3·2	1·13	1·31

TABLE 2.—THEORETICAL CRITICAL BENDING STRESSES CALCULATED BY "EXACT" AND SIMPLIFIED FORMULAE FOR UNSYMMETRICAL SECTIONS.

Section make-up			Type	$\frac{D}{T}$	$\frac{d}{t}$	$\frac{A_{web}}{A_{flange}}$	λ	k_2	$\frac{l}{r_y}$	Critical stress: ton/in.^{2}		C		$\frac{\text{Critical}}{C}$	
Web	Top flange	Bottom flange								Uniform bending moment	Mid-span pt load on top flange	Normal	Top flange loading	Normal	Top flange loading
7·36″ × ·31″	4″ × ·64″	—	T	13	23	·92	1·00	·50	100	48·4	—	43·0	—	1·13	—
									200	20·4	—	18·1	—	1·13	—
16″ × ·55″	6″ × ·85″	4″ × ·50″	I	20	29	1·24	·85	·35	100	42·0	—	31·4	—	1·34	—
									200	11·2	—	8·9	—	1·26	—
16″ × ·55″	6″ × ·85″	—	T	20	29	1·73	1·00	·50	100	51·6	—	34·0	—	1·52	—
									200	17·4	—	12·6	—	1·38	—
16″ × ·55″	4″ × ·50″	6″ × ·85″	I	20	29	1·24	·15	−·70	100	14·4	—	13·6	—	1·06	—
									200	7·8	—	7·4	—	1·05	—
76½″ × ¾″	24″ × 2″	16″ × 1½″	I	40	102	·80	·82	·32	100	29·6	—	25·0	—	1·18	—
									200	8·6	—	7·8	—	1·10	—
B.S.T. 118 6″ × 6″ × ½″ T			T	12	11	·92	1·00	·50	100	88·4	—	45·7	—	1·93	—
									200	37·5	—	20·4	—	1·84	—
7·36″ × ·31″	4″ × ·64″	—	T	13	24	·89	1·00	50	100	59·5	—	43·5	—	1·37	—
									200	22·9	—	16·8	—	1·36	—
12″ × 1″	12″ × 1″	—	T	13	12	1·0	1·00	·50	100	—	70·4	—	33·8	—	2·08
									200	—	32·4	—	14·6	—	2·22
16″ × ¾″	10″ × 1″	—	T	17	21	1·20	1·00	·50	100	—	46·2	—	28·3	—	1·64
									200	—	20·6	—	11·5	—	1·79
B.S.T. 118 6″ × 6″ × ½″ ⊥			⊥	12	11	·92	0	−1·00	100	38·3	—	20·2	—	1·90	—
									200	21·8	—	13·9	—	1·57	—
7·36″ × ·31″	—	4″ × 64″	⊥	13	24	·89	0	−1·00	100	20·1	—	18·0	—	1·12	—
									200	13·2	—	10·3	—	1·28	—
12″ × 1″	—	12″ × 1″	⊥	13	12	1·00	0	−1·00	100	—	30·0	—	16·2	—	1·85
									200	—	20·8	—	10·1	—	2·08
16″ × ¾″	—	10″ × 1″	⊥	17	21	1·20	0	−1·00	100	—	14·0	—	10·6	—	1·32
									200	—	10·8	—	7·0	—	1·54
~~24″ × 7½″ I~~	12″ × 3½″ [	—	I	24	39	·56	·96	·49	100	—	76·8	—	23·1	—	3·33
									200	—	26·0	—	8·5	—	3·04
13½″ × ⅜″	10″ × ¾″	5″ × ¾″	I	20	36	·33	·96	·49	100	—	43·4	—	25·6	—	1·71
	2/3″ × ¾″ lips								200	—	17·6	—	10·0	—	1·76
48″ × ⅝″	20″ × 1″	15″ × 1″	I	50	77	·70	·84	·34	100	—	25·2	—	19·3	—	1·30
	2/4″ × 1″ lips								200	—	7·8	—	5·3	—	1·47

TABLE 3.—THEORETICAL CRITICAL BENDING STRESSES CALCULATED BY "EXACT" AND SIMPLIFIED FORMULAE FOR UNSYMMETRICAL SECTIONS.

Section make-up			Type	$\frac{D}{T}$	$\frac{d}{t}$	$\frac{A\ \text{web}}{A\ \text{flange}}$	λ	k_2	$\frac{l}{r_y}$	Critical stress: ton/in.²		C		Critical / C	
Web	Top flange	Bottom flange								Uniform bending moment	Mid-span pt load on top flange	Normal	Top flange loading	Normal	Top flange loading
48" × ⅝"	20" × 1"	20" × 1"	I	50	77	·75	·50	0	100	20·9	16·9	18·6	13·4	1·12	1·26
									200	6·4	5·6	5·7	4·3	1·12	1·30
48" × ⅝"	20" × 1"	17½" × 1"	I	50	77	·80	·60	·10	100	23·8	18·8	20·3	14·6	1·17	1·29
									200	7·1	6·2	6·1	4·6	1·16	1·35
48" × ⅝"	20" × 1"	15" × 1"	I	50	77	·86	·70	·20	100	27·3	20·8	22·0	15·8	1·24	1·32
									200	8·0	6·8	6·6	4·9	1·21	1·39
48" × ⅝"	20" × 1"	10" × 1"	I	50	77	1·00	·89	·39	100	33·1	22·5	25·2	18·0	1·31	1·25
									200	9·4	7·5	7·4	5·5	1·27	1·36
48" × ⅝"	20" × 1"	7½" × 1"	I	50	77	1·09	·95	·45	100	34·8	22·4	26·2	18·7	1·33	1·20
									200	9·8	7·5	7·6	5·7	1·29	1·32
48" × ⅝"	17½" × 1"	20" × 1"	I	50	77	·80	·40	−·20	100	16·6	—	15·2	—	1·09	—
									200	5·3	—	4·8	—	1·10	—
48" × ⅝"	15" × 1"	20" × 1"	I	50	77	1·86	·30	−·40	100	12·6	12·3	11·8	8·7	1·07	1·41
									200	4·3	4·7	4·0	3·1	1·07	1·52
48" × ⅝"	10" × 1"	20" × 1"	I	50	77	1·00	·11	−·78	100	5·5	5·6	5·3	4·2	1·04	1·33
									200	2·5	2·3	2·3	2·0	1·09	1·15
48" × ⅝"	7½" × 1"	20" × 1"	I	50	77	1·09	·05	−·90	100	3·4	2·8	3·3	2·8	1·03	1·00
									200	1·9	1·8	1·8	1·6	1·05	1·12
13½" × ⅜"	10" × ¾"	10" × ¾"	I	20	36	·34	·50	0	100	27·0	26·6	25·5	19·8	1·06	1·34
									200	11·4	12·9	10·4	8·5	1·10	1·52
13½" × ⅜"	10" × ¾"	8¾" × ¾"	I	20	36	·36	·60	·10	100	30·1	28·5	27·2	21·0	1·11	1·36
									200	12·0	13·1	10·8	8·8	1·11	1·49
13½" × ⅜"	10" × ¾"	7½ × ¾"	I	20	36	·39	·67	·17	100	32·5	28·7	28·4	21·8	1·14	1·32
									200	12·5	13·2	11·1	9·0	1·13	1·47
13½" × ⅜"	10" × ¾"	6¼" × ¾"	I	20	36	·42	·80	·30	100	35·6	31·1	30·6	23·3	1·16	1·33
									200	13·2	14·0	11·7	9·4	1·13	1·49
13½" × ⅜"	10" × ¾"	5" × ¾"	I	20	36	·45	·88	·38	100	36·4	32·3	32·0	24·3	1·14	1·35
									200	13·2	14·0	12·0	9·6	1·10	1·46
13½" × ⅜"	10" × ¾"	—	T	20	36	·68	1·00	·50	100	47·0	37·0	34·0	25·7	1·38	1·46
									200	16·4	16·3	12·6	10·0	1·30	1·63
13½" × ⅜"	8¾" × ¾"	10" × ¾"	I	20	36	·36	·40	−·20	100	23·4	—	22·1	—	1·06	—
									200	10·4	—	9·5	—	1·10	—
13½" × ⅜"	7½" × ¾"	10" × ¾"	I	20	36	·39	·33	−·34	100	19·0	20·8	19·7	15·8	·97	1·32
									200	9·1	10·8	8·9	7·5	1·05	1·44
13½" × ⅜"	6¼" × ¾"	10" × ¾"	I	20	36	·42	·20	−·60	100	15·8	—	15·3	—	1·03	—
									200	8·2	—	7·8	—	1·05	—
13½" × ⅜"	5" × ¾"	10" × ¾"	I	20	36	·45	·12	−·76	100	12·6	11·5	12·6	11·8	1·00	·98
									200	7·2	8·0	7·1	6·2	1·02	1·29
13½" × ⅜"	—	10" × ¾"	⊥	20	36	·68	·0	−1·00	100	10·1	10·2	8·5	8·0	1·19	1·28
									200	7·2	7·9	6·1	6·5	1·18	1·21

This expression would be rather conservative for channel sections but that would not be a bad fault. Unfortunately, it was found impossible to simplify in the same manner the comprehensive expression for girders with unequal flanges. In these $I_y = \frac{1}{12}\left\{B_1{}^3T_1 + B_2{}^3T_2\right\}$ where B_1, T_1, B_2, and T_2, are the widths and thicknesses of the two flanges and substituting either $I_y = B_1{}^3T_1/6$ or $B_2{}^3T_2/6$ results in large errors. The accuracy obtainable for symmetrical beams was, in consequence, sacrificed to produce an all-embracing formula, applicable to all cases. It should be appreciated that the above empirically derived geometrical relations jointly give an approximately correct solution for different types of girders and cannot be altered individually without re-checking the solution for the whole range of girders.

Top-flange loading

Whilst providing pessimistic estimates of critical loading when load points are fully restrained against lateral movement, the above solutions may be unsafe when applied to design girders loaded at their top flanges by loads free to move sideways during buckling. Assuming the lines of action of such loads remain parallel to the y-axis, theoretical results are available for a number of cases of single, two-symmetrical, and uniformly distributed loads applied with vertical eccentricity to the shear centre, and these are summarized in Appendix IV. By introduction of the empirical constants

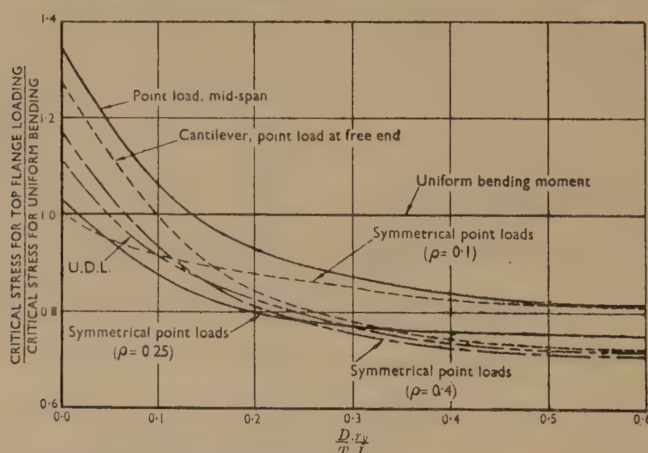


FIG. 6.—INFLUENCE OF TOP-FLANGE LOADING ON STABILITY

given above for symmetrical I-beams into these solutions, the relations between the ratio:

$$\frac{\text{critical stress for top flange loading}}{\text{critical stress for uniform bending}} \text{ and the parameter } Dr_y/TL$$

may be plotted as in Fig. 6. For slender shallow beams the effect of top-flange loading is slight and the above ratio may exceed 1·0 as a result of the change in the bending-moment distribution. In short-span deep girders, however, reductions of the order of 30% may occur in the critical stress, for which allowance must be made. By taking a lower envelope of the curves of Fig. 6 as a basis it is found that for the range of Dr_y/TL ratios envisaged the necessary correction may be brought about by

sing an effective length of compression flange equal to $1{\cdot}2L$. This correction is ccurate for deep girders and conservative for beams of the order of $D/T = 15$, in hich the factor ranges from $1{\cdot}08L$ to $1{\cdot}17L$, and has therefore been adopted.

Rather more serious reductions in critical loading may arise when, in addition to eing loaded at the top flange, the beam has smaller compression than tension flange. n the limiting case of an inverted tee section loaded through its toe the effective ertical eccentricity is equal to the depth of the web and the load-carrying capacity ay be very low indeed. The converse is true of the tee section loaded on its flange. ere again Winter's solution[4] is in error when compared to the analysis by Petterson,[8] wing to over-simplification. Petterson's solution, in the form of an infinite power ries, proves unwieldy for general application, however, and an explicit expression r critical loading was sought by using the Rayleigh energy method. In the nstance of a single point load at mid-span, the critical bending moment was obtained a form similar to equation (7) as given in Appendix IV. Agreement with the results r a particular section cited by Petterson was excellent and comparisons with Vinter's solution are made in Fig. 7. This expression was adopted for checking the

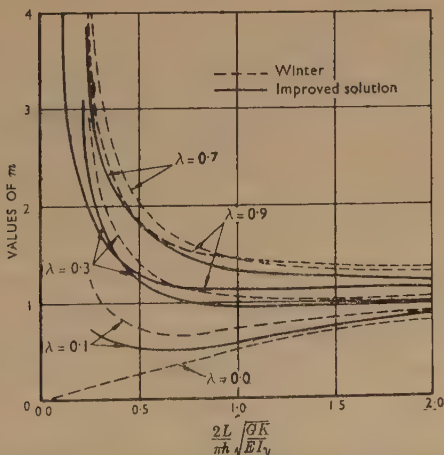


FIG. 7.—COMPARISONS OF SOLUTIONS FOR MONOSYMMETRIC BEAMS UNDER POINT LOAD AT TOP FLANGE—MID-SPAN

actors for the new formula, and the solution was extended to deal with lipped crane antry sections as shown in Appendix III. It has been found that provided the ncreased effective length is used, the proposed formula is satisfactory for both lipped nd plain sections with top flange loading.

Curtailed flanges

The practice of curtailing flange areas for economy where a member is to sustain a articular bending-moment distribution necessitates further modification to the ateral buckling clause. Owing to curtailment, both lateral flexural and torsional igidities will decrease towards the supports, as a result of which the critical value of tress at mid-span is reduced. The order of this reduction has been investigated by neans of energy solutions to cases of linear and parabolic curtailment of both

breadth and thickness at each flange as outlined in Appendix V. While necessarily erring on the unsafe side, this analysis had again the merit of providing an explicit solution, the critical moment being expressed as:

$$M_{crit} = \alpha\frac{\pi}{L}\sqrt{\left[EI_yGK\left\{1 + \frac{\beta\pi^2EI_c}{2GKL^2}\right\}\right]} \quad . \quad . \quad . \quad . \quad (12)$$

where the geometrical properties refer to the maximum section. The factors α and $\alpha\sqrt{\beta}$ are shown in Figs 8 and 9 for point loading at the shear centre, mid-span, and

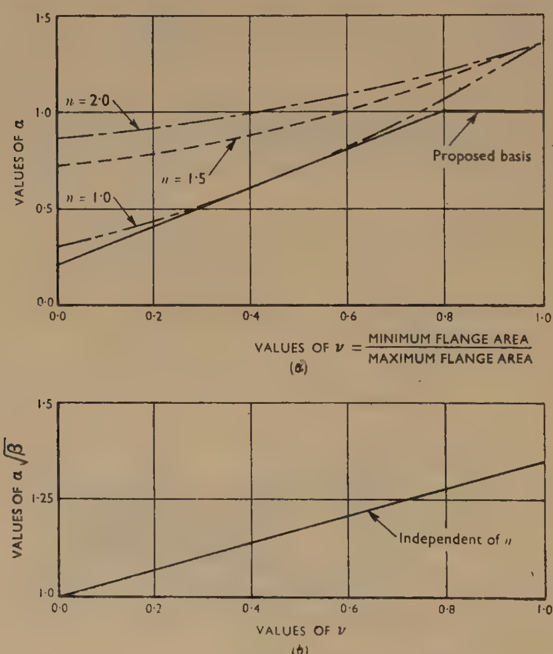


FIG. 8.—EFFECT OF LINEAR FLANGE-THICKNESS CURTAILMENT—POINT LOAD AT SHEAR CENTRE, MID-SPAN

linear curtailment of breadth and thickness, together with the assumed basis for B.S. 153. Here n is the ratio at mid-span of $\frac{\text{torsion constant of the whole section}}{\text{torsion constant of the flanges}}$

It will be seen that, while for shallow members, for which the α-term and the torsional rigidity predominate, thickness curtailment produces the greater reductions, breadth curtailment is the more serious for deep girders where flange stiffness is important and the $\alpha\sqrt{\beta}$-term predominates.

These solutions could only be extended to treat curtailed-flange cantilevers in cases where the warping rigidity was negligible. The variations in α with linear curtailment with warping terms neglected are shown in Figs 10 and 11. Whilst the proposed basis seems reasonable when applied to cantilevers, there is need for more detailed analysis of typical deep sections.

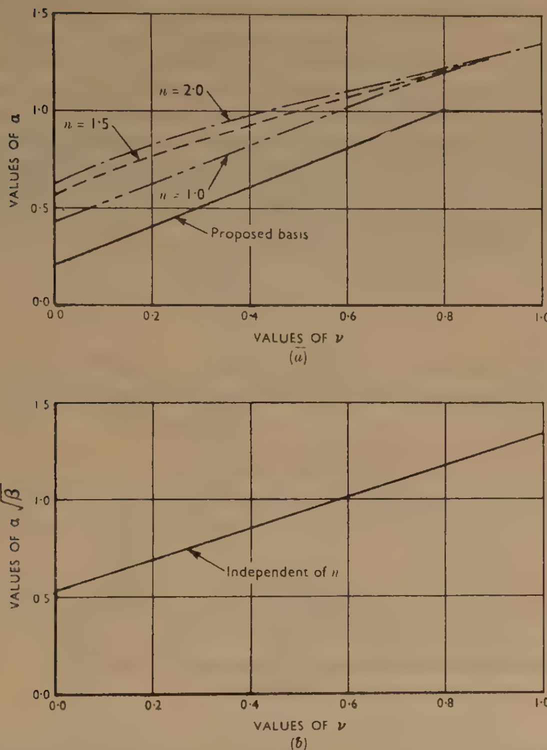


FIG. 9.—EFFECT OF LINEAR FLANGE-BREADTH CURTAILMENT—POINT LOAD AT SHEAR CENTRE, MID-SPAN

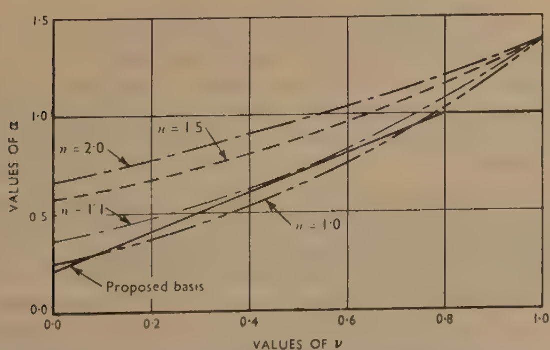


FIG. 10.—EFFECT OF LINEAR FLANGE-THICKNESS CURTAILMENT—POINT LOAD AT FREE END OF CANTILEVER

Comparing equations (2) and (12) it is evident that the formula given as equation (10) may be used to deal with curtailed flange beams if the flange thickness T be replaced by αT and $T/\sqrt{\beta}$. Thus an equivalent thickness T_e, may be used if an approximation can be found to satisfy $T_e = \alpha T = T/\sqrt{\beta}$. Coefficients k_1 have been determined by trial and error such that by using $T_e = k_1 \times$ (thickness of compression flange at point of maximum bending moment) the critical stress calculated by the Rayleigh solutions for simply supported or cantilevered girders carrying point or uniformly distributed loads with flanges curtailed to suit the bending-moment diagrams is in close agreement with the modified formula. It has been found that this coefficient varies with the ratio ν as shown in Table 7 (Appendix I).

In the case of curtailment of breadth of flange the ratio ν shall not be less than

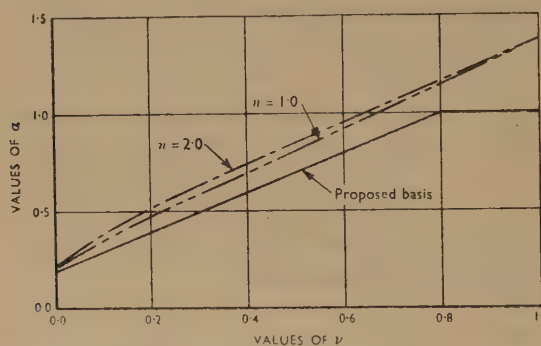


FIG. 11.—EFFECT OF FLANGE-BREADTH CURTAILMENT—POINT LOAD AT FREE END OF CANTILEVER

0·25, for it is considered that the empirical values of k_1 become unsafe for greater curtailment. It is assumed that this solution is approximately applicable also to girders with unequal flanges, although further analysis would be desirable.

INFLUENCE OF RESTRAINTS ON STABILITY

It is well known that the stability of any beam member may be ensured by judicious connexion of restraints along its length. Under these conditions design may be based on primary flexural failure, resulting in the use of the maximum working stress. Such restraints may consist for example of closely spaced loaded transverse members resting on the top flanges of main I-beams[12] (floor slabs being a limiting case), or interconnecting filler joists providing torsional restraint along the length of the main members, or lateral restraints tied back to some rigid support.[13]

In the case of the through-bridge girder intermittent restraint is provided by attachment of stiffeners connected to the transverse deck beams. The girder may be forced to buckle with nodes at these supports only if these restraints exceed a minimum stiffness. In such a case it is customary to treat the compression boom as a strut supported by a number of deflexional springs. The minimum stiffness requirements have been studied by Seide and others[14] and they have shown that in order that buckling shall occur with a half-wave-length equal to the spacing

tween supports S, with more than three intermediate supports within the span of e girder, the stiffness of each of these supports at flange level shall be defined by :

$$\delta = \frac{S^3}{40EI_c} \text{ (= deflexion/unit load)} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

hen fewer than three supports are used this definition of stiffness is conservative. he spacing S is generally fixed by other considerations and may be less than that abling a working compressive stress of 9 tons/sq. in. to be used. In this instance is inefficient to use equation (13) to estimate the required stiffness. On the basis the revised Standard this maximum working stress may be used in deep girders y D/T $\geqslant$ 50) provided that:

$$f_{b\,.\,crit} = \frac{\pi^2 E}{12}\left(\frac{B}{S}\right)^2 = 47{\cdot}7\mu \text{ tons/sq. in.} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where $\mu \geqslant 1$.

hen μ is greater than 1, the support stiffnesses may be reduced without altering the rmissible compressive stress. The modified stiffness that could be employed may

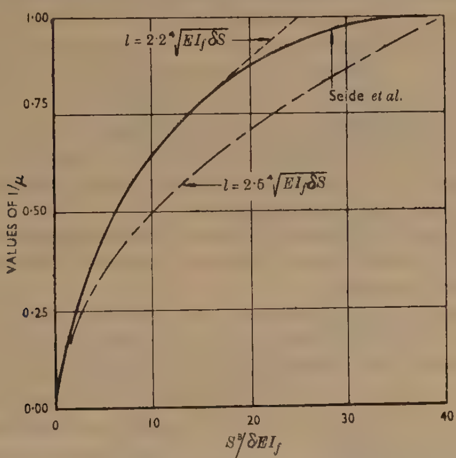


FIG. 12.—INFLUENCE OF SUPPORT STIFFNESS ON THE STABILITY OF COMPRESSION BOOMS OF THROUGH BRIDGES

estimated from the curve plotted in Fig. 12. Knowing μ, the modified value of $\frac{S^3}{\delta EI_c}$ may be derived from the curve.

The same curve may be used to deduce the effective length for buckling over a lf-wave-length greater than the spacing S, in which the stiffeners are deformed ring buckling. Putting the effective length as l, the critical stress may be written :

$$f_{b\,.\,crit} = \pi^2 E\left(\frac{r_c}{l}\right)^2 = \frac{\pi^2 E}{\mu}\left(\frac{r_c}{S}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad (15)$$

here μ is again greater than unity.

For a given value of $S^3/\delta EI_c$, μ may be derived from Fig. 12 and hence $l = S\sqrt{\mu}$.

Approximate treatment of this problem by Timoshenko[2] treating the supports as continuous elastic medium leads to the effective length given by:

$$l = 2{\cdot}2\sqrt[4]{EI_c\delta S} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

which applies accurately when the half-wave-length contains several stiffeners bu is in error as $l \to S$. This explicit expression has been modified by increasing th numerical constant in equation (16) to 2·5 and the effective length thus derive provides a safe lower limit to the critical stress as shown by the comparison in Fig. 1 This proposed basis is derived for the case of girders under uniform bending momer but will require no alteration when applied to long-span bridges under uniform loac ing. For short spans the basis will prove conservative and safe.

Provision is also made in the revised Standard for increasing permissible stresse in members restrained in lateral direction and against warping at their ends or agains lateral deflexion and twisting along their length. End fixity in direction of a com pression flange in practice implies restraint against sectional warping at the ends, anc when subjected to uniform moment, the effective length for a member restrained i this way is half its span.[15] Problems of elastic fixity have been treated and th solutions indicate that the ends are virtually fixed provided that the support stiffnes exceeds 20 times the lateral flexural stiffness of the member,[13] EI_y/L. In the wors case, of point loading at mid-span, the effective length becomes $0{\cdot}75L$ for a ver shallow beam and again tends to $0{\cdot}5L$ for deeper sections. Even where no speci provision is made for such restraint, considerable improvements in stability of plat girders may be attained as a result of using heavy tee-shape load-bearing stiffene welded to both flanges. These may provide end warping restraint and an increas in the effective torsional rigidity of the girders.

End fixity in the plane of loading requires no modification to the permissib stresses,[13] as a result of which the full benefit of increased load-carrying capacity c encastré members may be gained.

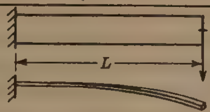
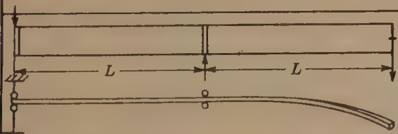
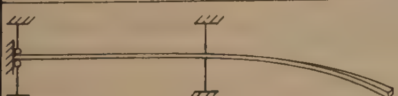
Intermediate restraints against sideways movement or twisting may be used t increase permissible stresses. Here again these must exceed a certain stiffness i order to be fully effective.[13] When nearly rigid, buckling will occur with nodes a restraints and in the worst case, where the member is under uniform bending momer between restraints, the effective length will equal the unsupported length, wit reduced values for other loading.

Similar restraints may be used to improve the stability of cantilevers. Analys of slender members (to be published elsewhere) has indicated the effective lengtl corresponding to buckling under point load at the "free" end with various typ of restraint at the load point and at points of support in a continuous syster These lengths are tabulated in Table 4. In the case of deep short members, whic are exceptional, no explicit solution may be obtained, but it is evident that the san effective lengths will provide a conservative basis when the compression flange restrained.

If the supporting system does not provide effective torsional fixity at the ends girders the buckling loads will be reduced. Shallow joists may be deemed to l satisfactorily held if resting on a horizontal seating with the top flange free. Tl torsional support may also be assumed to be provided by the load-bearing stiffene in the case of deep web girders, but it has been arbitrarily decided to specify that th top flange shall be held at the supports for all girders of d/t greater than 200 (*i.e.*, wit horizontal stiffeners).

TABLE 4.—CANTILEVER RESTRAINTS

Effective length l in equation $f_{b.crit} = \frac{4 \cdot 01}{Z_x l} \sqrt{\left(\frac{EI_y GK}{\gamma}\right)}$

	Restraint conditions	Effective length / L	
		Calculated	Test
	Free - encastre	1	1
	Loaded end prevented from twisting	0·72	0·795
	Loaded end prevented from lateral deflexion	0·57	0·575
	Loaded end prevented from lateral deflexion and twist	0·46	—
	Loaded end fixed in direction in lateral plane	0·48	0·477
	Loaded end fixed in direction and prevented from twisting	0·44	—
	Loaded end fixed in position, lateral direction, and against twisting	0·36	—
	Continuous over fulcrum; twisting prevented at fulcrum; loaded end free	1·0	1·0
	Continuous over fulcrum; no torsional restraint at fulcrum. Reaction end restrained from twisting and lateral movement. (With or without direction fixity)	2·7	2·95

Allowances for imperfections

All the foregoing analysis is concerned with the calculation of the critical loading of perfect beams under "ideal" conditions. Such conditions can never be achieved, even in the laboratory, and it is considered necessary to include allowances for imperfections. Such allowances may be based upon analysis including the effects of initial deformations of a member,[16, 17] lateral eccentricities to the shear centre of transverse loads,[17, 8] or loading inclined to the plane of greatest flexural rigidity.[8, 18]

In the case of a very deep girder, the compression flange behaves as a column and it is reasonable that its design should have the same basis as that for a strut. Moreover, the Perry–Robertson formula,[19] based on initial deformations, which is still widely used, has the advantage of having been evolved in conjunction with extensive tests and has a background of satisfactory application to design. In the complete absence of data on the eccentricities and inclinations of loading with beams and plate girders, the use of an assumed initial deformation to simulate the worst com-

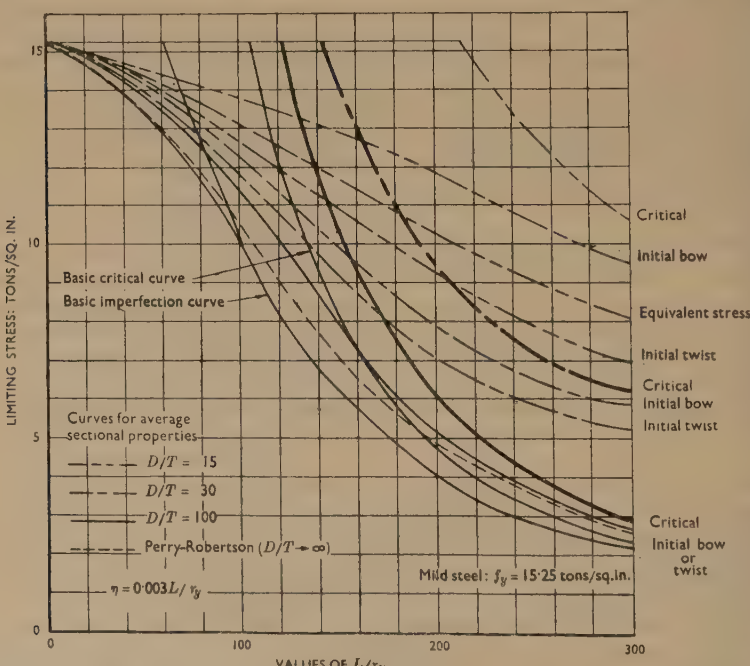


Fig. 13.—Influence of initial imperfections on load-carrying capacity—uniform bending moment

bined effects of all anomalies had distinct advantages. In adopting this approach the initial curvatures of the flanges were assumed to correspond to those given by the Robertson value of $\eta = 0{\cdot}003L/r_y$. The directions of curvature could be either the same, amounting to a general curvature in plan, or opposite, producing an initial twist.

Considering a symmetrical I-beam under uniform moment, the mean flange stresses t which a specified yield stress would be attained may be analysed by the solution utlined in Appendix VI. Experience has shown that these stresses provide realistic ɔwer bounds to the collapse loading of beams. In very deep girders collapse may ccur at stresses only slightly in excess of these limits, while the margin in stocky eams is greater owing to their increased shape factor. Employing the same pproximations to the geometrical properties of the sections as before, the critical nd limiting stresses for imperfect beams of $D/T = 100$, 30, and 15 are shown in 'ig. 13, together with the Perry–Robertson values (which coincide with the limit of $D/T \rightarrow \infty$). In deep members the deflexion of the compression flange only is the overning factor, and hence the curves for initial twist and initial deflexion are oincident. For joists, however, initial twist proves by far the more serious.

As a result of these investigations, and following the full-scale tests, the appli- ation of these results to design procedure was considered. It was found that a vorkable basis for all proportions of I-beams could be obtained by using limiting tresses based on the critical values. Taking the curves for $D/T \rightarrow \infty$ as basic, the imiting stress for any critical value may be derived. When applied to stockier nembers the same reduction from critical to limiting stress is made. Thus it is mplied that any given critical stress is always reduced by the same amount to ive the approximate limiting stress, for any values of l/r_y and D/T. The limiting tresses derived in this way for a member with $D/T = 15$ are shown in Fig. 13, and provide a compromise between the bases of initial curvature and twist. It is argued hat for shallow members the magnitude of the twist represented by the assumed lange imperfection (amounting to 5° or more in many cases) would not be tolerated nd that the equivalent limiting-stress basis should prove safe. This procedure has he advantage of being logically applicable to monosymmetric and curtailed-

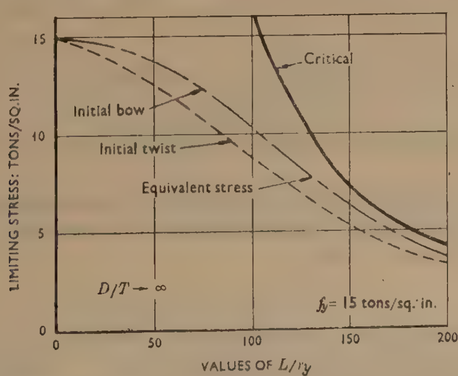


FIG. 14.—INFLUENCE OF INITIAL IMPERFECTIONS—POINT LOAD AT TOP FLANGE, MID-SPAN, OF DEEP GIRDERS

lange beams and cantilevers (in which the effects of imperfections remain to be ıvestigated).

As a check on the validity of the application of the method to deep girders under op-flange loading, the limiting stresses under mid-span point load have been evaluated or $D/T \rightarrow \infty$ and are compared with the approximate limiting stress curve in 'ig. 14.

The method has the disadvantage from the design viewpoint of eliminating the range of low l/r_y within which the full working stress may be used. For short-span shallow joists it will frequently be desirable to design by plastic methods and there is a case for assuming that the limiting stress coincides with the yield value up to an arbitrary value of l/r_y. It has been shown that, provided the stress-strain relation is linear to yield, the flanges of stocky beams may become fully plastic before lateral buckling occurs under uniform bending moment.[15, 20, 21] Where the shape factor is large, the limitations of the "equivalent limiting stress" basis would seriously penalize the use of such members. In deep plate girders, however, the shape factor may be only 1·0 and the rolled material may have low limits of proportionality.

As a compromise it was decided to take the yield stress as basis up to $l/r_y = 60$ for deep girders and to join this point to the original limiting stress curve at $l/r_y = 100$. This procedure at the worst provides a 13% over-estimate of the limiting stress. The allowable stresses are then obtained by using a suitable load factor on the

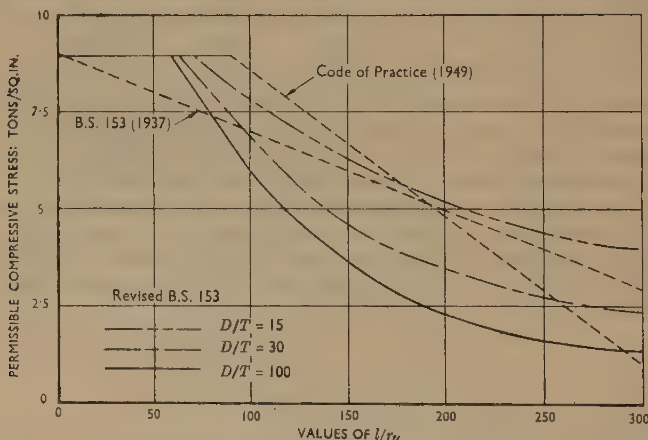


FIG. 15.—PERMISSIBLE COMPRESSIVE STRESSES IN BENDING—MILD STEEL

modified limiting curve for $D/T = \infty$ and relating them to the appropriate values of the theoretical critical stress C. Typical allowable stress curves for different values of l/r and D/T are shown in Fig. 15.

FACTORS OF SAFETY

The modified limiting curve shown in Fig. 13 gives a *minimum* value of limiting stress for a girder subjected to any reasonable loading. The stress must be reduced to produce a permissible working stress by the introduction of factors of safety comparable in all respects to those already selected for axially loaded tension and compression members.

Flanges of deep girders are stressed almost uniformly across their thickness and can be taken as comparable to a tie and a strut except that some degree of additional stability is usually provided by the web and the stiffeners. The web itself does not add much to the strength of the girder after the yield stresses have been exceeded. However, in shallow girders with relatively thick webs, such as all rolled sections and plate girders with unstiffened webs, the stresses in flanges are not uniform across their

hickness and also the web resists a considerable part of the bending moment so that here is a reserve of strength after the extreme fibre stress reaches yield (shape actor).

In the case of tension members, when yield stress develops over the whole cross-ection plastic elongation takes place and this is taken as the criterion of failure lthough actually rupture would not occur until the load is further increased by bout 80%. The adopted safety factors are based on guaranteed yield stresses otwithstanding the fact that in the vicinity of rivet holes, or because of the presence of secondary stresses, the allowable average stresses are often very much exceeded.

In the case of compression members, the criterion of failure is taken as yield for ery stocky struts or buckling for slender ones. The modified Perry-Robertson ormula gives limiting values of average axial stress for solid pin-ended struts taking nto account probable imperfections of material and workmanship and possible ccentricity of loading. These stresses, however, may be further reduced when the nember is made up of separate elements laced or battened together, so that the ssumed basic factors of 1·7 may reduce to only 1·6. The safety (or load) factors or axial stress in mild steel adopted in the draft B.S. 153 are as follows:

Load combination	Tension or compression yield	Buckling or elastic instability
(*a*) Dead load + live load	1·7	1·6
(*b*) Combination (*a*) + wind and others .	1·36	1·27
(*c*) Erection	1·31	1·23

Since secondary stresses do not materially affect the ultimate strength of the tructure, no additional margin of safety is provided on their account and all the bove factors may in fact be further reduced by about 20% when the unavoidable econdary stresses develop.

The factors for tension members have been in use since 1937 and presumably nust be considered as reasonable. The above factors for compression members ave been adopted only recently but the straight-line formula in B.S. 153 (1937), ives nearly the same working stresses for medium and long struts as Perry–Robertson's limiting values divided by these factors.

It may be of interest to attempt to justify the adopted factors by the philosophical pproach given in Appendix IV of the Report of the Committee on Structural Safety[22]:

X-factor (minimum factor)

Allow for variation in quality of material say ± 5% giving a factor of	1·05
Allow for inaccuracy of fabrication and erection, say	1·05
Allow for inaccuracy of calculations (slide rule and approximations), say	1·05
Then Factor for material, fabrication, and calculations	1·16

Allowance for overloading:

Dead load, say	± 5%
Normal live load, say	+ 30%
Wind and other loads, say	+ 10%

For different bridges the relation between dead load and live load would vary, giving factors varying from about 1·15 to 1·25 for combined dead and live load; when combined with wind and all other loadings, these factors can be reduced.

Suggested factors for:

Load combination	(*a*):	1·2
,, ,,	(*b*):	1·1
,, ,,	(*c*):	1·05

From the above, the minimum essential load factors (X-factor) to cover sub-standard material, workmanship, and calculations, and also possible overloading, all occurring together are:

Combination	(*a*) factor:	$1{\cdot}16 \times 1{\cdot}2$	$= 1{\cdot}48$
,,	(*b*) ,,	$1{\cdot}16 \times 1{\cdot}1$	$= 1{\cdot}28$
,,	(*c*) ,,	$1{\cdot}16 \times 1{\cdot}05$	$= 1{\cdot}22$

It will be seen that these factors approximately are the same as the ones actually adopted for allowable axial compressive stresses except that for combination (*a*) loading, i.e., for the normal function of the bridge, a further factor of safety (Y-factor) of at least 1·15 is provided.

The adoption of different factors of safety for different combinations and kinds of loadings is based on the theory of probability. Although explicit recognition has been given to the practice for only a fairly short time, and most extensively in aircraft design, it has been adopted without being specifically recognized almost as long as structures have been designed. It is reasonable to consider that the chance of a slightly under-designed member made from inferior material, inaccurately fabricated, carelessly erected, and then subjected to forces produced by a most unfavourable disposition of every kind of loading, which jointly exceed the designed values—is very small, and that should such an unlikely coincidence take place, it would be satisfactory if the structure just survived.

Incidentally, if the safety factor for the combined effects of all applied loads was the same as that for any one load, stresses due to all the possible loadings would have to be computed in full each time a structure is designed. The reduction of the factor of safety for combined loading in many cases permits the designer to calculate only the stresses due to dead and live loads, because he knows that in certain structures the additional stress due to wind temperature, etc., will always be smaller than the allowable increase in stress resulting from the reduction of the safety factor.

A considerable economy of the designer's time is thus often achieved. In fact it is possible that the desire to save work had more to do with the introduction of the variable factors of safety than the theory of probability. However this may be, it would appear that the factors of safety have now been cut to the bone and further reductions are not very likely.

It is desirable to adopt approximately the same factors for the fibre stresses in flanges of girders as those adopted for direct stresses. Thus in tension flanges and in compression flanges of short girders ($l/r_y < 60$) the proposed allowable stresses are obtained by dividing the yield stress by 1·7 (1·6 in the case of girders with unstiffened webs to allow for the favourable shape factor) and in the case of compression flanges of slender girders ($l/r_y > 60$) the desired load factors are approximately realized if the allowable stresses are obtained by using a load factor of 1·7 on the modified limiting curve for $D/T = \infty$

ANALYSIS OF TEST RESULTS

Existing data

Sufficient experimental evidence is available to confirm the analysis of elastic critical loading of symmetrical beams under near ideal conditions.[17, 23, 24] Corresponding tests on monosymmetric beams have been few, confined to those on a light beam under uniform bending carried out by Hill[5] and confirmatory tests under eccentric point loading by Petterson.[8] All these investigations were carried out under laboratory conditions on very small beams and provide no indication of the probable behaviour of full-scale members. The only data relating to tests on rolled steel joists are collected in a Paper by Moore,[25] but the results quoted are not directly valid for comparison as a result of the restraints afforded by the test rig to twisting and lateral movement. If, however, the effective length be taken as the distance between load points times 0·85 (as suggested by the revised clause) the test results appear as shown in Fig. 16. These apply to beams of $D/T \simeq 20$ and the

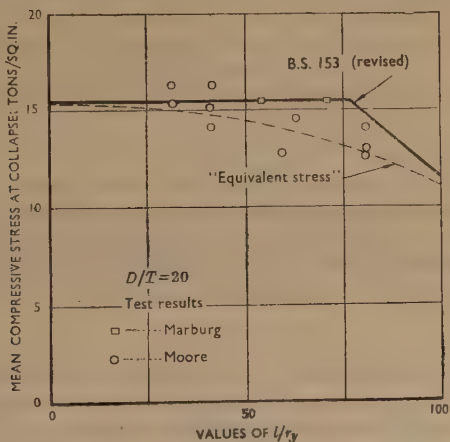


FIG. 16.—ANALYSIS OF SOME EXISTING TEST RESULTS

equivalent limiting stress basis plotted for comparison shows reasonable agreement. While limited in range, the results do show the tendency for collapse of beams of short unsupported span to occur at stresses rather lower than yield.*

Tests on an unsymmetrical I-beam and gantry section

As a further check on the solution due to Winter and Goodier, two model monosymmetric beams were tested under uniform bending moment over various spans. The models, made from xylonite, had the proportions shown in Fig. 17 and were tested in the rig described in reference 23. The I-section was tested in turn with its broad and narrow flanges in compression and the buckling moments are compared with estimated values in Fig. 17. For the proportions chosen the two theoretical solutions provide identical results, being somewhat below the observed values with the broad flange in compression. Measured values of flexural and torsional rigidity were used in the analysis.

* A recent series of tests on rolled steel beams carried out by Hechtman *et al.*,[37] has added further support to this conclusion.

The crane-gantry section could not be made to buckle without material failure except in the inverted position. Here again the Goodier–Timoshenko solution proved satisfactory with the Winter values rather high.

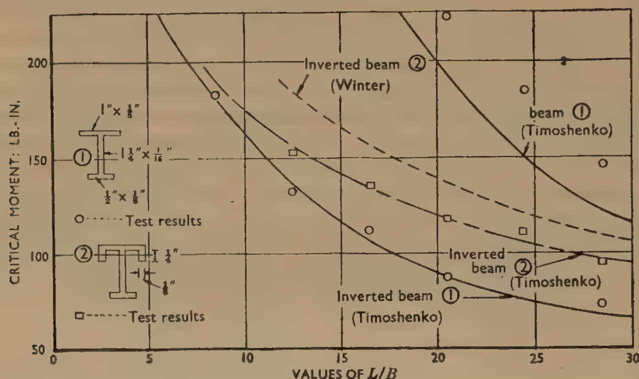


FIG. 17.—TEST RESULTS FOR MONOSYMMETRIC I BEAMS—UNIFORM BENDING MOMENT

Tests on restrained cantilevers

In order to check the analysis of cantilevers having restrained ends, tests were carried out on narrow rectangular aluminium alloy models, 1 in. × $\frac{1}{8}$ in. in section, loaded at their "free" end and held in the assumed manner. Comparisons between the calculated and experimentally derived effective lengths are made in Table 4, from which it is seen that reasonable agreement was obtained.

Tests on model girders

Two of the model girders designed to the revised B.S. 153, and described in the first part of the companion Paper,[26] collapsed by lateral buckling. The test rig, shown in Fig. 38, p. 464, prevented lateral movement of the load points, and the central span may be considered to have partial end fixity.

The limiting stresses for these models are shown in Fig. 18, the B.S. 153 curve for $D/T = 92$ strictly applying to beam E but being only slightly low for $D/T = 68$ The corresponding load factors were 2·13 and 1·99, the material properties satisfying B.S. 15 and there being no data on initial imperfections.

Full-scale tests

The tests described in the companion paper [26] were designed to check the reliability of the revised clauses under a variety of conditions, using commercial materials. The results obtained for the four experimental girders have been summarized in Table 15 (facing p. 486) and the behaviour of each girder will be discussed from the viewpoint of lateral stability of the flanges in the following section.

Girder No. 1.—This represented the limiting case of girders with unstiffened webs, in which web distortion might influence lateral buckling. With $D/T = 17$, the torsional rigidity contributed greatly to stability. The initial flange shapes, shown in Fig. 50, pp. 472–475, provided both twist and deflexion to the girder with the bow of the compression flange equivalent to $\eta = 0{\cdot}0015L/r_y$. The flange materials had compressive limits of proportionality approaching the yield stress but a very low tensile limit.

Owing to the large imperfections, the girder deflected laterally under increasing load, as shown in Fig. 60, facing p. 487, and the stresses in the edges of the compression flange diverged in the characteristic "fountain" plot, as in Fig. 55, facing

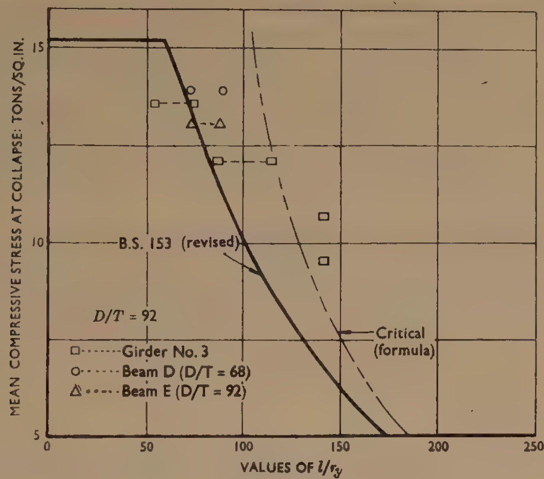


FIG. 18.—COMPARISON WITH TEST RESULTS—GIRDER NO. 3

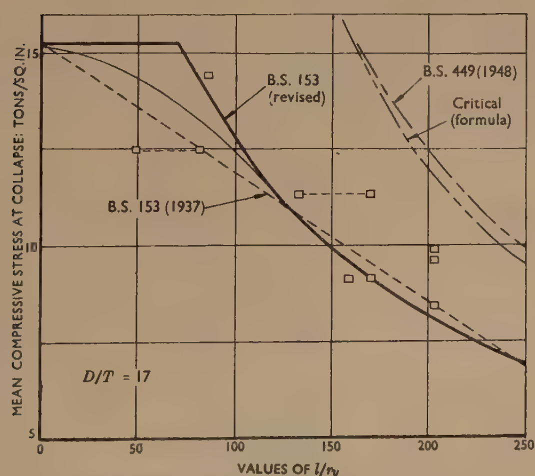


FIG. 19.—COMPARISON WITH TEST RESULTS—GIRDER NO. 1

p. 486. In order to preserve the girder, each test was stopped when the peak stress reached the proportional limit. It will be seen from these curves that at such a stage the girder was near collapse and may be considered to have failed. The estimated mean flange stresses at this stage, tabulated in Table 15, have been plotted in Fig. 19 against the collapse stresses based on the revised Standard. When the effective length was reduced as a result of restraints, the range between the theoretical lower

limit to this length and that permitted by the Standard is shown by a dotted line joining the two extreme positions for the test result. The net effective length is taken as the product of the span and the factors covering both restraints and top-flange loading where applicable.

It may be considered that this girder represents a severe case of combined imperfection and poor materials, and approximates realistically to the cases envisaged in drafting the revised formula. It is clearly shown that collapse may occur at stresses well below the critical values and the results appear to justify the proposed basis within tolerable limits. The assumptions regarding effective lengths also seem acceptable. The actual load factors, representing the ratio between observed collapse loads and working values based on permissible stresses are shown in Table 15, ranging from 1·50 to 2·10.

Girder No. 2.—This again had appreciable flange imperfections roughly approximating to the initial forms assumed in analysis, as shown in Fig. 50, pp. 472–475. In deep girders the initial shape of the compression flange mainly governs behaviour, any bow of the tension flange causing twist and curvature of the centroidal axis which counteract each other in their influence. In this instance, with $D/T = 52$, the compression flange shape corresponded to $\eta = 0{\cdot}0012L/r_y$. The material properties were similar to those for girder No. 1, with rather lower tensile proportional limits but higher yield stresses.

The lateral displacements and stresses again built up in non-linear relation with the applied loads until the tests were stopped at the limiting stresses, as shown

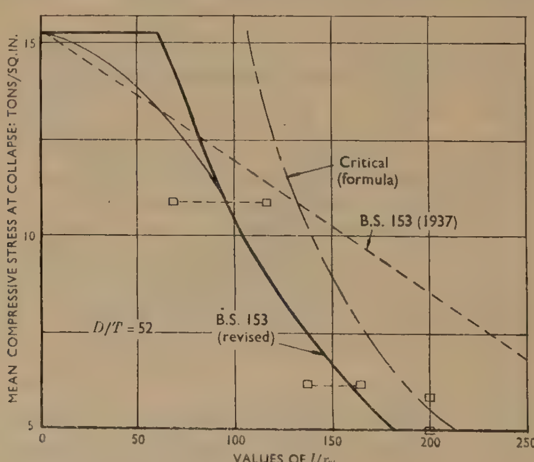


FIG. 20.—COMPARISON WITH TEST RESULTS—GIRDER NO. 2

in Figs 56 and 60, facing p. 487. Comparison with the proposed basis, made in Fig. 20, showed load factors ranging from 1·78 to 2·24. Web buckling appeared in no way detrimental to overall lateral stability.

Girder No. 3.—Flange imperfections in this girder amounted to an initial twist about the centroid corresponding to a value of $\eta = 0{\cdot}0006L/r_y$ (only one-fifth of that assumed) and the resulting lateral displacements were very small until near collapse.

s a result of difficulty experienced in following up these slight movements, the tests n this girder were carried out with the load linkage pivoted, the effect of which is ieoretically to reduce the effective vertical eccentricity of loading to nearly zero, nd this has been allowed for in estimating the effective lengths. The flange materials ad tensile and compressive proportional limits approaching yield stresses in excess f the specified minimum and values of E were 3% in excess of that assumed.

Lateral flange bending was slight until near collapse in most tests, as seen from ie curves of Fig. 56, facing p. 487. The mean limiting flange stresses, plotted gainst the B.S. 153 basis in Fig. 18, represented load factors ranging from 1·73 to 2·74 rith the high values in the slender range. It was significant in both these tests and iose on the deepest girder that collapse loads exceeded the critical values estimated y means of the approximate formula. More detailed analysis has shown that these icreases were largely due to the influence of the welded load-bearing stiffeners which, lthough partly accounted for by reducing the effective length, may have the effect of early doubling the load-carrying capacity. Such influence is of greater relative importance in the higher range of slenderness and less marked in the range where the rarping rigidity is the governing term, accounting in part for the diminishing load ictor.

Web buckling again had no influence on overall stability of the girder.

'irder No. 4.—This, the deepest permissible girder with two horizontal web stiffeners, ras fabricated so accurately that the initial imperfections of the compression flange rere negligible, as shown in Fig. 50, pp. 472–475, the flange having an initial S-shape rith little mid-span displacement. The flange materials had again nearly "ideal" roperties.

Under load the lateral displacements and stresses were, as expected, very small ntil near collapse, the results being shown in Fig. 56, facing p. 487. In the first test iere was little warning of impending failure, collapse being quite sudden. The slight ermanent set resulting provided an initial bow in subsequent tests. The observed miting stresses, plotted in Fig. 21, represented load factors ranging from 1·42 to 2·66.

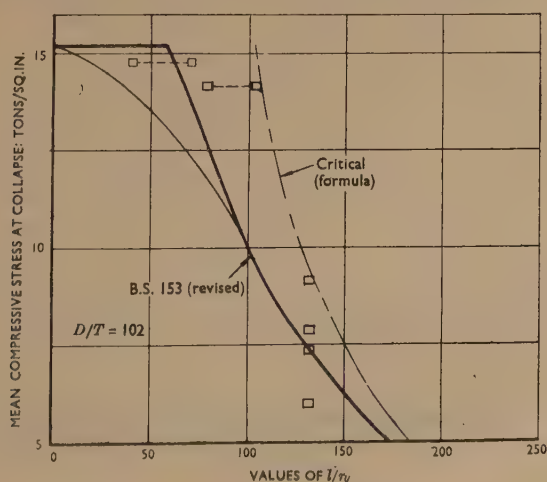


FIG. 21.—COMPARISON WITH TEST RESULTS—GIRDER NO. 4

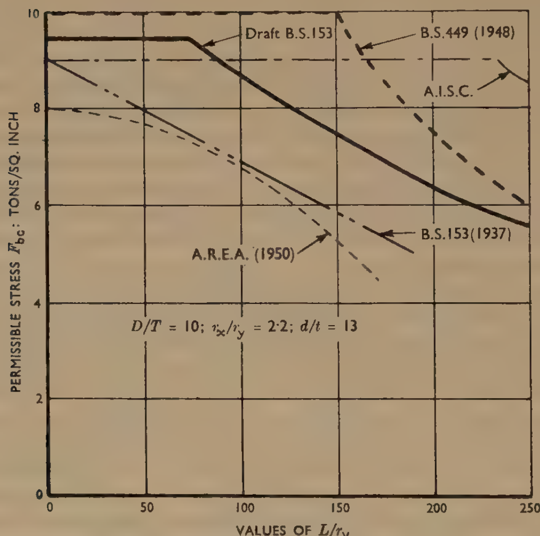


Fig. 22.—Permissible stresses in joist (B.S.B. 16: 9″ × 7″ × 58 lb.) for effective lengths up to 34 ft

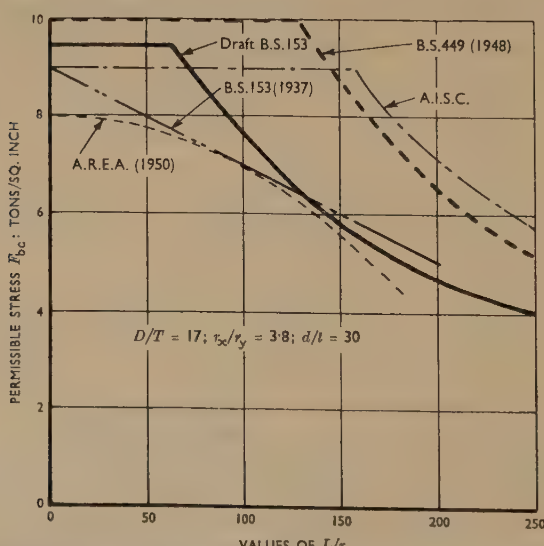


Fig. 23.—Permissible stresses in joist (B.S.B. 133: 16″ × 8″ × 75 lb.) for effective lengths up to 37 ft

The lowest load factor was obtained with the top flange free to move at the sup-orts, a condition prohibited in girders with horizontal stiffeners. Attachment of estraints to prevent such movement increased the critical stress by 45% in contrast o an increase of only 13% in girder 3, of 20% in girder 2, and no increase in girder 1.

Comparison of permissible flange stresses

Permissible compressive stresses have been calculated for a representative selection f symmetrical beams and plate girders on the basis of B.S. 449 (1948); B.S. 153 1937); American Institute of Steel Construction, 1948; American Railway Engineer-ng Specification (1950); and the proposed method. These are shown in Figs 22–27. t will be seen that in all instances the B.S. 449 values exceed those proposed, whilst he other specifications give lower values for stocky members and considerably higher alues for the more slender ones.

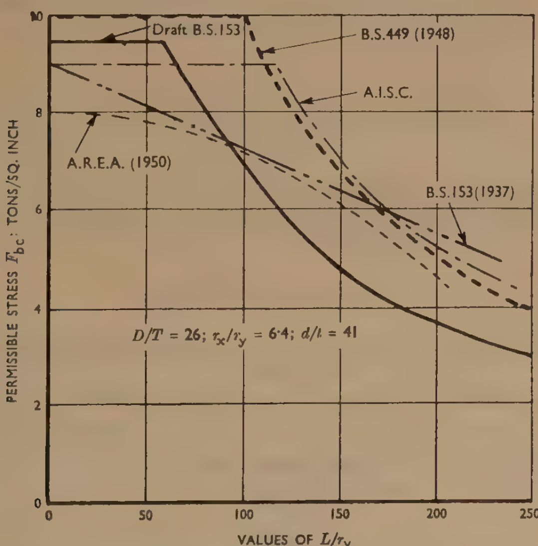


Fig. 24.—Permissible stresses in joist (B.S.B. 138: 22″ × 7″ × 75 lb.) for effective lengths up to 28 ft

The permissible stresses for the beams and girders of Figs 22–27 are as follows:

B.S. 153 (1937): $F_{bc} = 9(1 - 0{\cdot}01l/B)$ tons/sq. in.

B.S. 449 (1948): $F_{bc} = \dfrac{1{,}000}{l/r_y} K_1$ tons/sq. in.

A.I.S.C. (1948): $F_{bc} = \dfrac{12{,}000{,}000}{\dfrac{lD}{BT}}$ lb/sq. in.

A.R.E.A. (1950): $F_{bc} = 18{,}000 - 5(l/B)^2$ lb./sq. in.

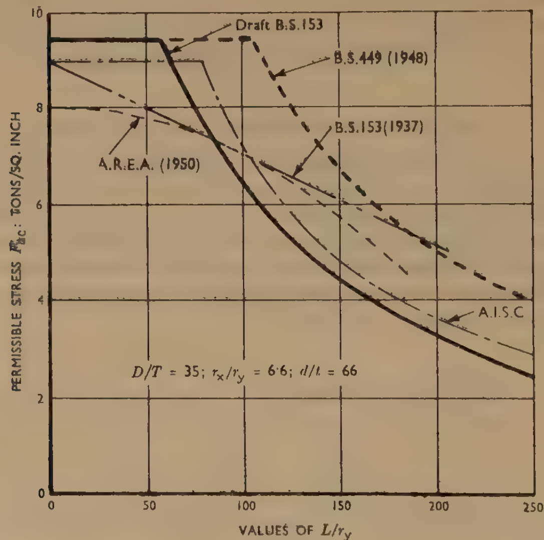


FIG. 25.—PERMISSIBLE STRESSES IN WELDED PLATE GIRDER (WEB: 33″ × ½″; TWO FLANGE PLATES: 10″ × 1″) FOR EFFECTIVE LENGTHS UP TO 44 FT

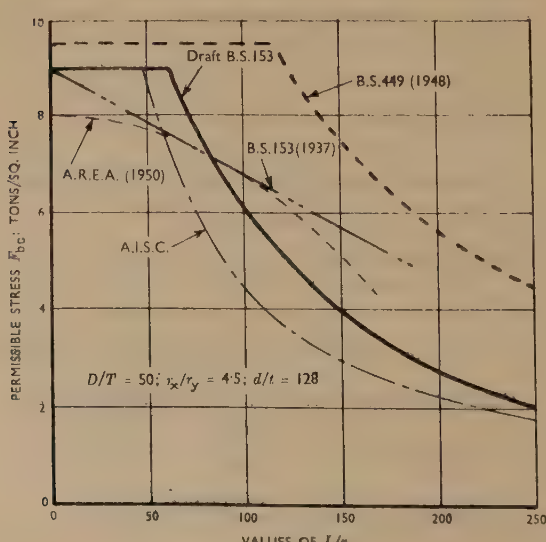


FIG. 26.—PERMISSIBLE STRESSES IN WELDED PLATE GIRDER (WEB: 48″ × ⅜″; TWO FLANGE PLATES: 20″ × 1″) FOR EFFECTIVE LENGTHS UP TO 100 FT

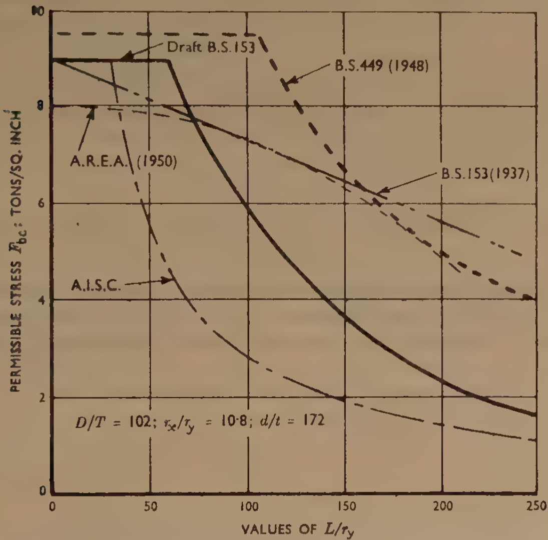


FIG. 27.—PERMISSIBLE STRESSES IN WELDED PLATE GIRDER (WEB: 150″ × $\frac{7}{8}$″; TWO FLANGE PLATES: 30″ × $1\frac{1}{2}$″) FOR EFFECTIVE LENGTHS UP TO 115 FT

Permissible web stresses

ALLOWABLE SHEAR STRESSES IN EXISTING BRITISH STANDARDS

Existing specifications for the design of girder webs seem to have been based on e following basic assumptions:

(1) For thick webs (*i.e.*, those which yield in shear before buckling) the allowable average shear stress should be limited to a definite proportion of the shear yield stress.

(2) Thin webs (*i.e.*, those which will buckle before the shear yield stress of the material is reached) should be stiffened with suitably spaced vertical stiffeners and either the same stress as in (1) above should be used, or the stress should be obtained from the theoretical critical value as derived by Timoshenko. In the latter case, the edges of the web are assumed to be either simply supported or partially restrained. B.S. 153 (1937) and B.S. 449 (1937) allowed the same average stress throughout the entire range of depth/thickness ratios, whereas C.P. 113 and B.S. 449 (1948) base the allowable stresses for stiffened webs on the critical value, the latter assuming some edge fixity.

Whilst assumption (1) is reasonable, assumption (2) would be valid only if the eoretical critical value was of paramount importance, as in the case of the Euler ess for very slender struts.

However, the critical stress for web plates having normal structural dimensions is small direct importance. Unless the web plate was absolutely flat prior to loading,

no critical behaviour would be observed as the load on the plate is increased, and stresses greatly in excess of the critical value could be carried. This excess increases with higher clear depth/thickness ratios, and for the very high ratios used in aircraft construction (400) the web carries its load, not in shear, but in a manner analagous to an N-type truss, with the intermediate stiffeners carrying an axial load. In normal steel girders, however, such high ratios are not adopted and the full "tension field" stress condition is therefore rarely achieved.

Behaviour of plates subject to shear loads in excess of the critical value

If an absolutely flat simply supported plate were subjected to shearing forces, it would remain flat until the critical stress was reached and would then buckle out of its original plane. This movement would be barely discernible, however, and the

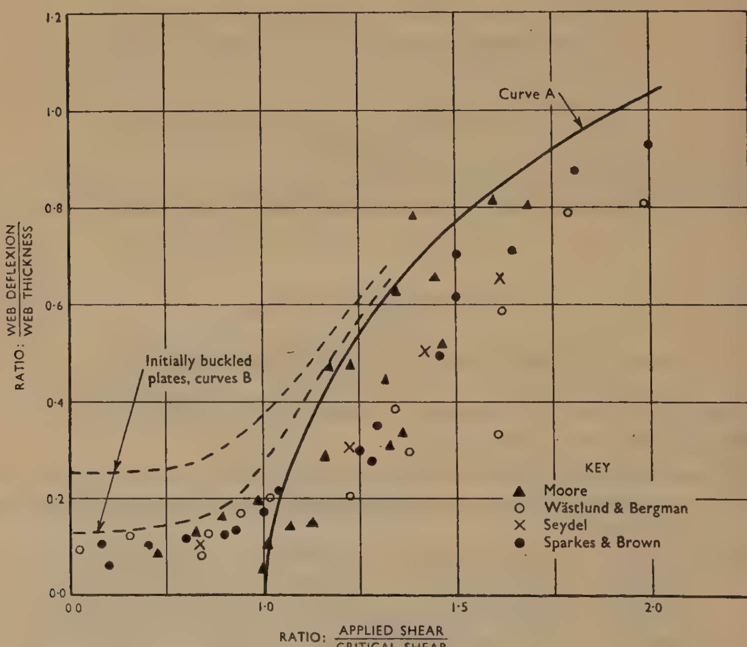


Fig. 28.—Behaviour of initially buckled plates (curves B) compared to that of initially flat plates (curve A)

load would have to be increased twofold before it became appreciable. Actuall with a practical plate, some small lateral deflexions take place as soon as the plate loaded and "critical" behaviour is very rare.

At loads in excess of the critical value, the buckled form is similar to that predicte by the theory of initially flat plates and the theoretical analysis in this range therefore directly applicable to practical cases.

Fig. 28 shows the theoretical and experimental maximum deflexions for squa simply supported plates; curve A indicates the values for absolutely flat plates, whi

rves B indicate those for initially buckled plates. From curve A it will be observed at the maximum lateral web deflexion at a load twice the critical value is approxi-tely equal to the plate thickness.

This means that the lateral deflexion of a web plate, $\frac{3}{8}$ in. thick and 6 ft deep, essed to $4\frac{1}{2}$ tons/sq. in., would be approximately $\frac{3}{8}$ in., or 1/190th of the depth.

Before the plate buckles, the stresses in it are due to shear only, but after buckling, nding stresses are also introduced, and, in addition, the middle plane of the plate leformed and extended, introducing membrane tensile stresses which react on the mpressive ones, minimizing their influence. The introduction of these tensile esses provides the fundamental difference between the buckling of web plates in ders and the buckling of columns. Any analogy between the Euler load for thin uts and the critical loads on web plates is therefore misleading.

The intensity of the bending and membrane stresses in the buckled plate vary er the area of the plate, the bending stresses being greater near the centre of the ite where the buckles are larger, while the distribution of shear stress is practically iform.

If these stresses are combined according to the Huber–von Mises–Hencky theory, en the maximum ratio of $\sqrt{(f_x^2 + f_y^2 - f_x f_y + 3f_s^2)} \div \sqrt{3}f_s$ in relation to the io $f_s/f_{s\,.\,crit}$ has been deduced by Bergman [29, 30] and is as shown in Fig. 29, where:

$\sqrt{(f_x^2 + f_y^2 - f_x f_y + 3f_s^2)}$ represents the equivalent axial stress, or $\sqrt{3}\times$ apparent shear stress.

om this it will be observed that the increase in apparent shear stress with loads excess of the critical value is relatively small and does not exceed 30%.

Thus, if such loads are applied, the plate will have buckled, and the maximum parent shear stress will have increased, but unless the fibre stress on the surface the plate has exceeded its yield value, the lateral deflexion will be small and will appear when the load is removed.

Derivation of new design formulae

eb plates subject to shear only

The proposed allowable shear stress on web plates is derived by adopting a load tor of 1·45 on the theoretical shear yield stress (*i.e.*, tensile yield stress $\div\sqrt{3}$) or a shear stress which will cause yielding in the fibre at the most highly stressed rt of the buckled plate. That is, the proposed shear stress provides a constant d factor of 1·45, not against buckling as in the case of B.S. 449 (1948), but against ld or the formation of permanent buckles.

The plate is assumed to be simply supported, whereas in practice there is a restrain-g influence by the flanges and the load factor is therefore somewhat greater. Some ler of this increase can be seen from Fig. 30, which indicates the restraining in-ence of the flanges on the web of I-section columns.

Using Fig. 29 and the approximate value (in tons/sq. in.) of the critical stress ren by Timoshenko as $f_{s\,.\,crit} = (256 \div b/t)^2(1 + \frac{3}{4}(b/a)^2)$, Figs 31 and 32 are con-ucted. Curves A give shear stresses which, for all values of d/t up to 240, exceed critical stress by an amount which will cause an increase in apparent shear stress to the yield value. This yield would not occur over the whole of the plate, but ly at the crest of the buckle.

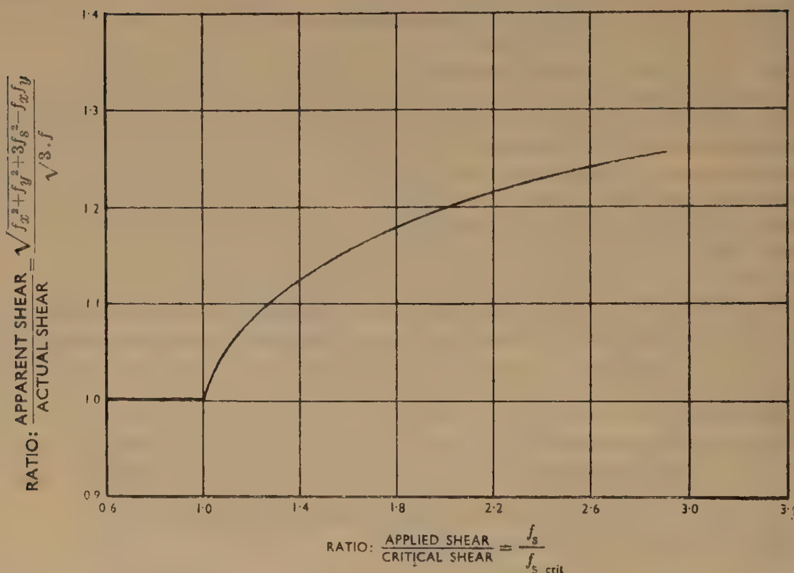


FIG. 29.—THE INCREASE OF APPARENT SHEAR STRESS (FAILURE CRITERION) WITH INCREASE OF RATIO OF APPLIED SHEAR TO CRITICAL SHEAR

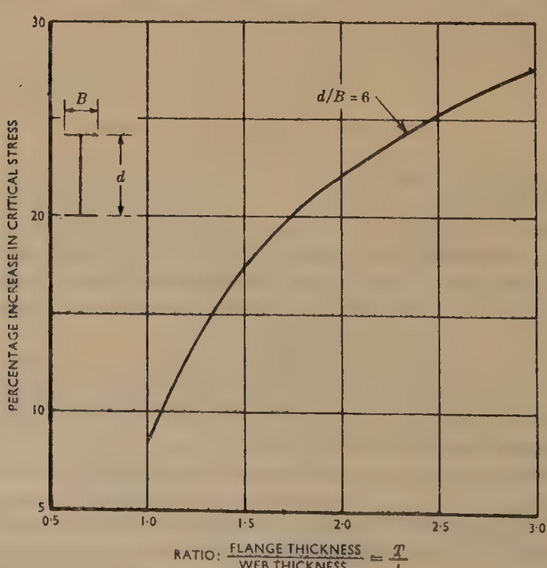


FIG. 30.—RESTRAINING INFLUENCE OF FLANGES ON WEB OF I-SECTION IN COMPRESSION

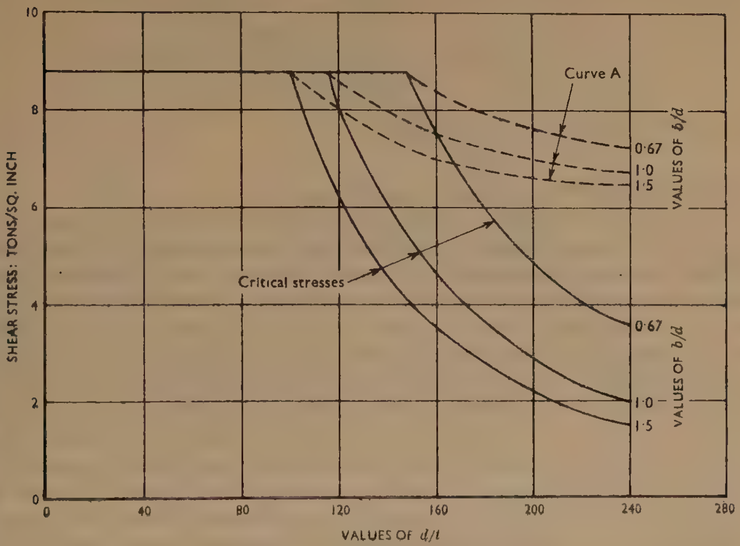


FIG. 31.—TIMOSHENKO'S CRITICAL SHEAR STRESSES AND THE CORRESPONDING SHEAR FAILURE STRESSES FOR DIFFERENT VALUES OF d/t AND b/d

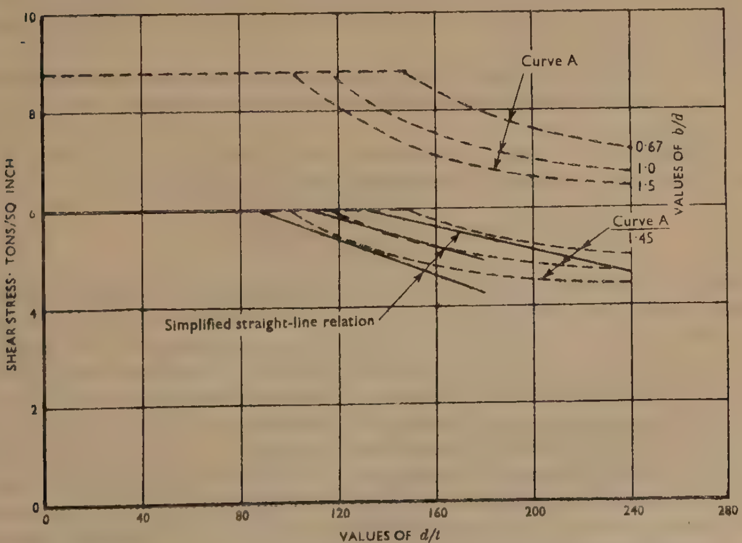


FIG. 32.—SHEAR FAILURE STRESSES (CURVES A) AND ALLOWABLE SHEAR STRESSES FOR VARIOUS VALUES OF d/t AND b/d

Allowable stresses are obtained by dividing curves A by the required load factor of 1·45 and these approximate to the straight-line relationship given by:

$$F_s = 6{\cdot}0\left[1{\cdot}3 - \frac{b/t}{250\left[1 + \frac{1}{2}\left(\frac{b}{a}\right)^2\right]}\right] \text{(in tons/sq. in.)}$$

and are tabulated in Table 11, Appendix 1.

Web plates subject to combined bending and shear

It is customary to design unstiffened plate girders and rolled sections, by considering their behaviour under bending and shear independently. If web buckling in stiffened webs were considered as a criterion, however, such a simplified procedure would be impossible and the co-existent combination of bending and shear would have to be determined for various points in the girder in order that their influence upon the critical stress could be evaluated. Such a procedure would lead to unnecessarily thick web plates or excessively close spacing of intermediate stiffeners.

The behaviour of web plates carrying combined bending and shear stresses in excess of the critical values is similar to that for webs subjected to shear only, and if some elastic buckling of the web is again permitted, considerable economy results.

The combination of shear and bending stresses without buckling occurs in unstiffened plate girders and rolled sections in which the equivalent direct stress at the junction of web and flange given by the Huber–von Mises–Hencky theory is as high as 14 tons/sq. in. Thus, under this combination, a factor of safety against "local" yielding of only 1·1 is provided.

In the past this rather low factor of 1·1 has proved acceptable and is therefore proposed as the minimum for the combination of maximum bending and shear stresses. There is, of course, a further reserve of strength in the girder, since the maximum stress is not produced over the entire plate initially, and tests indicate that average girders would withstand a further 30–40% increase in load before a general yielding of the plate occurs.

The allowable shear stresses shown in Fig. 32 are now used as a basis for combining with the bending stresses to determine the equivalent stress and the load factor against yield.

The critical stress for a simply supported plate, subjected to bending stresses only, has been derived by Timoshenko and is approximately equal to

$$f_{b\,.\,crit} = \frac{25\pi^2 E}{12(1-\sigma^2)} \cdot \left(\frac{t}{d}\right)^2$$

The relation between the ratios $f_s/f_{s\,.\,crit}$ and $f_b/f_{b\,.\,crit}$ required to produce a theoretical critical combination is almost independent of the side ratios of the stiffened panel and is shown in Fig. 33. Using this relation in conjunction with the above value for the web critical bending stress, the curve A on Fig. 34, is constructed giving the the revised critical shear stress for a web plate subjected to the full value of the allowable bending stress, i.e., 9·0 tons/sq. in. Operating on this curve by the relation given in Fig. 29, the apparent shear stress in the plate is as indicated on curve B, and has a maximum value of 6·5 tons/sq. in. This maximum stress occurs near the crest of the buckle, and is therefore not co-existent with the maximum bending stress.

The actual position of the crest of the buckle is rather indeterminate, depending largely on the initial form of the web plate. However, taking the co-existing stress as 90% of the maximum bending stress, and combining this with the above apparent

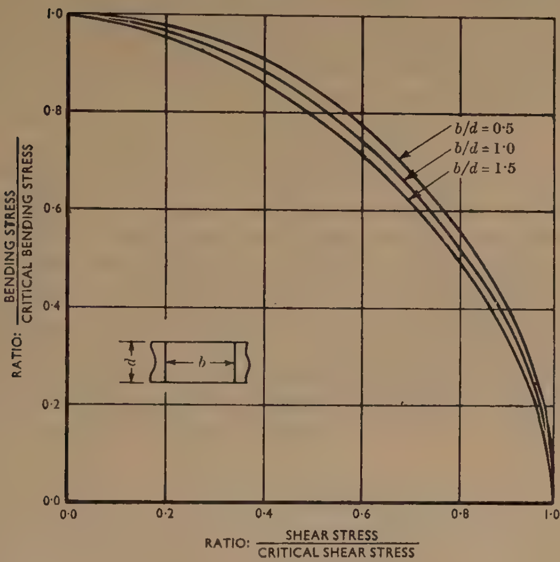


FIG. 33.—INTERACTION CURVES FOR CRITICAL WEB BENDING AND SHEAR STRESSES FOR DIFFERENT VALUES OF b/d

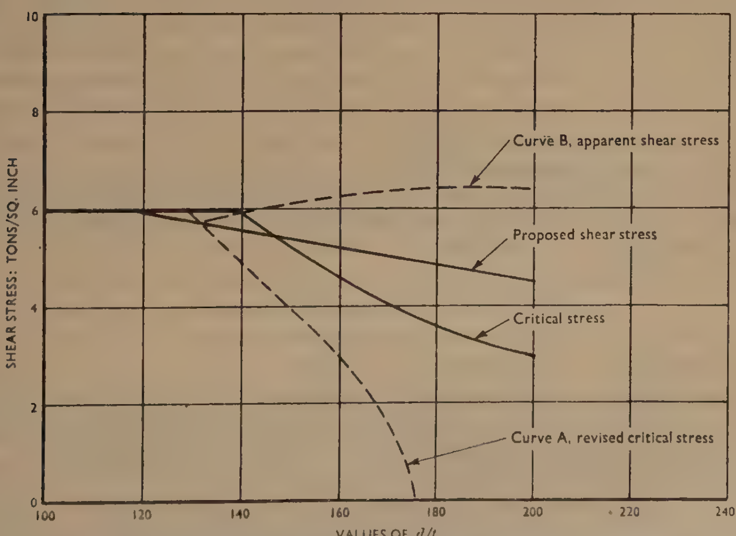


FIG. 34.—APPARENT SHEAR STRESS (CURVE B) FOR A TYPICAL WEB PANEL SUBJECTED TO MAXIMUM ALLOWABLE SHEAR AND BENDING STRESSES

shear stress according to the Huber–von Mises–Hencky theory as $f_e = \sqrt{(f_b^2 + 3f_s^2)}$ the resulting equivalent stress in no case exceeds the limiting values of 14 tons/sq. in.

It should be noted, however, that when the allowable bending and shear stresses occur together in a girder, it is proposed to prohibit any additional increase in stress for wind or abnormal loading.

Limits to sides of web panels

Unstiffened webs

The critical value for shear stress on web plates, as given by Timoshenko for simply supported plates, is a function of $(t/d)^2$ and therefore decreases rapidly with increases in the d/t-ratio of the web plate. A web plate subjected to shear stresses will buckle before it yields if the above critical stress is reached before the shear yield stress.

Taking the shear yield stress as $15{\cdot}25/\sqrt{3} = 8{\cdot}8$ tons/sq. in. leads to a proposed limit of $85t$ for the depth of unstiffened webs. These values are in keeping with experimental results. Sparkes and Heyman have shown from a series of tests that mild-steel webs having a d/t-ratio as high as 90 usually yield before buckling.

Stiffened webs

When the d/t-ratio exceeds the above values, it is economical to stiffen the web, either by vertical or horizontal stiffeners. Vertical stiffeners are more effective in increasing the shear strength of the web, and horizontal stiffeners are used only on very slender webs where the web deflexions tend to become visible and the spacings of vertical stiffeners too close.

Webs stiffened with vertical stiffeners only.—As the d/t-ratio of the web increases, so does the ratio of allowable stress to the theoretical critical stress, and, unless a limit were placed on this ratio, the deflexion of the web would be noticeable under normal working loads. Taking as a limit a lateral deflexion equivalent to the plate thickness, reference to Fig. 28 indicates a maximum ratio for $f_s/f_{s.cr}$ under working loads of about 1·7 to 1·8. This is achieved by limiting the depth to $200t$ for mild steel.

Webs stiffened horizontally and vertically.—By using horizontal stiffeners, the overall d/t-ratio can be increased further; with one such stiffener it is proposed to raise it to 300, and with two stiffeners up to 400.

The spacing of the vertical stiffeners along the girder is also limited so that the smaller panel dimension never exceeds $180t$. This limitation is introduced to improve the ultimate load-carrying capacity of the girder, as well as for stiffening the web for fabrication and transport purposes.

Intermediate web stiffeners

Vertical stiffeners

Minimum rigidity required.—In order to ensure that intermediate vertical stiffeners effectively restrain the web plate and make it act as a rectangular plate supported at four edges, they need a certain minimum rigidity, *i.e.*, their moment of inertia about the plane of the web must have a minimum value.

For an initially plane plate, subject to shear, Timoshenko derives $I_s = 0{\cdot}3d^4t^3/b^3$. For practical web plates which contain some small initial buckles, this is known to be too small and Moore [32] and others recommend $I_s = \frac{4}{3}d^3t^3/b^2$ if the maximum stiffener deflexion is to remain small in comparison with that of the web.

Ultimate-load tests on girders have shown this value of inertia to be adequate, but t is proposed to increase the inertia further and to adopt $I_s = 1{\cdot}5d^3t^3/b^2$.

It will be observed that this value exceeds the Timoshenko theoretical value by amounts varying from 3·35 times for long panels to 10 times for short panels. It is the stiffness required to develop the full allowable shear strength of the panel, and s therefore independent of the actual shear load on the web, once the stiffener spacing has been fixed. However, for a given web thickness, the higher the shear the closer the spacing and, as would be expected, the heavier the stiffener.

Table 5 compares the values of I_s for stiffeners designed in accordance with B.S. 449 and the proposed rule.

Axial load in stiffeners.—As previously explained, tensile membrane stresses are set up in the web plate when it buckles, and these have to be resisted by compressive forces in the intermediate stiffeners, with the girder tending to act as an N-type truss.

The magnitude of these forces, and hence the load in the stiffener, is a function of the ratio $f_s/f_{s\,.\,crit}$. The relation between that ratio and f_c/f_s (where f_c is the compressive stress in the intermediate stiffener) is shown in Fig. 35.

This indicates the order of the direct stress in the stiffeners. For normal structural steel plate girders designed by the proposed rules, the ratio $f_s/f_{s.cr}$ rarely exceeds 2 and the direct load can be safely carried by all practical stiffeners. From this it will be seen that the controlling requirement for the design of intermediate stiffeners on steel girders is one of rigidity rather than of direct stress, though, of course, at loads much in excess of the design load some axial stress does develop.

So far as stiffener design is concerned, the curves of Fig. 35 also emphasize the difference between the use of steel and aluminium as structural materials. Although an aluminium alloy may have a shear proof stress equal to that of steel, its modulus of elasticity E is considerably smaller, and the critical stress for similarly dimensioned web plates is only one-third that for steel. Hence the stiffeners in aluminium plate girders may have to be designed to withstand direct axial loads, as is usual in aircraft construction.

Horizontal stiffeners

For economic and aesthetic reasons, it is desirable to increase the range of the web d/t-ratio to the maximum. New clauses therefore are proposed to permit the use of horizontal stiffeners. Little experimental data is available for the design of this type of stiffener, and the new rules are based on theoretical analyses with a suitable safety factor.

One horizontal stiffener.—The most effective position for a horizontal stiffener on a deep web subjected to bending and shear is at 2/5 of the distance from the compression flange to the neutral axis of the girder. This horizontal stiffener must withstand axial loads due to bending moments in addition to restraining the web plate from buckling. The minimum inertia required is therefore influenced by the ratio of the area of the stiffener to area of web plate.

In practice this ratio is unlikely to exceed 0·1 (corresponding to a 6-in. × 3-in. × $\frac{3}{8}$-in. angle stiffener on a 108-in. × $\frac{3}{8}$-in. web plate) and theoretical values of minimum stiffener requirements are indicated in Fig. 36. Unlike vertical stiffeners, the horizontal stiffeners are not called upon to carry increases in axial load when loads in excess of the theoretical critical values are reached, and they are further restrained by the dissimilar buckled forms of the adjacent panels. The theoretical values[31] have therefore been increased by a factor somewhat smaller than in the case

TABLE 5.—MINIMUM PERMISSIBLE INERTIAS (IN.^{4}) OF VERTICAL STIFFENERS

d: in.	3/8-in. web					1/2-in. web					3/4-in. web				
	d/t	B.S. 449* (1948)	New rule			d/t	B.S. 449* (1948)	New rule			d/t	B.S. 449* (1948)	New rule		
			$b = 1{\cdot}5d$	$b = d$	$b = d/1{\cdot}5$			$b = 1{\cdot}5d$	$b = d$	$b = d/1{\cdot}5$			$b = 1{\cdot}5d$	$b = d$	$b = d/1{\cdot}5$
30	80	4	1	2·4	5										
45	120	7	1·6	4	8	90	8	4	9	19					
60	160	11	2	5	11	120	13	5	11	25	80	14	17	38	85
75	200	19	3	6	14	150	20	7	14	32	100	22	22	48	107
90						180	30	8	17	38	120	33	26	58	130
105						210	40	9	20	44	140	47	30	67	150
120											160	65	34	76	171
135											180	86	38	85	190

* Assuming plates of minimum thickness placed symmetrically on the web.

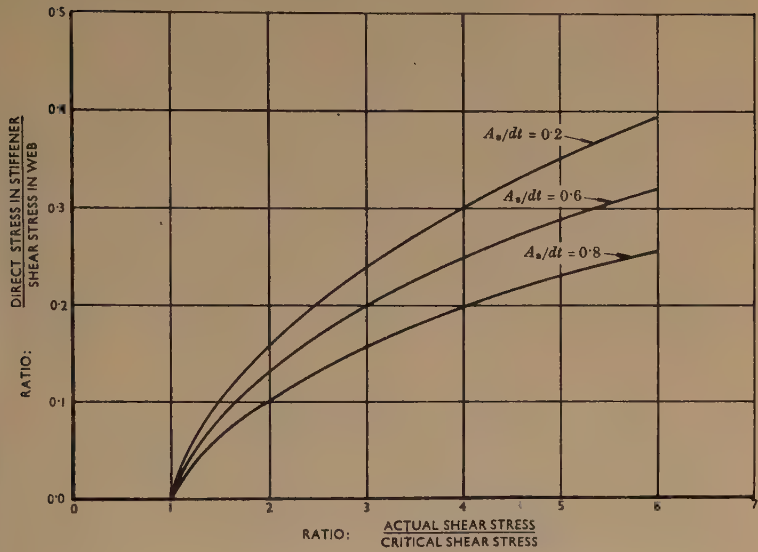


FIG. 35.—INCREASE OF STRESS IN STIFFENERS WITH THE INCREASE OF RATIO OF ACTUAL SHEAR TO CRITICAL SHEAR

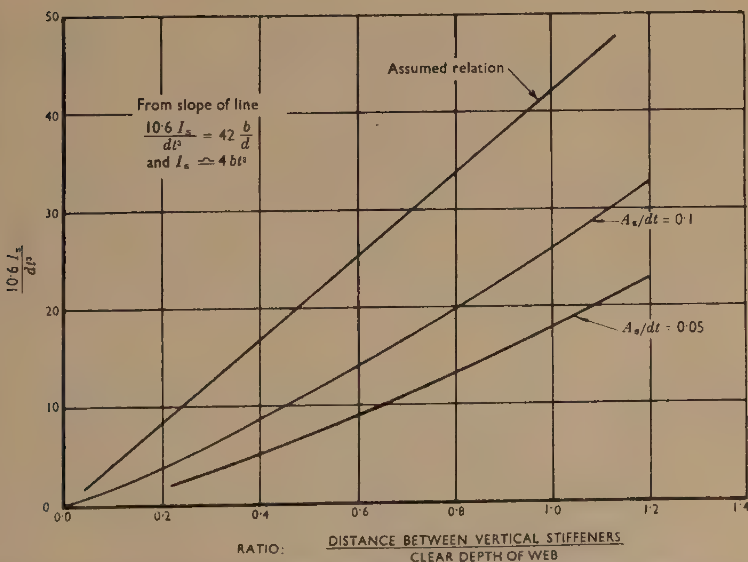


FIG. 36.—THE REQUIRED THEORETICAL AND THE PROPOSED STIFFNESS OF THE HORIZONTAL STIFFENER (AT 1/5 DEPTH FROM COMPRESSION FLANGE) RELATIVE TO THE STIFFNESS OF THE WEB PLATE, SUBJECTED TO BENDING, FOR DIFFERENT SPACING OF VERTICAL STIFFENERS

TABLE 6.—MINIMUM PERMISSIBLE INERTIA (IN.^{4}) OF SINGLE HORIZONTAL STIFFENER

Stiffener spacing, b inches	3/8-in. web				1/2-in. web				3/4-in. web			
	Proposed	German D.I.N. 4114			Proposed	German D.I.N. 4114			Proposed	German D.I.N. 4114		
		$b = 0{\cdot}4d$	$b = 0{\cdot}6d$	$b = 0{\cdot}8d$		$b = 0{\cdot}4d$	$b = 0{\cdot}6d$	$b = 0{\cdot}8d$		$b = 0{\cdot}4d$	$b = 0{\cdot}6d$	$b = 0{\cdot}8d$
60	12·6	7·1	8·0	8·3	30·0	16·8	19·0	20·0	101	60	68·0	71·5
80	16·8	9·4	10·6	11·1	40·0	22·4	25·2	26·5	134	78	88·3	93·0
100	21·0	11·7	13·2	13·8	50·0	28·0	31·4	33·0	168	94	108·6	114·5
120					60·0	33·6	37·6	39·5	202	112	129	136
140									235	132	150	158
160									269	150	170	179

Proposed B.S. 153: $I_s = 4bt^3$.

D.I.N. 4114: $I_s = 0{\cdot}092dt^3(21{\cdot}3 + 112{\cdot}6\gamma)(b/d - 0{\cdot}1)$ ($\gamma = A_s/dt$ and is taken as 0·1).

vertical stiffeners and the value of minimum rigidity given by $I_s = 4bt^3$ is suggested. is is shown in Fig. 36, and compared with the requirements of the German standard Table 6. The latter standard was the minimum requirement for perfect plates, ereas the proposed rule makes some allowance for possible imperfections.

vo horizontal stiffeners.—When a further increase in d/t-ratio is required it is prosed that an additional stiffener be placed on the neutral axis of the girder. This ffener serves solely to limit the panel dimensions and carries no direct load; it can erefore be smaller than the upper horizontal stiffener. A value of minimum inertia $I_s = dt^3$ is proposed.

In the case of girders with unequal flanges the proportions of the web panels should carefully checked to ensure that in no case does either the greater unsupported ar dimension of a web panel exceed $270t$ or the smaller unsupported clear dimension the same panel exceed $180t$.

Discussion of test results

The tests described in Structural Paper No. 49 (p. 426) were devised to prove e validity of the proposed design rules. These girders have maximum flange and ffener spacing as originally proposed in 1954. The arrangement of loads was ch that maximum allowable bending and shear stresses were combined, *i.e.*, a itical stress of nearly 14 tons/sq. in. developed with a margin against local yield of ly about 10%.

Results of the tests carried out by Heyman on model girders are given in Tables 12 d 13.

eb

Girders of the A series (unstiffened webs) developed, at collapse loads, yield stress shear on the whole web. The ratios of collapse to allowable load were 1·44 or gher. Girder C4 with d/t-ratio of 240, however, collapsed prematurely under a ad only 1·24 times the allowable load. This was considered too low a factor against llapse and the permitted limit of d/t was therefore reduced from 240 to 200. It ust be pointed out, however, that this particular girder had large initial web ckles and, unless these coincide with the wave form developed under load, the tting up of tensile membrane stresses would have been postponed and some duction in the load-carrying capacity therefore could be expected.

Girders D and E failed by flange buckling, but at the time of collapse the loads the webs exceeded the allowable ones by 50 and 35% respectively, and there was ly slight deflexion of the web at this stage, clearly indicating a further reserve of rength.

The tests on the full-size girders carried out at the Military Engineering Experiental Establishment were intended primarily to furnish data on flange buckling, it the results (see Table 15, facing p. 486) provide additional information on the haviour of webs. Girder 1 ($d/t = 85$) failed by flange buckling at a load of 37·5 ns when the shear stress was 10% in excess of the design value. The absence buckles again indicates a reserve of strength.

Girder 2 ($d/t = 200$) exhibited web buckling in the course of test Q as shown by e results plotted in Figs 57 and 59; the web plating was initially appreciably storted. In this instance the theoretical critical shear stress (ignoring bending) as 3·2 tons/sq. in. and it will be seen that the readings for diagonal gauges on either de of the plate at the position of maximum buckle diverge rapidly once this stress

is exceeded. The proposed permissible 4·1 tons/sq. in. and the web shear stress was profile taken at about this working stage showed an additional peak lateral deflexion of 0·6 of the plate thickness corresponding to a total deflexion of $1{\cdot}35t$. The peak stress in the buckles occurred at the neutral axis and attained the nominal yield value at a mean shear stress of 5·2 tons/sq. in. compared to an estimated value of 7·1, the discrepancy being due to initial distortions and the neglect of the bending stresses in calculation. The web finally collapsed at a load of 117 tons, providing a load factor of 1·75; it should be remarked that the material yield stress was in excess of the specified minimum (see Table 14) and a reduced value of load factor of the order of 1·50 would be applicable.

The third girder ($d/t = 300$) had badly distorted web plating and vertical stiffeners in the larger panels and in consequence did not behave as predicted. The nominal yield stress was reached in the lower end panel at a load of 67% of the working value and of 0·45 of the theoretical basis in test H. Web profiles taken at this load indicate additional web deflexions of the order of $2\frac{1}{2}$ times the web thickness corresponding to a total deflexion of $1{\cdot}7t$. With the addition of extra vertical stiffeners, in test J, the nominal yield stress was reached in the upper panel adjacent to the load at a mean shear stress of 5·0 tons/sq. in., compared to a predicted value of 5·9 tons/sq. in. The girder collapsed by flange buckling at a load ensuring a web load factor (reduced to account for material properties) of at least 1·4. It would appear that the maximum d/t should be reduced for this type of girder.

The deepest girder, No. 4, had a nearly flat web initially. In test C the web buckled in the lower end panel at a theoretical critical shear stress of 3·2 tons/sq. in. as shown in Fig. 57. Nominal yield was attained at a stress of 7·5 tons/sq. in., in contrast to a predicted value of 7·1. The maximum total deflexion of the panel under twice the working shear stress was about twice the web thickness. Collapse of the girder in test C occurred by flange buckling at a load ensuring a minimum reduced load factor of 1·6 in shear. Upon addition of extra vertical stiffeners, as in Test D, nominal yield was reached in the upper panel nearest the loads at a shear stress of 6·1 tons/sq. in. compared to an estimate of 5·9.

Stiffeners

The web stiffeners for the model and full-size girders were designed in accordance with the most unfavourable interpretation of the proposed rules. Thus the intermediate stiffeners were not attached to either of the flanges and the longitudinal stiffeners were made discontinuous at their intersection with the vertical ones.

The edges of some of the flats used in the model girders were badly distorted and prematurely buckled. As a precaution against such failure in practice the allowable thickness of such flanges has been increased from 1/16th to 1/12th of the outstand.

All other stiffeners in both sets of test girders stood up to the test loads without showing any sign of distress whatsoever and seemed to fully justify the present method of design.

From Fig. 58, facing p. 487, it will be seen that the top horizontal stiffener carries its share of compression stress in the web and the stiffener on the neutral axis is practically unstressed until large buckles are formed at the estimated collapse load. Both stiffeners then show rapid increase in stress.

Design

General aims

The aim of the designer is to produce the lightest girder compatible with economy
d easy maintenance.

This will generally require that both the net tension flange and the gross compres-
on flange are working at the respective maximum permissible stresses and the depth
the girder is such that the overall weight of the whole girder is least.

Procedure

It is assumed that the spans are known and the moments, shears, etc., have already
en calculated. The first step is to determine from previous knowledge, or by
her methods, the approximate depth and area of web required, decide whether or
t it will be stiffened,[35] and guess the probable flange areas, D/T, and l/r_y. Then
e procedure for the more accurate calculations is as follows:

elded girders, without holes

(1) *For small values of* $l/r_y(F_{bc} =$ yield/factor) then: $Z_c = Z_t =$ Moment$/F_{bc}$ and the procedure is normal.

(2) *For large values of* l/r_y ($F_{bc} <$ yield/factor), obtain from Table 9, Appendix I, the approximate value of C, and hence from Table 10, Appendix I, the value of F_{bc}. The required gross value of Z_c is moment $\div F_{bc}$. If the girder is to have equal flanges the subsequent procedure is normal, but for maximum economy, the compression flange should be sufficiently larger than the tension flange that the stress in the compression flange is F_{bc} and in the tension flange is F_{bt}. This can be achieved by reducing the tension flange and possibly slightly increasing the compression flange (as obtained for a symmetrical girder) until the ratio $y_c/y_t = F_{bc}/F_{bt}$ and the two section moduli are of correct magnitude.

The girder must now be checked as one with unequal flanges. To do this, deter-
ine the values of l/r_y, D/T, and λ, and hence from Table 8, Appendix I, the value
k_2. Then obtain the new value of C from $A + k_2B$ (see Appendix I) and hence F_{bc}.
e process is somewhat involved, but the final result will show a substantial
onomy.

iveted or welded girders with holes in tension flanges

(1) *For small values of* $l/r_y(F_{bc} =$ yield/factor) then:

$$\text{Stress in tension flange} = \frac{\text{Moment}}{\text{Gross } Z_t} \times \frac{\text{Gross area of flange}}{\text{Net area of flange}}$$

and must not exceed yield/factor.

If the girder is to have equal flanges, as laid down in B.S. 153 (1937), both flanges have to be increased to allow for holes in the tension flange. In this connexion it may be mentioned that it seems to be proper to assume that normal holing of the tension flange does not influence the position of the neutral axis as derived from the gross section of the girder. For maximum economy the tension flange only should be increased. Usually this cannot be done locally at the holes and the design becomes that for a girder with unequal flanges. For normal type of girders, the net area of the tension flange should be increased by about $1\frac{1}{2}$ times the area

of the holes deducted. As the neutral axis moves towards the tension flange, the actual stress in the compression flange increases, while the value of F_{bc} may have to be decreased because, now, $C = (A - k_2B)y_c/y_t$. A complete recheck will be necessary, again making the design somewhat complicated, but, as before, there will be an economy, although in this case, only a small one.

(2) *For large values of l/r_y (F_{bc} < stress in tension flange on net area)*
In this case, if the girder is to have equal flanges the compression flange determines the size. However if F_{bc} is much smaller than F_{bt}, then the areas of both flanges should be adjusted until maximum allowable stresses are reached in each flange and to achieve this the procedure will be similar to that for case (2) of welded girders, without holes.

Curtailment of flanges

The curtailment of flanges weakens the girder and when l/r_y is large, achieves little, if any, economy. However, when l/r_y is small, curtailment of flanges is economic and either one or both flanges may be curtailed. The procedure is approximately as follows:

Estimate the minimum and maximum flange sections and obtain ν = minimum flange areas ÷ maximum flange areas and hence from Table 7, Appendix I, find k_1. Then the effective flange thickness $T = k_1t$, where t is the maximum mean flange thickness = maximum flange area ÷ width of flange.

With the modified value of D/T proceed checking the girder as before.

This procedure is somewhat cumbersome and will operate against curtailment of flanges, which is all to the good.

Examples

Fully worked-out examples of the design of girders are given in Appendix II.

Conclusions

It will be seen that the complicated expressions derived for critical stresses in flanges and webs of single-web symmetric and unsymmetrical girders have been reduced to a more usable form. With the assistance of tables the design of girders with unequal flanges is made reasonably simple. The range of depth of girders has been considerably increased by the introduction of horizontal stiffeners and of a more rational design of the vertical ones. These improvements will greatly extend the economic range of plate girders.

It should be noted that the allowable stresses apply equally well to channels, tees, and angles, but not to double-web box-girders, whose torsional stiffness is usually such that for most practical spans and proportions the question of lateral instability would not arise. If an approximate check is required, appropriate geometric properties can be introduced into the fundamental equation for critical stress.

It is estimated that if and when the proposed method of design is adopted and designers have had time to get familiar and skilful with it, an overall economy of about 10–15% in the weight of steel used in plate girder and compound beam work will be realized.

Acknowledgements

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APPENDIX I

PROPOSED CLAUSES IN REVISED B.S. 153

CLAUSE V

asic permissible stresses in structural steel
Except where variations are specified in other Clauses of the specification, structures all be so designed that the calculated stresses in structural steels do not exceed the values ven in the table below:—

Description	Basic permissible stresses (tons/sq. in.) Steel to B.S. 15
arts in tension and compression	
On the effective sectional area for extreme fibre stress	
(i) for plate girders with unstiffened webs and for rolled beams	9·5 (also see Clause W)
(ii) for plate girders with stiffened webs	9·0 (also see Clause W)
arts in shear	
Maximum stress	7·0
Average on the gross effective sectional area of webs of girders	6·0* (also see Clause X)

* As a result of the discussion this figure should be changed to 5·5.

CLAUSE W

WORKING STRESSES IN FLANGES OF GIRDERS WITH SINGLE SOLID WEBS

) *On the gross section of the compression flange*
The bending stress f_{bc} calculated on gross section of the girder shall not exceed the propriate values given in clause V, nor shall it exceed F_{bc}, given in Table 10, in which C obtained as follows (where the ratio of thicknesses of flanges does not exceed 3):—

(i) For girders with flanges of equal moment of inertia about y–y axis

$$C = \frac{170,000}{(l/r_y)^2}\sqrt{1 + \frac{1}{20}\left(\frac{l}{r_y}\cdot\frac{T}{D}\right)^2} \qquad [= A]$$

Where l = effective length, in inches, of compression flange (see Clause Z).
r_y = radius of gyration, in inches, of the gross section of the whole girder, about the y–y axis, at the point of maximum bending moment.
D = overall depth of girder, in inches, at the point of maximum bending moment.
T = effective thickness of flange, in inches
= $k_1 \times$ mean thickness of the horizontal portion of the flange at the point of maximum bending moment. (For rolled sections, thickness given in reference books.)
The co-efficient k_1 allows for the curtailment of the thickness and/or the breadth of the flanges and depends on ν, the ratio of the area of flanges at the point of minimum bending moment to the area of flanges at the point of maximum bending moment. For flanges of constant area $k_1 = 1$. Values of k_1 for different values of ν are given in Table 7.

TABLE 7.—VALUES OF k_1

ν	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	0·0
k_1	1·0	1·0	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2

Note:—(*a*) In the case of curtailment of breadth of flange, the ratio ν shall not be less than 0·25.

(*b*) Where the value of N for the compression flange, considered alone, is smaller than that when both flanges are combined, this smaller value of N shall be used.

(ii) For girders with compression flange of greater moment of inertia about the y–y axis than that of the tension flange:—

$$C = \frac{170{,}000}{(l/r_y)^2}\sqrt{1 + \frac{1}{20}\left(\frac{l}{r_y}\cdot\frac{T}{D}\right)^2} + k_2\frac{170{,}000}{(l/r_y)^2} \qquad [= A + k_2B]$$

Where l, r_y, and D are as defined in (i) above, and

T = effective thickness of flange, in inches

= $k_1 \times$ mean thickness of the horizontal portion of the flange of greater moment of inertia, about the y–y axis of the girder, at the point of maximum bending moment.

The coefficient k_1 being as in Table 7.

k_2 = a coefficient to allow for inequality of tension and compression flanges and depends on λ, the ratio of the moment of inertia, about the y–y axis of the girder, of the *compression* flange to that of the whole section, at the point of maximum bending moment.

For equal flanges $k_2 = 0$.

Values of k_2 for different values of λ are given in Table 8A:—

TABLE 8A.—VALUES OF k_2

λ	1·0	0·9	0·8	0·7	0·6	0·5
k_2	0·5	0·4	0·3	0·2	0·1	0·0

(iii) For girders with compression flanges of smaller moment of inertia about y–y axis than that of the tension flange:—

$$C = \left\{\frac{170{,}000}{(l/r_y)^2}\sqrt{1 + \frac{1}{20}\left(\frac{l}{r_y}\cdot\frac{T}{D}\right)^2} + k_2\frac{170{,}000}{(l/r_y)^2}\right\} \times \frac{y_c}{y_t} \qquad \left[= (A + k_2B) \times \frac{y_c}{y_t}\right]$$

Where l, r_y, D, T, and k_2 are as defined in (ii) above, and

y_c = distance from neutral axis of girder to extreme fibre in compression.

y_t = distance from neutral axis of girder to extreme fibre in tension.

Values of k_2 for different values of λ are given in Table 8B.

TABLE 8B.—VALUES OF k_2

λ	0·5	0·4	0·3	0·2	0·1	0·0
k_2	0·0	−0·2	−0·4	−0·6	−0·8	−1·0

For all cases, values of A and B are given in Table 9.

(*b*) *On the net section of the tension flange*

The bending stress f_{bt} calculated on gross section of the girder, and increased in the ratio of gross area of flange ÷ net area of flange (see Clause Y), shall not exceed the appropriate values given in Clause V.

TABLE 9.—VALUES OF A AND B

Values of C are given by: (i) A

(ii) $A + k_2B$

or (iii) $(A + k_2B)y_c/y_t$

	A										B
	Values of D/T										
	10	15	20	25	30	35	40	45	50	100	∞
50	102·0	84·8	77·9	74·3	72·6	71·4	70·6	70·1	69·7	68·4	68·0
60	79·0	63·3	56·9	53·5	51·7	50·6	49·8	49·3	48·7	47·7	47·2
70	64·4	50·2	44·1	40·9	39·1	38·0	37·2	36·7	36·4	35·1	34·7
80	54·4	41·3	35·6	32·7	30·9	29·8	29·1	28·6	28·2	27·0	26·6
90	47·2	35·1	29·8	26·9	25·3	24·2	23·5	23·0	22·6	21·4	21·0
100	41·7	30·5	25·5	22·8	21·2	20·2	19·5	19·0	18·6	17·4	17·0
110	37·3	27·0	22·3	19·7	18·2	17·2	16·5	16·0	15·7	14·5	14·1
120	33·8	24·2	19·8	17·3	15·8	14·9	14·2	13·8	13·4	12·2	11·8
130	30·9	21·9	17·8	15·4	14·0	13·1	12·4	12·0	11·6	10·5	10·1
140	28·5	20·1	16·1	13·9	12·5	11·7	11·0	10·6	10·2	9·1	8·7
150	26·5	18·5	14·8	12·6	11·3	10·5	9·9	9·4	9·1	8·0	7·5
160	24·7	17·1	13·6	11·6	10·3	9·5	8·9	8·5	8·2	7·1	6·6
170	23·1	16·0	12·6	10·7	9·5	8·7	8·1	7·7	7·4	6·3	5·9
180	21·8	15·0	11·8	9·9	8·8	8·0	7·4	7·0	6·7	5·7	5·3
190	20·6	14·2	11·1	9·3	8·2	7·4	6·9	6·5	6·2	5·1	4·7
200	19·5	13·3	10·4	8·7	7·6	6·9	6·4	6·0	5·7	4·7	4·3
210	18·5	12·7	9·8	8·2	7·2	6·5	6·0	5·6	5·3	4·3	3·9
220	17·6	12·1	9·3	7·8	6·7	6·1	5·6	5·2	4·9	3·9	3·5
230	16·8	11·5	8·9	7·3	6·4	5·7	5·2	4·9	4·6	3·6	3·2
240	16·1	11·0	8·5	7·0	6·1	5·4	4·9	4·6	4·3	3·4	3·0
250	15·4	10·5	8·1	6·7	5·7	5·1	4·7	4·3	4·1	3·1	2·7

TABLE 10.—PERMISSIBLE STRESS F_{BC} (TONS/SQ. IN.) FOR DIFFERENT VALUES OF C

C	F_{BC}	C	F_{BC}	C	F_{BC}
2	1·0	12	4·6	35	8·2
3	1·5	14	5·1	40	8·6
4	2·0	16	5·5	45	8·9
5	2·4	18	5·9	50	9·2
6	2·8	20	6·3	55	9·4
7	3·2	22	6·7	60	9·5
8	3·5	24	7·0	& over	
9	3·8	26	7·3		
10	4·1	28	7·5		
		30	7·7		

CLAUSE X

Working stresses in solid web plates

(*a*) The average shear stress f_s calculated on the effective sectional area of the web shall not exceed the values of F_s given by (i) for unstiffened webs, or the lesser of the values given by (i) and (ii) for stiffened webs.

(i) $F_s = 6{\cdot}0$ tons/sq. in.*

(ii) $F_s = 6\left\{1{\cdot}3 - \dfrac{b/t}{250\{1 + \frac{1}{4}(b/a)^2\}}\right\}$ tons/sq. in.

where a = the greater unsupported clear dimension of the web in a panel, not greater than $270t$.

b = the lesser unsupported clear dimension of the web in a panel, not greater than $180t$.

t = thickness of web.

The values of F_s for different ratios of d/t and various spacings of stiffeners, satisfying the requirements of (i) and (ii) above, are given in Table 11.

Where depth of panel d is taken as—

(i) For webs without horizontal stiffeners the clear distance between flange angles or, where there are no flange angles, between flanges (ignoring fillets).

When "tongue plates," *i.e.*, vertical plates of not less than twice the thickness of the web plate inserted between flanges and web plate, are used d shall be taken as the distance between the edges of the tongue plates.

(ii) For webs with horizontal stiffeners (as described in other Clauses) the clear distance between tension flange (angles or flange plate or tongue plates) and the horizontal stiffener.

(*b*) *Combined shear and bending stresses*

Irrespective of permissible increases of stress (as given in other Clauses) the equivalent stress f_e due to bending and shear as given by:—

$$f_e = \sqrt{f_{bt}^2 + 3f_s^2} \quad \text{or} \quad \sqrt{f_{bc}^2 + 3f_s^2}$$

shall not exceed 14 tons/sq. in., where f_{bt} or f_{bc} and f_s are the co-existent bending and shear stresses.

(*c*) *Combined bearing, bending, and shear stresses*

When a bearing stress is combined with tensile bending and shear stresses approaching the maximum allowable values under any combination of loading, the equivalent stress f_e shall not exceed 14 tons/sq. in.

where $$f_e = \sqrt{f_{bt}^2 + f_p^2 + f_{bt}f_p + 3f_s^2}$$

in which f_{bt}, f_s, and f_p are the numerical values of co-existent bending, shear, and bearing stresses.

* Subsequently changed to 5·5 tons/sq. in.

d/t	Stress F_s (tons/sq. in.) for different distances between stiffeners.												
	0·33d	0·4d	0·5d	0·6d	0·7d	0·8d	0·9d	d	1·1d	1·2d	1·3d	1·4d	1·5
90	6·00	6·00	6·00	6·00	6·00	6·00	6·00	6·00	6·00	6·00	6·00	6·00	6·00
100	6·00	6·00	6·00	6·00	6·00	6·00	6·00	6·00	6·00	6·00	5·95	5·89	5·84
110	6·00	6·00	6·00	6·00	6·00	6·00	6·00	6·00	5·93	5·84	5·76	5·70	5·64
120	6·00	6·00	6·00	6·00	6·00	6·00	5·96	5·88	5·76	5·66	5·58	5·51	5·44
130	6·00	6·00	6·00	6·00	6·00	5·91	5·80	5·72	5·59	5·48	5·39	5·31	5·25
140	6·00	6·00	6·00	6·00	5·91	5·76	5·65	5·56	5·42	5·31	5·21	5·12	5·05
150	6·00	6·00	6·00	5·97	5·78	5·62	5·49	5·40	5·25	5·13	5·02	4·93	4·85
160	6·00	6·00	6·00	5·85	5·64	5·47	5·34	5·24	5·08	4·95	4·84	4·74	4·66
170	6·00	6·00	5·99	5·73	5·51	5·33	5·19	5·08	4·91	4·77	4·65	4·55	4·46
180	6·00	6·00	5·88	5·60	5·37	5·18	5·03	4·92	4·74	4·59	4·47	4·36	4·27
190	6·00	6·00	5·77	5·48	5·24	5·04	4·88						
200	6·00	6·00	5·67	5·36	5·10	4·89	4·73						
210	6·00	5·93	5·56	5·24	4·97	4·75							
220	6·00	5·84	5·45	5·12	4·83	4·60							
230	6·00	5·76	5·35	4·99	4·70								
240	6·00	5·67	5·24	4·87	4·56								

Note: 6·0 tons/sq. in. has been subsequently reduced to 5·5 tons/sq. in.

Clause Y

Sectional properties of solid web girders (plate girders and rolled beams)

(*a*) Solid web girders shall be proportioned on the basis of the moment of inertia of the gross cross-section with the neutral axis taken at the centroid of that section. In computing the maximum stress, the stresses calculated on this basis shall be increased in the ratio of gross to effective area of the flange section. For this purpose the flange section in riveted construction shall be taken to be the flange plates, flange angles, and that portion of the web and side plates, if any, between the flange angles; in welded construction the effective area shall be taken to be the flange plates and the tongue plates, *i.e.*, thick vertical plates connecting flanges to web, if any.

(*b*) The effective sectional area of compression flanges shall be the gross area less the specified deductions for excessive width or unsupported projections of plates and the maximum deductions for open holes and holes for black bolts occurring in a section perpendicular to the axis of the member.

(*c*) The effective sectional area of tension flanges shall be the gross sectional area less:—

(i) *Deductions for rivet and bolt holes*

(A) Where rivets or bolts are not staggered, the area to be deducted shall be the sum of the sectional areas of the maximum number of holes in any cross-section at right angles to the direction of stress in the member.

(B) Where rivets or bolts are staggered, the area to be deducted shall be the sum of:—
either the sectional area of holes in a cross-section as in (A) above,
or the sectional area of all holes on any zig-zag line extending progressively across the member, less the product of the thickness of the material, and the quantity $p^2/4G$ for each gauge space in the chain.

Where p = the pitch, *i.e.*, the distance between the centre lines of consecutive rivets measured parallel to the direction of stress in the member.
G = the gauge, *i.e.*, the distance between consecutive rivets in a chain measured at right angles to the direction of stress in the member.

For sections such as angles with holes in both legs the gauge shall be measured along the centre of the thickness of the angle.

Note. In built-up members where the chains of holes considered in individual parts do not correspond with a common chain of holes for the members as a whole, the value of any bolts or rivets joining the parts between such chains of holes shall be taken into account in determining the strength of the member.

(*d*) The effective sectional area for parts in shear shall be as follows:—

(i) *Webs of plate girders*
The effective sectional area shall be the gross sectional area of the full depth of the web plate. Where webs are varied in thickness in the depth of the section by the use of tongue plates or the like, the above approximation is not permissible and the maximum shear stress shall be computed.

(ii) *Rolled beams and channels*
The effective sectional area shall be the gross sectional area of the web calculated on the full depth of the section.

(iii) *Rectangular and asymmetrical sections*
The maximum shear stress shall be computed from the whole area of the cross-section having regard to the distribution of flexural stresses.

(*e*) Webs which have openings larger than those used for rivets or other fastening, require special consideration and the provisions of this clause are not applicable.

(*f*) The slenderness ratio l/r_y of the girder shall not exceed 250; further it shall not exceed 125 for cantilevers.

Clause Z

Effective length of compression flanges

(*a*) For simply supported girders where there is no lateral bracing between compression anges and no cross-frames but with each end restrained against torsion (see Note below) e effective length to be used in the equation given in Clause W, shall be taken as follows:—

(i) With ends of compression flanges unrestrained against lateral bending . $l =$ span (*i.e.*, free to rotate in plan at the bearing)

(ii) With ends of compression flanges partially restrained against lateral bending . . . . $l = 0{\cdot}85 \times$ span

(iii) With ends of compression flanges fully restrained against lateral bending $l = 0{\cdot}7 \times$ span (*i.e.*, not free to rotate in plan at the bearing)

Note. Restraint against torsion at the supports is sufficiently provided by stiffness of the section in the case of rolled beams (to B.S. 4) seated on the bearings, and by bearing stiffeners in the case of plate girders seated on a bearing. The ends of compression flanges of girders with horizontal web stiffeners shall in addition be held against lateral deflection.

(*b*) For simply supported girders where there is no lateral bracing of the compression ange, but where cross-members and stiffeners (see Note below) provide lateral restraint, e effective length and the allowable working stress shall be calculated as follows:—

(i) when δ is not greater than $S^3/40EI$ then $l = S$

where $\delta =$ the virtual lateral displacement of the compression flange at the frame nearest mid-span of the girder, taken as a horizontal deflexion of the stiffener at the point of its intersection with the centroid of the compression flange, under the action of a unit horizontal force applied at this point to the frame only. This deflexion shall be computed, assuming that the cross-member is free to deflect vertically and the tangent to the deflexion curve at the centre of its span remains parallel to the neutral axis of the unstrained cross-member.

$S =$ distance between frames.

$I =$ maximum moment of inertia of compression flange about the y–y axis of the girder.

e allowable working stress shall be obtained from Clause W.

(ii) When δ is greater than $S^3/40EI$ as defined above, then:

$$l = 2{\cdot}5\sqrt[4]{EIS\delta}$$

but not greater than the effective span.

Note:—The connexions of members forming U-frames shall be designed to resist the ect of a horizontal force acting normal to the compression flange of the girder at the vel of the centroid of this flange and having a value equal to $1\frac{1}{4}\%$ of the force in e flange at the point concerned in addition to the effects of wind and other applied rces.

(*c*) For all girders where there is effective lateral bracing to the compression flange the ective length to be used in the equation given in Clause W, shall be taken as:—

$l =$ the distance between centres of intersection of the bracing with the compression flange.

(*d*) For all girders where compression flanges are unbraced but supported laterally by embers controlled by an effective bracing system or anchorage the effective length to used in the equation given in Clause W shall be taken as:—

$l =$ the distance between centres of lateral supports.

(*e*) For cantilever beams the effective length to be used in the equation given in Clause W ll be taken as follows:—

(i) Built in at the support, free at the end . . . . $l = 0{\cdot}85L$

(ii) Built in at the support, restrained against torsion at the free end by contiguous construction. . . . . $l = 0{\cdot}75L$

(iii) Built in at the support, restrained against lateral deflection and torsion at the end . . . . $l = 0{\cdot}5L$

(iv) Continuous at the support, unrestrained against torsion at the support and free at the end . . . . $l = 3L$

(v) Continuous at the support with partial restraint against torsion at the support and free at the end . . . . $l = 2L$

(vi) Continuous at the support, restrained against torsion at the support and free at the end . . . . $l = L$

Where in cases (iv), (v), and (vi) there is a degree of fixity at the "free" end the effective length shall be reduced by 0·75/0·85 and 0·5/0·85 for degrees of fixity corresponding to cases (ii) and (iii) respectively.

L = length of cantilever.

The Note to Clause *Za* also applies here.

(*f*) All the above effective lengths shall be increased by 20% when the load is applied at the compression flange and is free to move laterally.

APPENDIX II

Design examples

Example 1: Plate girder

Data: Girder of 38 ft span carrying point load of 50 tons at mid-span and a total uniformly distributed load of 5 tons (including own weight).
Maximum bending moment: 6,000 tons-in. and max. shear = 27·5 tons.
Depth of girder (to be) approximately 4 ft.
All-welded construction.[35]

(*a*) *Unstiffened web; equal flanges*
Min. thickness of web ($d/t < 85$) = 48″/85 = 9/16 in.
Area of web = 27 sq. in. (ample for shear).

Try 20″ flange width, whence approx. $r_y = 4''$ and $l/r_y = 110$.
Assume $T = 1''$, whence approx. $D/T = 50$.
Then from Table 9: $C = 15{\cdot}7$ tons/sq. in.
and from Table 10: $F_{bc} = 5{\cdot}5$ tons/sq. in.
Estimate area of flange A_F from:

$$6{,}000 = 5{\cdot}5\left\{\frac{48 \times 9/16}{6} + A_F\right\} \times 50''$$

whence $A_F = 17{\cdot}3$ sq. in. approx.

Try $\left\{\begin{array}{l}\text{Flanges } 20'' \times 7/8'' \\ \text{Web } 48'' \times 9/16''\end{array}\right\}$ Then $\left\{\begin{array}{l} I_x = 26{,}080 \text{ in.}^4 \\ I_y = 1{,}168 \text{ in.}^4 \\ Z = 1{,}050 \text{ in.}^3\end{array}\right.$

whence $f_{bc} = \dfrac{6{,}000}{1{,}050} = \underline{5{\cdot}7 \text{ tons/sq. in.}}$

Check: $r_y = 4{\cdot}35''$; $l/r_y = 105$; $D/T = 57$.
From Table 9: $C = 16{\cdot}9$ tons/sq. in.
" Table 10: $F_{bc} = 5{\cdot}7$ tons/sq. in.
So the design is satisfactory (weight/ft = 210 lb.).

(*b*) *Unstiffened web; unequal flanges*
Use web 48″ × 9/16″. Area = 27 sq. in. as before.
Top flange 20″ wide.
Assume approximate values $F_{bc} = 6$ and $F_{bt} = 9{\cdot}5$ tons/sq. in.
Position of neutral axis can be estimated from: $y_t + y_c = 49''$ and $y_t/y_c = 9{\cdot}5/6{\cdot}0 = 30''/19''$

Approximate moment resisted by web $= \dfrac{27}{6} \times 48 \times \left(\dfrac{6{\cdot}0 + 9{\cdot}5}{2}\right)$
$= 1{,}700$ tons-in.

∴ Approx. " " " each flange $= \dfrac{6{,}000 - 1{,}700}{2} = 2{,}150$ tons-in.

and ,, area of top flange $= \dfrac{2{,}150}{19 \times 6{\cdot}0} = 19$ sq. in.

and ,, ,, ,, bottom ,, $= \dfrac{2{,}150}{30 \times 9{\cdot}5} = 7{\cdot}5$ sq. in.

After trial, the following make-up is obtained:

Top flange: $20'' \times 1''$
Web: $48'' \times 9/16''$
Bottom flange: $14'' \times \frac{1}{2}''$

Then: $I_x = 19{,}370$ in.^{4}, $y_c = 19{\cdot}1$ in., $I_y = 782$ in.^{4}, $Z_c = 1{,}015$ in.^{3}, $Z_t = 638$ in.^{3}

whence $f_{bc} = 5{\cdot}9$ and $f_{bt} = 9{\cdot}4$ tons/sq. in.

Check: $r_y = 3{\cdot}85''$; $l/r_y = 119$; $D/T = 49{\cdot}5$; $\lambda = 667/782 = 0{\cdot}85$.

From Table 8A: $k_2 = 0{\cdot}35$.
,, Table 9: $C = 13{\cdot}7 + 0{\cdot}35 \times 12{\cdot}0 = 17{\cdot}9$ tons/sq. in.
,, Table 10: $F_{bc} = 5{\cdot}9$ tons/sq. in.
and from Clause V: $F_{bt} = 9{\cdot}5$ tons/sq. in.

So the design is satisfactory (weight/ft = 184 lb.).

Comparing results for designs (*a*) and (*b*) it is seen that the girder with equal flanges is 14% heavier than the one with unequal flanges.

(*c*) *Stiffened web; equal flanges*

Use web $48'' \times 5/16''$. Area = 15 sq. in. and $d/t = 154$.
$f_s = 1{\cdot}8$ ton/sq. in. (satisfactory).
Try $20'' \times 7/8''$ flange plates as in case (*a*).

Main section

Two flanges: $20'' \times 7/8''$
Web: $48'' \times 5/16''$

Then: $I_x = 23{,}780$ in.^{4}, $I_y = 1{,}167$ in.^{4}, $Z = 955$ in.^{3}

whence $f_{bc} = 6{,}000/955 = 6{\cdot}3$ tons/sq. in.

Check: $r_y = 4{\cdot}83''$; $l/r_y = 94$; $D/T = 57$.

From Table 9: $C = 20{\cdot}6$ tons/sq. in.
,, Table 10: $F_{bc} = 6{\cdot}4$ tons/sq. in.

So the design is satisfactory (weight/ft = 170 lb.).

Intermediate stiffeners

Shear is small (27 tons), so use max. permissible spacing:

$$1{\cdot}5d = 1{\cdot}5 \times 4 = 6 \text{ ft approx.}$$

Span of 38 ft gives 7 spacings of $65'' = b$.

Required stiffener inertia $I_s = 1{\cdot}5d^3t^3/b^2 = \dfrac{1{\cdot}5 \times 48^3}{65^2} \times \left(\dfrac{5}{16}\right)^3 = 1{\cdot}2$ in.^{4}

This is too small for practical purposes so adopt $3'' \times 2'' \times 5/16''$ angles on alternate sides of web.

$$I_s = 1{\cdot}29 + 1{\cdot}46 \times 2^2 = 7{\cdot}13 \text{ in.}^4$$

End stiffeners

Load = 27 tons; $l = 0{\cdot}7 \times 48'' = 34''$.
F_c is about $8{\cdot}5$ tons/sq. in.
Use end plate $12'' \times \frac{1}{2}''$; $f_c = 2{\cdot}2$ tons/sq. in.

Weight of stiffeners = 2 × 80 lb. + 6 × 20 lb. = 284 lb.
Total weight per foot of girder = 170 + 8 = 178 lb.

Note: Girder with unstiffened web is 18% heavier than girder with stiffened web.

(*d*) *Stiffened web; unequal flanges*

Use web: $48'' \times 5/16''$. Area = 15 sq. in. as before.
Top flange: 20″ wide.
Assume F_{bc} and F_{bt} to be about 6·5 and 9·0 tons/sq. in. respectively.

Position of neutral axis can be estimated from $y_t + y_c = 49''$
and $y_t/y_c = 9{\cdot}0/6{\cdot}5 = 28{\cdot}5''/20{\cdot}5''$.

Approximate moment resisted by web $= \frac{15}{6} \times 48 \times \left(\frac{9{\cdot}0 + 6{\cdot}5}{2}\right) = 950$ tons-in.

∴ Approx. „ in each flange $= \frac{6{,}000 - 905}{2} = 2{,}525$ tons-in.

and „ area of top flange $= \frac{2{,}525}{6{\cdot}5 \times 20{\cdot}5} = 19$ sq. in.

„ „ „ „ bottom „ $= \frac{2{,}525}{9{\cdot}0 \times 28{\cdot}5} = 10$ sq. in.

After trial, the following make-up is obtained:

$$\left.\begin{array}{ll}\text{Top flange:} & 20'' \times 15/16''\\ \text{Web:} & 48'' \times 5/16''\\ \text{Bottom flange:} & 14'' \times 3/4''\end{array}\right\}\ \text{Then:}\left\{\begin{array}{l} I_x = 19{,}420 \text{ in.}^4\\ y_c = 20{\cdot}4 \text{ in.}\\ I_y = 797 \text{ in.}^4\\ Z_c = 955 \text{ in.}^3\\ Z_t = 665 \text{ in.}^3\end{array}\right.$$

whence $f_{bc} = 6{\cdot}3$ and $f_{bt} = 9{\cdot}0$ tons/sq. in.

Check: $r_y = 4{\cdot}25''$; $l/r_y = 107$; $D/T = 53$; $\lambda = 625/797 = 0{\cdot}78$.
From Table 8A: $k_2 = 0{\cdot}28$.
„ Table 9: $C = 16{\cdot}4 + 0{\cdot}28 \times 15{\cdot}0 = 20{\cdot}6$ tons/sq. in.
„ Table 10: $F_{bc} = 6{\cdot}4$ tons/sq. in. and from Clause V, $F_{bt} = 9{\cdot}0$ tons/sq. in.
So the design is satisfactory (Weight/ft = 150 + 8 = 158 lb. This is the lightest girder.

The comparative weights of the four girders are as follows:—

Girder	(*a*)	(*b*)	(*c*)	(*d*)
Weight	1·33	1·17	1·13	1·00

So select girder (*d*) as the lightest and cheapest.

Example 2: Slender compound beams

Data: Span of 40 ft, carrying uniformly distributed load of 1,600 lb/ft. (incl. own weight).
Top flange unsupported.
Maximum bending moment = 1,700 tons-in. and max. shear = 14·3 tons.

(*a*) *Symmetrical beam*

Try $22'' \times 7'' \times$ 75-lb. R.S.J. with two $12'' \times 13/16''$ plates.
Properties: $I_x = 4{,}217$ in.^{4}; $I_y = 275$ in.^{4}; $r_y = 2{\cdot}58$ in.
Using 15/16″ rivet holes:

$$\frac{\text{Gross area bottom flange}}{\text{Net}\quad ,,\quad ,,\quad ,,} = \frac{15{\cdot}6}{12{\cdot}5} = 1{\cdot}25.$$

Check: $Z_c = 358$ in.^{3} ∴ $f_{bc} = 4{\cdot}75$ tons/sq. in.
$f_{bt} = 1{\cdot}25 \times 4{\cdot}75 = 5{\cdot}95$ tons/sq. in.
$T = 15{\cdot}6/12 = 1{\cdot}30''$; $D/T = 18{\cdot}2$; $l/r_y = 186$.
From Table 9: $C = 12{\cdot}5$ tons/sq. in.
„ Table 10: $F_{bc} = 4{\cdot}75$ tons/sq. in.
„ Clause V: $F_{bt} = 9{\cdot}5$ tons/sq. in.
So the design is satisfactory (Weight/ft = 141 lb.).

Flange curtailment

In this case, the flange plates cannot be curtailed without at the same time considerably increasing their maximum thickness, and no further economy is obtained.

(*b*) *Unsymmetrical beam; top flange plate only*

Guess approximate values $F_{bc} = 5$ and $F_{bt} = 9{\cdot}5$ tons/sq. in.
Required ratio $y_t/y_c = 9{\cdot}5/5{\cdot}0 = 1{\cdot}9$.
Try $22'' \times 7'' \times$ 75-lb. R.S.J. with $12'' \times 1''$ plate on top flange only.

$$\text{Properties:}\left\{\begin{array}{l} I_x = 2{,}705 \text{ in.}^4\\ I_y = 185 \text{ in.}^4\\ r_y = 2{\cdot}34 \text{ in.}\end{array}\right\}\text{and}\left\{\begin{array}{l} y_c = 7{\cdot}95 \text{ in.}\\ Z_c = 340 \text{ in.}^3\\ Z_t = 180 \text{ in.}^3\end{array}\right. \qquad \frac{y_t}{y_c} = \frac{15{\cdot}05}{7{\cdot}95} = 1{\cdot}9$$

$T = 17{\cdot}85/12 = 1{\cdot}49$ in.

Check: $f_{bc} = 5\cdot0$ and $f_{bt} = 9\cdot45$ tons/sq. in.
$l/r_y = 206$; $D/T = 23/1\cdot49 = 15\cdot5$; $\lambda = 164/185 = 0\cdot89$.
From Table 8A: $k_2 = 0\cdot39$.
„ Table 9: $C = 12\cdot6 + 0\cdot39 \times 4\cdot1 = 14\cdot2$ tons/sq. in.
„ Table 10: $F_{bc} = 5\cdot2$ tons/sq. in.
„ Clause V: $F_{bt} = 9\cdot5$ tons/sq. in.
So the design is satisfactory (weight/ft = 116 lb.).

Note: The beam with equal flanges is 22% heavier than the one with top flange only and is more expensive to fabricate. So select beam (*b*) as lightest and cheapest.

xample 3: Stocky compound beams

Data: Span of 20 ft, carrying a point load of 41 tons at mid-span (including own weight) with top flange held at mid-span.
Maximum bending moment: 2,460 tons-in. and max. shear = 20·5 tons.

(*a*) *Symmetrical beam; uncurtailed flanges*
Try 22″ × 7″ × 75-lb. R.S.J. with 10″ × 7/8″ flange plates.
Properties: $I_x = 3{,}967$ in.^{4} $r_y = 2\cdot18$ in.
$I_y = 187$ in.^{4} $Z = 335$ in.^{3}
Using 15/16″ rivet holes:

$$\frac{\text{Gross area bottom flange}}{\text{Net} \quad ,, \quad ,, \quad ,,} = \frac{14\cdot55}{11\cdot35} = 1\cdot28$$

and $T = 14\cdot55/10 = 1\cdot46$ in.
Check: $f_{bc} = 2{,}460/335 = 7\cdot35$ and $f_{bt} = 1\cdot28 \times 7\cdot35 = 9\cdot4$ tons/sq. in.
$l/r_y = 55$; $D/T = 23\cdot75/1\cdot46 = 16$.
From Table 9: $C = 74$ tons/sq. in.
„ Table 10: $F_{bc} = 9\cdot5$ tons/sq. in.
„ Clause V: $F_{bt} = 9\cdot5$ tons/sq. in.
So the design is satisfactory (weight of beam = 2,690 lb.)

(*b*) *Unsymmetrical beam; uncurtailed flanges*
Try 22″ × 7″ × 75-lb. R.S.J. with 10″ × 7/16″ top and 10″ × 1″ bottom flanges.
Properties: $I_x = 3{,}440$ in.^{4} $y_c = 13\cdot25$ in.
$I_y = 160$ in.^{4} $Z_c = 260$ in.^{3}
$r_y = 2\cdot1$ in. $Z_t = 338$ in.^{3}
Using 15/16″ rivet holes.

$$\frac{\text{Gross area bottom flange}}{\text{Net} \quad ,, \quad ,, \quad ,,} = \frac{15\cdot85}{12\cdot42} = 1\cdot27.$$

$T = 15\cdot85/10 = 1\cdot58''$.
Check: $f_{bc} = 2{,}460/260 = 9\cdot5$ and $f_{bt} = 1\cdot27 \times 2{,}460/338 = 9\cdot3$ tons/sq. in.
$l/r_y = 57$; $D/T = 14\cdot8$; $\lambda = 56/160 = 0\cdot35$.
From Table 8B: $k_2 = 0\cdot3$.
„ Table 9: $C = (70 - 0\cdot3 \times 53\cdot7)13\cdot25/10\cdot19 = 70$ tons/sq. in.
„ Table 10: $F_{bc} = 9\cdot5$ tons/sq. in.
„ Clause V: $F_{bt} = 9\cdot5$ tons/sq. in.
So the design is satisfactory (weight of beam = 2,480 lb.).

Note: The beam with equal flanges is 8½% heavier than the beam with unequal flanges.

(*c*) *Unsymmetrical beam; curtailed flanges*
Try make-up as in design (*b*) and curtail both flange plates.
Then: $\nu = 11\cdot7/26\cdot0 = 0\cdot45$ and, from Table 7, $k_1 = 0\cdot65$.
$\therefore T = 0\cdot65 \times 15\cdot85/10 = 1\cdot03''$ and $D/T = 23$.
l/r_y, λ, and k_2 are as before for (*b*).
From Table 9: $C = (61 - 0\cdot30 \times 53\cdot7)13\cdot25/10\cdot19 = 59$ tons/sq. in.
„ Table 10: $F_{bc} = 9\cdot5$ tons/sq. in.
and, as before, $F_{bt} = 9\cdot5$ tons/sq. in.
So the design is satisfactory

Curtailment of both flanges:—

Moment of resistance of holed joist = 1,050 tons-in.

$$\therefore \text{ Length of flange plates} = \frac{2{,}460 - 1{,}050}{2{,}460} \times 20 + 1\cdot5 \text{ (lap)}.$$
$$= 13\cdot0 \text{ ft.}$$

Weight of beam = 2,155 lb.

Note: Beam (*b*) with uncurtailed flanges is 15% heavier than beam (*c*), and beam (*c*) is cheaper to fabricate.

Select beam (*c*) as lightest and cheapest.

APPENDIX III*

Monosymmetric beams under uniform bending moment

The critical terminal moment, applied about the x-axis, required to buckle a beam of monosymmetric cross-section of one of the types shown in Fig. 3 has been derived by Engel[28] and may be expressed as:—

$$M_{crit} = \frac{\pi}{L}\sqrt{\frac{EI_yGK}{\gamma}}\left[\sqrt{\left\{1 + \frac{\pi^2}{GKL^2}\left(C + \frac{EI_yQ^2}{4\gamma}\right)\right\}} + \frac{\pi}{2L}Q\sqrt{\frac{EI_y}{GK\gamma}}\right] \quad . \quad (17)$$

where $Q = \frac{1}{I_x}\{y_0(2I_x - I_y) - {}_A\iint y^3 dA\}$

The origin of co-ordinates is taken as the centroid of the section, positive directions of axes being as shown in Fig. 3. Then y_0 is the co-ordinate of the shear centre. This solution is identical with that due to Goodier[6] and Timoshenko[7] when $\gamma = 1$.

Unsymmetrical I-*section*

The shear centre for the section shown in Fig. 3a is defined by $s = h(1 - \lambda)$.

The warping constant may be written: $C = h^2EI_y\lambda(1 - \lambda)$ and for this particular section:

$$y_0 = -(h - c - s)$$
$$= -h\{\lambda - \tfrac{1}{2}(2\omega_1 + \omega_2)\}$$

and
$$_A\iint y^3 dA = \left[\omega_3c^3 - \omega_1(h - c)^3 + \frac{\omega_2}{4h}\{c^4 - (h - c)^4\}\right]A$$

where $\omega_1 A$, $\omega_3 A$ are the areas of the respective flanges and $\omega_2 A$ is the web area. Then, in equation (7), p. 3:

$$\tau_1 = 4\lambda(1 - \lambda) + (Q/h)^2 \quad \text{and} \quad \tau_2 = Q/h$$

Lipped I ("*gantry*") *section*

When equal lips are added to one flange, as in Fig. 3*b*, the shear centre moves towards this flange. The shear centre of the flange itself clearly lies at a distance e above the flange centre-line, and, treating this flange as a thin-walled channel, the distance is given by $e = D_L{}^2B^2T_L/4\lambda I_y$. The location of the shear centre for the entire section may then be defined by $s = h(1 - \lambda) - e\lambda$ and the warping constant is given by:

$$C = EI_y\{\lambda(s + e) + (1 - \lambda)(h - s)\}$$
$$= EI_y\lambda(1 - \lambda)h^2(1 + e/h)^2$$

The geometric properties for determining Q in equation (17) are found to be:

$$y_0 = c - \lambda(h + e)$$

and

$$_A\iint y^3 dA = [\omega_3c^3 - \omega_1(h - c)^3 + \{c^4 - (h - c)^4\}\omega_2/4h]A - T_L\{(h - c)^4 - (h - c - D_L)^4$$

where $\omega_1 A$ = area BT of compression flange.

* The notation used throughout Appendices III–VI is given on pp. 460, 461.

APPENDIX IV

TOP FLANGE LOADING

mmetrical point loads

Consider a uniform beam of cross-section symmetrical about the y-axis, loaded by two nt loads P applied to the top flange at distances ρL from the respective supports. The mber is supported as shown diagrammatically in Fig. 1 and the loads are assumed to free to move laterally during buckling, remaining parallel to the y-axis. Let the points load application be at heights $(y_0 + s + \frac{1}{2}T_C) = p$ above the shear centre.

When the critical values of P are reached the system is in a state of neutral equilibrium, ined by the condition that the total potential energy of the sytem is stationary (*i.e.*, does t vary with small displacements of the beam into some deflected form, as in Fig. 1). e critical loads may then be derived from:

$$U + V) = \int_0^L \left\{ GK(\theta')^2 + C(\theta'')^2 + Pp[\theta]^2_{\rho L} - \frac{\gamma M_x{}^2\theta^2}{EI_y} - QM_x(\theta')^2 \right\} dz = 0 \quad . \quad . \quad (18)$$

Assuming a twisted form given by $\theta = \phi \sin \pi z/L$, where the origin is taken at one pport, and introducing the relevant expressions for M_x, the critical loading is found m:

$$\frac{{}^2_{crit}}{I_y} \cdot L^3 f_1(\rho) + P\{p \sin^2 \pi\rho + \pi^2 Q f_2(\rho)\} = \frac{\pi^2 GK}{4L}\left\{1 + \frac{\pi^2 C}{GKL^2}\right\} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

ere
$$f_1(\rho) = \tfrac{1}{2}\left\{\rho^2(\tfrac{1}{2} - \tfrac{2}{3}\rho) + \frac{1}{4\pi^3}(\sin 2\pi\rho - 2\pi\rho \cos 2\pi\rho)\right\}$$

d
$$f_2(\rho) = \tfrac{1}{2}\left\{\tfrac{1}{2}\rho(1 - \rho) + \frac{1}{4\pi^2}(\cos 2\pi\rho - 1)\right\}$$

hen $\rho \to 0$ the critical bending moment is given by $P\rho L$ and equation (19) produces the ution for uniform moment given in Appendix III. When $\rho = 0{\cdot}5$ the loading corre- onds to mid-span point loading of $2P$, the critical value of which is found to be:

$$P]_{crit} = \frac{17{\cdot}2}{L^2}\sqrt{\frac{EI_yGK}{\gamma}}\left[-\frac{1{\cdot}74}{L}(p + 0{\cdot}366Q)\sqrt{\frac{EI_y}{GK\gamma}} \pm \sqrt{1 + \frac{1}{GKL^2}\left\{\pi^2 C + 3{\cdot}02\frac{EI_y}{\gamma}(p + 0{\cdot}366Q)^2\right\}}\right] \quad . \quad (20)$$

This solution provides excellent agreement with the analytical solution of Pettersson.[8]

Where the loading is applied through a linkage that pivots about some point at a distance below the point of load application, there is a restoring component of force Pu_L/R ting when the beam is slightly displaced laterally. Thus the system is similar to that an elastic point restraint of stiffness P/R. The additional work done during buckling thus $Pu_L{}^2/2R$ at each load point. In the case of a deep girder the tension flange remains actically undeflected during buckling and $u_L = \theta_L D$. Thus in equation (19) the term p replaced by $(p - D^2/R)$ and the effect of the linkage is to reduce the effective vertical centricity of loading. In the case of a symmetrical I-girder $p = D/2$ and the effective centricity is $\frac{1}{2}D(1 - 2D/R)$, becoming zero when $R = 2D$.

The approximate influence of top-flange loading with symmetrical free loads is shown Fig. 6 for several values of ρ when applied to a beam of symmetrical I-section.

ntilever loaded at its free end

Consider a narrow symmetrical-section cantilever, of length L, and of negligible warping- idity, loaded by a point load on its top flange at the free end. Taking the origin at the castré end, the equilibrium equations of the slightly buckled member necessary to define critical load are:

$$\left.\begin{aligned} EI_y u'' &= -\gamma P(L - z) \\ GK\theta' &= P(L - z)u' - P(u_L - u) + \tfrac{1}{2}PD\theta_L \end{aligned}\right\} \quad . \quad . \quad . \quad (21)$$

tting
$$\frac{D}{2}\sqrt{\frac{EI_y}{\gamma GK}} = v$$

d letting
$$P_{crit} = q[P_{crit}]_{v=0} = \frac{4{\cdot}01q}{L^2}\sqrt{\frac{EI_yGK}{\gamma}}$$

30

then equations (21) provide the following relation between the critical load and vertical eccentricity:

$$v = \frac{\left\{1 - \frac{(4 \cdot 01q)^2}{3.4} + \frac{(4 \cdot 01q)^4}{3.4.7.8} - \ldots + \frac{(-1)^n(4 \cdot 01q)^{2n}}{3.4.7.8\ldots(4n-1)4n}\right\}}{4 \cdot 01q\left\{1 - \frac{(4 \cdot 01q)^2}{4.5} + \frac{(4 \cdot 01q)^4}{4.5.8.9} - \ldots \frac{(-1)^n(4 \cdot 01q)^{2n}}{4.5.8.9\ldots(4n+1)4n}\right\}}$$

When represented graphically this solution may be seen to have the approximate form $q = \sqrt{(1 + 4v^2)} - 2v$. This approximation may be extended to provide a conservative estimate of the buckling loads for cantilevers of symmetrical I-section by analogy with the comparable cases of beams, and the critical load may be written as:

$$P_{crit} = \frac{4 \cdot 01}{L^2}\sqrt{\frac{EI_yGK}{\gamma}}\left[\sqrt{\left\{1 + \frac{EI_y}{4GKL^2}(\pi^2h^2 + 4D^2/\gamma)\right\}} - \frac{D}{L}\sqrt{\frac{EI_y}{GK\gamma}}\right] \quad . \quad . \quad . \quad (22)$$

The error, on the pessimistic side, is largely due to that in the warping restraint term as shown in Fig. 5, p. 401. This approximate expression has been used to plot the curve in Fig. 6, p. 406.

Uniformly distributed load

Timoshenko has solved the corresponding problem of a symmetrical I-beam under uniformly distributed loading by means of an energy method.[36] The curve in Fig. 6 is based on this result, which is applicable when $\gamma \doteqdot 1$.

APPENDIX V

The influence of flange curtailment

Symmetrical I-beams under point load at mid-span

Let a beam of span L, subjected to a load P at the shear centre, mid-span, have flanges of constant breadth B but of thickness varying linearly from T at mid-span to νT at the supports. Loading and support conditions will be considered as "ideal." The neutral equilibrium condition of equation (18), Appendix IV is then satisfied if:

$$\int_0^{L/2}\left\{GK_z(\theta')^2 + \frac{Eh^2}{4}I_{y_z}(\theta'')^2 - \frac{\gamma P^2}{4EI_{y_z}}z^2\theta^2\right\}dz = 0 \quad . \quad . \quad . \quad (23)$$

where the origin is taken at one support.

The geometric properties are:

$$I_{y_z} = I_y\{\nu + 2(1 - \nu)z/L\}$$

$$K_z = \frac{K}{n}[(n-1) + \{2(1-\nu)z/L + \nu\}^3]$$

and

$$K = \tfrac{2}{3}nBT^3$$

Substituting these variable properties into equation (23) and assuming a twisted form as $\theta = \phi \sin \pi z/L$, an approximate solution for the critical load, in terms of the mid-span proportions, may be derived. The expression containing P has been integrated graphically for a number of values of n, assuming $\gamma = 1$, and the solutions may be written as:

$$P_{crit} \doteqdot \frac{4\pi\alpha}{L^2}\sqrt{EI_yGK}\sqrt{1 + \frac{\beta\pi^2EI_yh^2}{4GKL^2}} \quad . \quad . \quad . \quad . \quad . \quad (24)^*$$

The coefficients α and β being plotted against ν in Fig. 8, p. 408.

In the alternative instance, when the flange thickness is uniform but the breadth is curtailed linearly from B to νB, the lateral second moment of area and the torsion constant may be expressed as

$$I_{y_z} = I_y\{\nu + 2(1-\nu)z/L\}^3$$

$$K_z = \frac{K}{n}\{(n-1) + 2(1-\nu)z/L + \nu\}$$

* Corrected following discussion. See p. 494, line 1, and p. 513, line 1.

Again employing equation (23) and integrating the last function graphically, a solution the form of equation (24) may be obtained. The coefficients α and β for this case are otted in Fig. 9, p. 409.

Similar solutions have been derived for members subject to uniform bending moment, ving parabolic and linear flange curtailment. These solutions are necessarily in error the high side as a result of the assumption concerning the twisted form. Any attempt use the Ritz method by the introduction of an improved form containing two or more rms has the disadvantage of making the solution applicable only to a specific beam.

ntilevers loaded at the free end

Here again the energy method proves only of limited value where a general solution is quired. Any simple trigonometric form may only partially satisfy the end conditions here a member has a finite warping rigidity. The solutions for both thickness and eadth curtailment have been derived from equation (18), Appendix IV, for a symmetrical section cantilever having negligible warping rigidity and are given in Figs 10 and 11, . 409, 410.

Further analysis and experimental investigation are being carried out in this field.

APPENDIX VI

Limiting stresses for imperfect beams

niform bending moment

Consider an elastic I-beam as shown in Fig. 1, p. 399, having initial displacements ven by:

$$u_0 = \epsilon_0 \sin \pi z/L; \qquad \theta_0 = \phi_0 \sin \pi z/L$$

has been shown[17] that the mean flange stress f_L, at which some limiting stress f_y is tained in the most highly stressed fibres, may be obtained from:

$$^{3} - f_L^2\{\pi J\psi(B/L) + \tfrac{1}{2}\pi JN(B/L)^2\phi_0 + f_y\}$$
$$- f_L\{\tfrac{1}{2}\pi^2 NJ\psi f_c(B/L)^2 + J\phi_0 f_c(B/L) + f_c^2\} + f_y f_c^2 \qquad = 0 \quad . \quad . \quad (25)$$

here f_c is the critical stress for a similar member without imperfections.

Equation (25) may be simplified to the form:

$$^{3} - f_L^2\left\{f_y + \frac{\pi^2 BhE}{2L^2}(\phi_0/2 - \epsilon_0/h)\right\}$$
$$- f_L f_c\left\{f_c + \frac{\pi EB}{2L}\phi_0\sqrt{\frac{GK'\gamma}{EI_y}} + \frac{\pi^3 Eh\epsilon_0 B}{4L^3}\sqrt{\frac{EI_y}{GK'\gamma}} + f_y f_c^2 \qquad = 0 \quad . \quad . \quad (26)\right.$$

hen both flanges are initially deflected in the same direction,

$$\phi_0 = 0 \quad \text{and} \quad \epsilon_0 = -2r_y^2\eta/B,$$

utting $\eta = jL/r_y$ and substituting the approximations for section properties given in the xt, the final equation for determining the limiting stresses becomes:

$$^{3} - f_L^2\{f_y + \pi^2 jE(r_y/L)\}$$
$$- f_L f_c[f_c + 47{\cdot}5jE(D/T)(r_y/L)^3\{1 + 20(Dr_y/TL)^2\}^{-\frac{1}{2}}] + f_y f_c^2 \qquad = 0 \quad . \quad . \quad (27)$$

milarly when the flange deformations are in opposite directions, $\epsilon_0 = 0$ and $= 4jr_yL/Bh$. Hence the limiting stress equation becomes:

$$^{3} - f_L^2\{f_y + \pi^2 jE(r_y/L)\}$$
$$- f_L f_c\{f_c + 2{\cdot}06jE(T/D)\sqrt{1 + 20(Dr_y/TL)^2}\} + f_y f_c^2 \qquad = 0 \quad . \quad . \quad (28)$$

op flange loading

No explicit solutions are available for the limiting stresses in a deep girder having perfections and loaded by point loads above the shear centre. As a check on the validity the approximate approach proposed in the text, the case of mid-span top-flange loading s been studied. By introducing approximate relations between load and rotation

parameters, derived from solutions in terms of infinite series, the following solutions have been obtained:—

When $\phi_0 = 0$

$$f_L^2\{f_c - 2J\psi(B/L)\} - f_L\{f_y f_c + f_c f_c' + 2{\cdot}3JN\psi f_c(B/L)^2\} + f_y f_c f_c' = 0 \quad (29)$$

where f_c' = critical stress for top-flange loading

$$\simeq 50{,}000(Tr_y/DL)\{\sqrt{1 + 26(Dr_y/TL)^2} - 2{\cdot}46(Dr_y/TL)\}$$

and f_c = critical stress for shear-centre loading

$$\simeq 50{,}000(Tr_y/DL)\sqrt{1 + 20(Dr_y/TL)^2}$$

Similarly, when $\epsilon_0 = 0$:

$$f_L^2\{f_c + 0{\cdot}36J\phi_0(B/L)\}$$
$$- f_L\{f_y f_c + f_c f_c' + 1{\cdot}35J\phi_0(B/L)(0{\cdot}85Nf_cB/L + f_c')\} + f_y f_c f_c' = 0 \quad . \quad . \quad (30)$$

APPENDIX VII

Notation

a	Length of longer supported edge of web plating
A	Area of cross-section; B.S. 153 coefficient (Appendix I)
b	Length of shorter supported edge of web plating; stiffener spacing
B	Flange breadth; B.S. 153 coefficient (Appendix I)
c	Distance between centroid of section and tension flange
C	Warping constant; B.S. 153 critical bending stress (Appendix I)
d	Depth of web
d'	Distance between horizontal stiffener and tension flange
D	Overall depth of section
D_L	Depth of lip of gantry section
e	Distance of shear centre of top flange from its centre-line, gantry section
E	Young's modulus
f_{bc}	Compressive stress due to bending
f_{bt}	Tensile ,, ,, ,, ,,
$f_{b.crit}$	Critical bending stress
f_L	Limiting ,, ,,
f_e	Equivalent stress
f_s	Average shear stress
$f_{s\cdot crit}$	Critical shear stress
f_p	Bearing stress
f_x	Longitudinal stress in web
f_y	Yield stress; transverse stress in web
F_{bc}	Permissible compressive stress due to bending
F_{bt}	,, tensile ,, ,, ,, ,,
F_s	,, average shear stress
G	Modulus of rigidity
h	Distance between flange centroids
I_x	Second moment of area of section about x-axis
I_y	,, ,, ,, ,, ,, ,, ,, y-axis
I_c	,, ,, ,, ,, ,, compression flange about y-axis
I_s	,, ,, ,, ,, ,, pair of stiffeners about centre of web or of a single stiffener about the face of the web
j	$\eta r_y/L$
J	$\frac{\pi}{2}E\sqrt{\frac{GK'\gamma}{EI_y}}$
k_1, k_2	Constants used in B.S. 153 (Appendix I)
K	Torsion constant
K'	$K\left(1 + \frac{\pi^2 EI_c h^2}{2GKL^2}\right)$
K_1	Bending-stress factor in B.S. 449 (1948)
l	Effective length of compression flange

Span
Buckling stress coefficient
Applied moment
r Moment about x-axis at any section
$3K/2BT^3$
$\frac{h}{B}\sqrt{\frac{EI_y}{GK'\gamma}}$
Vertical eccentricity of loading
Point load
Ratio $P_{crit}/[P_{crit}]_{v=0}$
Coefficient for monosymmetric beam
Radius of gyration
Pivotal distance of load linkage
Distance of shear centre from centroid of compression flange
Spacing of supports to through-bridge girder
Thickness of web
Effective thickness of flange
Thickness of lip of gantry section
Lateral displacement of centroid
Lateral displacement of point of application of load
Internal strain energy
$\frac{D}{2L}\sqrt{\frac{EI_y}{GK'\gamma}}$
External potential energy
, z Cartesian co-ordinates
Ordinate of extreme fibre of compression flange
„ „ „ „ „ tension „
„ „ shear centre
Modulus of section about x-axis
Flange-curtailment coefficients
$(I_x - I_y)/I_x$
Stiffness of supports to through-bridge girder
Initial lateral deflexion of centroid at mid-span
Perry–Robertson constant
Angle of rotation
I_c/I_y
Numerical coefficient
Ratio: minimum flange areas/maximum flange areas
Distance of point load from support $\div L$
Poisson's ratio
Critical stress coefficient for monosymmetric beam
Additional rotation at mid-span
Initial „ „ „
$\frac{\epsilon_0}{L}\sqrt{\frac{EI_y}{GK'\gamma}}$
Area ratio of section components

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he Paper, which was received on the 2nd December, 1955, is accompanied by thirty-six rams, from which the Figures in the text have been prepared, six Tables, and seven endices.

Structural Paper No. 49

EXPERIMENTAL VERIFICATION OF THE STRENGTHS OF PLATE GIRDERS DESIGNED IN ACCORDANCE WITH THE REVISED BRITISH STANDARD 153: TESTS ON FULL-SIZE AND ON MODEL PLATE GIRDERS

by

*** Eric Longbottom, B.Sc., A.M.I.C.E., and**

Jacques Heyman, M.A., Ph.D., A.M.I.C.E.

SYNOPSIS

The first part of this Paper describes two series of tests on miniature welded plate girders made of mild steel having webs of seventeen-gauge sheet and flanges of $\frac{1}{4}$-in. strip.

The first series of tests were designed to provide as many checks as possible on the draft clauses of the new B.S. 153 covering the design of plate girders having d/t ratios ranging from a maximum of 84 for unstiffened webs to 400 for webs having both vertical and horizontal stiffeners.

The results of the second series of tests, in addition to checking the new design method also agree remarkably well with the simple rules given for the plastic design of plate girders with unstiffened webs, and show that the rules are applicable for web-depth to web-thickness ratios (d/t) up to 90. As the analysis does not allow for lateral instability, however, a plastic design should be checked for stability under working loads.

The miniature tests largely corroborated the validity of the proposed new rules, but supporting evidence from full-scale tests was considered desirable. To this end four full-size girders were made and tested. This work is described in the second part of this Paper.

The largest girder had a 75-in.-deep $\times$ $\frac{3}{16}$-in.-thick web, 10-in. $\times$ $\frac{3}{4}$-in. flange plates, vertical and horizontal web stiffeners, and was 32 ft 1 in. long. The tests were carried out at M.E.X.E., Christchurch, under a 250-ton capacity loading rig, so arranged that two equal point loads could be applied to the top flanges.

The tests were arranged to check successively the criterion for flange stability, web buckling, and the effect of the simultaneous occurrence of maximum bending and shear stresses.

The Paper gives details of the girders tested, a description of the test rig, test procedure, and the test results.

INTRODUCTION

In the early summer of 1954 the British Standards Committee responsible for the revision of B.S. 153 were considering the clauses governing the design of plate girders. Since the previous publication of B.S. 153, Part 3, in 1937, a great deal of additional information had become available and although B.S. 449, published in 1948, and the Code of Practice for Simply Supported Steel Bridges, published in 1949, embody

* Mr Longbottom is Head of a Design Group in Ministry of Supply, Military Engineering Experimental Establishment, Christchurch, and Dr Heyman is a University Lecturer in Engineering, University of Cambridge.

ne notable advances, more recent research and experiment indicated that a further p forward might be made. Perhaps the most significant advance was in the ablishment of criteria for the design of webs with both vertical and horizontal feners.

Part I of this Paper describes tests on scale-model girders, carried out by Dr yman, which largely corroborated the validity of the proposed new B.S. 153 rules, hough supporting evidence from tests on full-size structures is still lacking. wever, the co-operation of the Ministry of Supply was obtained and arrangements re made for a number of tests on full-size plate girders to be carried out at the litary Engineering Experimental Establishment (M.E.X.E.), Christchurch. These ts are the subject of Part II of the Paper.

Part I.—Tests on miniature plate girders

Four tests were carried out at the limiting d/t-ratios proposed by the original draft S. 153, namely $d/t = 85$, 240, 300, 400. The lowest value was the limit for un-ffened webs; the next for webs with vertical stiffeners; $d/t = 300$ was the limit one horizontal stiffener; and $d/t = 400$ was the maximum value permitted (with o horizontal stiffeners).

A previous Paper[38] reported tests on small (3-in. deep) plate girders with thick bs. The results enabled rules to be formulated for the plastic design of such plate ders, providing the d/t-ratio for the web was not greater than about 72. These sign rules are given below with some discussion. The figure of 72 was arrived at considering reports by other investigators, but it was felt at the time that the ure might well be increased, certainly up to the figure of 85 proposed in the draft S. 153 for unstiffened webs. Accordingly a new series of tests on girders with = 94 were performed, and these are also reported here.

In both series of tests, the girders were fabricated from 17-gauge (0·056-in.) mild-el sheet, and the flanges were cut from $\frac{1}{4}$-in. black mild-steel strip. Both materials d reasonably uniform properties, as will be seen from the control tests below. All ders were welded (electric arc), particular attention being paid to the order of lding so as to avoid distortion. The flanges remained remarkably straight in most es, although one or two required manual bending about the minor axis of the der. However, except for the shallowest specimens, the web showed great tortion, initial deflexions being sometimes several times the web thickness. In test ($d/t = 240$) this initial web distortion may have contributed to the pro-tion of collapse, but in the other tests failure occurred in some other manner.

Test results (i)—control tests

Tests were carried out on specimens cut from the flanges and the webs of the lapsed girders. Additional specimens were also cut from material adjacent to t used for the girders.

The flange specimens were of $\frac{1}{4}$-in. × $\frac{1}{4}$-in. cross-section, and were tested as simply pported beams under two-point loading over a $4\frac{1}{2}$-in. span. In all, twenty such ts were carried out, and from the collapse loads the yield stress of the flange mater- was determined as 18·6 tons/sq. in. with a standard deviation of 0·3 ton/sq. in.

[38] Jacques Heyman and V. L. Dutton, "Plastic Design of Plate Girders with Unstiffened bs." Weld. & Met. Fabrn, vol. 22, p. 265 (July 1954).

Simple statistical tests showed that there was no significant variation from girder to girder, and the value of 18·6 tons/sq. in. has been used in the analysis of all the results.

The web specimens gave results showing rather more scatter, but again no significant variation could be determined between girders. The specimens were cut to a standard profile for the Hounsfield tensile testing machine, and the fourteen results gave a mean yield stress of 16·7 tons/sq. in. with a standard deviation of 0·8 ton/sq. in.

TEST RESULTS (ii)—B.S. GIRDERS

The four miniature girders had d/t ratios of 85, 241, 299, and 401 compared with the nominal limits of 85, 240, 300, and 400. All girders were tested simply supported under two-point loading. The girders were designed at a time when the relevant clauses in B.S. 153 had just been drafted, and it was decided to test as many of the design rules as possible in each girder. Partially as a result of the tests, some of the draft clauses have been revised; in particular, the outstand of the flat web stiffeners and the limiting ratio d/t for webs with vertical stiffeners have been reduced.

To start with, the depth of the girder was fixed to give the maximum d/t ratio for that particular class, and the web stiffeners given the minimum section and the maximum permitted spacing. The corresponding maximum shear stress then gave the total load on the beam. The flange section was next fixed, in relation to the length of the girder, to give the maximum permitted bending stress under the same load. Finally, the distance between the two point loads was determined from the flange-stability clauses. The web stiffeners were not attached to the flanges, and the horizontal stiffeners were discontinuous between panels, and on the opposite side of the web to the vertical stiffeners.

It is now felt that too many variables were introduced into these tests, and that it would have been better to concentrate, for example, on the shear and bending clauses. In fact, the two deeper girders both failed by lateral instability, and it is not known how much reserve of strength they had against failure in bending. The limit for l/r_y of 90 or more is now agreed to be unsafe, and the draft B.S. has been modified.

The dimensions of the girders are given in Table 12, p. 468, together with collapse loads and load factors; Fig. 37 gives the overall dimensions.

All tests were carried out in an Amsler 500-ton compression testing machine; it was necessary to use this machine because of the large spans involved. Fig. 38 shows the deepest girder in position and after testing. The load is actually applied from the bottom, and is resisted by the upper cross-head. It will be seen that the ends of the girders are supported on rollers, while the load comes on to the top flange through two balls. Considerable difficulty was experienced in setting up the girders so that loading was in the plane of the web; several trial runs were made with light loads and the girders adjusted for very slight out-of-plumbness so that the minimum lateral deflexion was experienced in the elastic range. Fig. 39 shows the shallowest girder in position in the machine before testing. The eight dial gauges used in all the tests to record minor-axis deflexions and twist can be seen, and some of the vertical deflexion gauges bearing on the top flange are also visible.

Girder A.4 ($d/t = 85$)

The girder is shown in Fig. 39 and after collapse in Fig. 40. In the final stages of collapse, large buckles developed in the end panels, as may be seen from Fig. 40. Collapse finally occurred at 5·2 tons, and at this stage it is estimated that the whole

Fig. 38.—Girder E after test

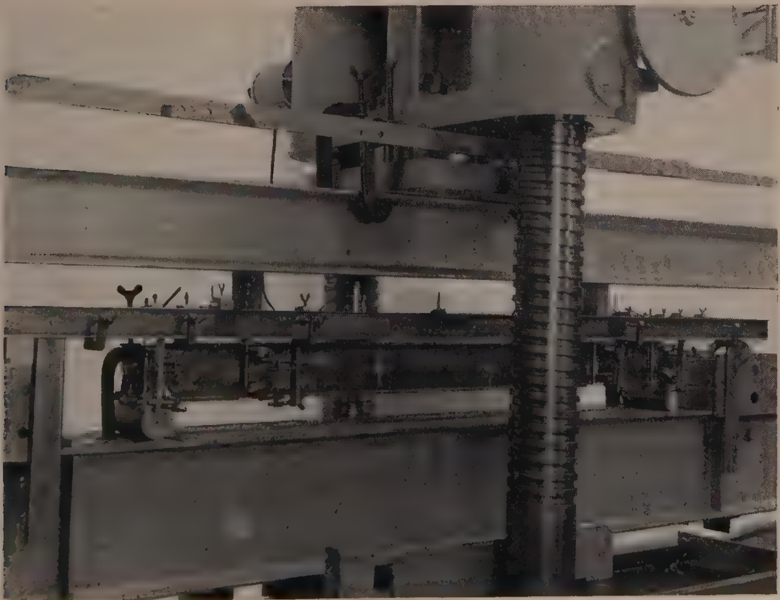


Fig. 39.—Girder A.4 under test

Fig. 40.—Girder A.4 after test

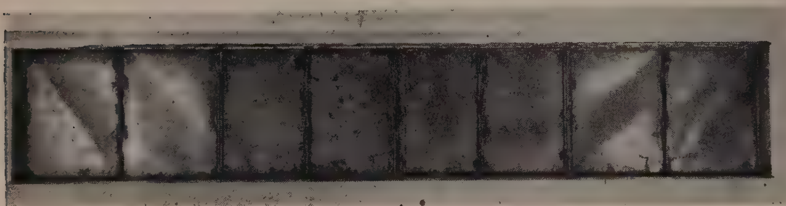


Fig. 41.—Girder C.4 after test

Fig. 42.—Girder D after test

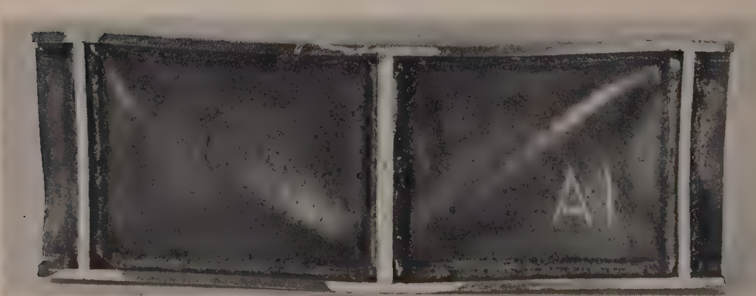


Fig. 47.—Girder A.1 after test

FIG. 49.—GIRDER A.2 AFTER TEST

FIG. 53.—GIRDER No. 4 AFTER TEST

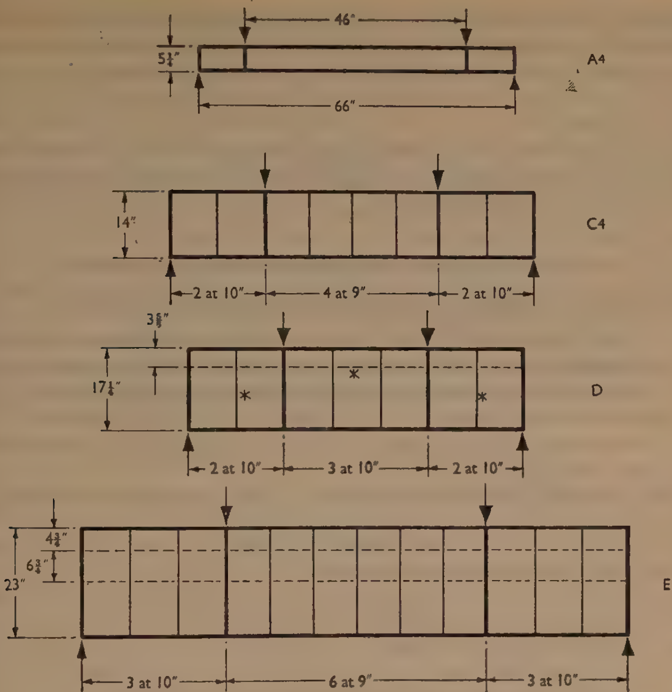


FIG .37.—PRINCIPAL DIMENSIONS OF GIRDERS TESTED

f the web was subjected to the full yield stress in shear. This is discussed below ith reference to the plastic design method.

The lateral deflexions of the unsupported compression flange were becoming large ; loads of more than 5 tons, but after collapse occurred in the end panels, the top ange recovered its straightness on removal from the machine. The observed load ctor was 1·68; the significance of the adjusted factor in Table 12 is again explained elow.

irder C.4 ($d/t = 241$)

This test gave the lowest load factor in the series. Collapse was very sudden at 3 tons, being promoted by the development of large buckles in the panels subjected shear. Fig. 41 shows these buckles in the girder after testing. Very little lateral ovement of the girder was observed.

irder D ($d/t = 299$)

Three tests were carried out on this girder. In the first, at a load of 9 tons, one f the vertical stiffeners marked with an asterisk in Fig. 37 buckled at the neutral xis of the girder, and large deflexions of the girder resulted. The load was removed, nd ¼-in. × 0·056-in. flanges were added to the two vertical stiffeners. In the econd test, a slight buckle was observed in the centre horizontal stiffener at 11·0 tons, nd this again led to large overall deflexions. This stiffener was also flanged, so that the final test the three stiffeners marked with asterisks in Fig. 37 were all

strengthened. At about 11 tons slight buckles were observed in the end panels, but the girder went on to carry 12·6 tons, failure being promoted by lateral instability of the compression flange. Fig. 42 shows the girder after testing.

Girder E ($d/t = 401$)

Failure of this girder again occurred by top flange instability. The web buckling, which can be seen in Fig. 38, was very slight, and was not apparent until the instruments had been removed after the test.

Plastic analysis for girders with thick webs

The design rules formulated from the earlier series of tests (i) were:

1. A collapse analysis shall be undertaken of the plate girder under consideration, the applied loads being multiplied by a suitable load factor, and the bending moment M and shear force F calculated at the plastic hinges.
2. The average shear stress $f_s = F/dt$ where d and t are the depth and thickness of the web, shall not exceed $1/\sqrt{3}$ times the guaranteed yield stress (f_y) of the web material.
3. The permissible plastic bending stress (f_p) in the web shall be determined from
$$f_p = \sqrt{f_y^2 - 3f_s^2}$$
4. The flanges shall be designed so that
$$M = M_f + M_w$$
where $M_f = BT(d + T)f_y$
$M_w = \frac{1}{4}td^2f_p$
B and T denote the width and thickness of each flange.

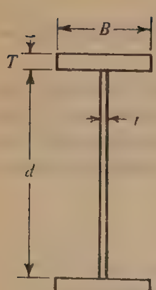


FIG. 43.

FIG. 44.

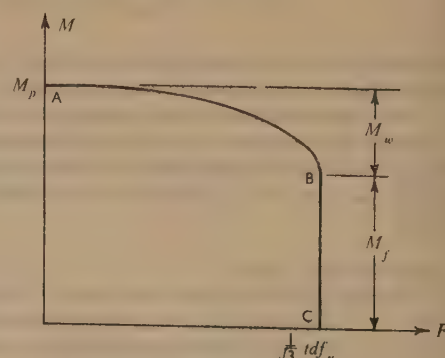


FIG. 45.—MOMENT AND SHEAR FORCE FOR FULL PLASTICITY (SCHEMATIC)

These rules can of course be operated in reverse to analyse a given section. The girder cross-section is shown in Fig. 43, and it is assumed that the stress distribution on the web is as shown in Fig. 44, the flanges carrying no shear force. For a given girder, the above rules give a design curve as shown in Fig. 45. From A to B in this figure, the design rules apply; the vertical line BC at $F = \frac{1}{\sqrt{3}}tdf_y$ implies that the

› fails in shear, and that the girder can carry any moment up to M_f, the moment resistance of the flanges above. That this curve represents the behaviour of a ler with low d/t was demonstrated by the earlier tests.

A point of immediate significance is the load factor associated with the maximum ar stress of 6 tons/sq. in. allowed by B.S. 153. A steel to B.S. 15 having a yield ess of 15·25 tons/sq. in. will yield in shear at $15{\cdot}25/\sqrt{3} = 8{\cdot}80$ tons/sq. in., so that load factor of a simply supported girder, the web of which is designed to ons/sq. in., will have a load factor not greater than $8{\cdot}80/6 = 1{\cdot}47$. This compares h the load factor in bending (to 9·5 tons/sq. in.) of about 1·85 for a simply supported ler. The actual load factor will lie somewhere between these two limits depending the actual ratio of M to F at the critical section of the girder, and providing that ure of the girder as a whole does not occur in some other way (i.e., by lateral tability) at a lower load factor.

A second series of miniature tests were carried out to check this variation of load tor implied in B.S. 153 for girders with thick webs.

Test results (iii)—girders with $d/t = 94$

Three tests were carried out on simply supported girders carrying a central point d, using an adapted Denison tensile testing machine; no support was given to the flanges. All girders had 1-in.-wide flanges and the results are summarized in ble 13. In this Table, the B.S. working load is calculated for the attainment of maximum permitted bending or shear stress, whichever is critical for the par-ular girder. The plastic collapse loads are calculated using the yield stresses from control tests and are according to the analysis given above. The observed lapse load divided by the B.S. working load gives the observed load factor; in the t column, these load factors have been adjusted (on the basis of the plastic analysis) a yield stress of 15·25 tons/sq. in. instead of those actually obtaining for the ders. This adjustment has also been made in the last column of Table 12 for der A.4.

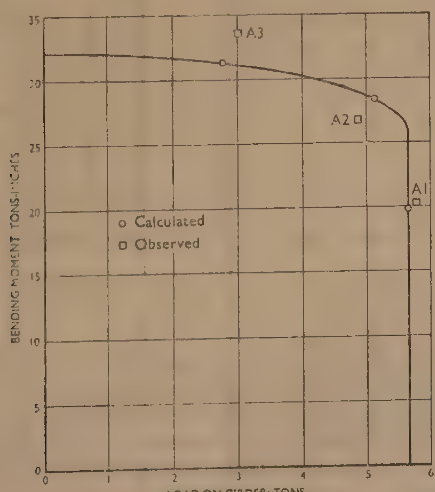


Fig. 46.—Moment and shear force for full plasticity (girders A)

TABLE 12

Beam	d/t	D/T	Distance between loads: in.	l/r_y†	Flanges in. × in.	Web (17ga.): in.	Vertical stiffs (17ga.): in.	Horizontal stiffs (17ga.)	B.S. 153 working load: tons	Observed collapse load: tons	Observed load factor	Adjusted load factor
A.4	85	21	46	116	$1\frac{3}{8} \times \frac{1}{4}$	$4\frac{3}{4}$	—	—	3·1W	5·2	1·68	1·44
C.4	241*	56	36	92	$1\frac{5}{8} \times \frac{1}{4}$	$13\frac{1}{2}$	$\frac{3}{4}$	—	6·7W	8·3	1·24	—
D	299	68	30	91	$1\frac{1}{2} \times \frac{1}{4}$	$16\frac{3}{4}$	$\frac{7}{8}$	$\frac{3}{4}''$ at $\frac{D}{5}$	5·9F	12·6	2·13	—
E	401	92	54	89	$2\frac{1}{2} \times \frac{1}{4}$	$22\frac{1}{2}$	$1\frac{1}{16} \times \frac{1}{4}$ fl.	$\frac{3}{4}''$ at $\frac{D}{5}$ $\frac{5}{8}''$ at N.A.	8·2F	16·2	1·99	—

TABLE 13

Beam	d/t	D/T	Flanges	Web	Span: in.	B.S. 153 working load: tons	Plastic collapse load: tons	Observed collapse load: tons	Observed load factor	Adjusted load factor
A.1	94*	23	1 in. × $\frac{1}{4}$ in.	$5\frac{1}{4}$ in. × 0·056 in.	14	3·53W	5·67	5·76	1·63	1·49
A.2					22	2·68F	5·14	4·85	1·81	1·55
A.3					45	1·31F	2·78	2·98	2·27	1·91

* d/t outside proposed permissible limits.

† The effective length l is taken as 0·85 × distance between loads.

der A.1

his had the shortest span, and plastic analysis showed that failure was to be ected along the part of the curve BC in Fig. 45, that is, by pure shear in the web. s curve is replotted accurately, using the nominal section modulus and observed d stresses, in Fig. 46, where the tests are marked. The calculated collapse load ees well with that observed, and the adjusted load factor is 1·49 (*cf.* 1·47 above for ure in shear). In the final stages of collapse large buckles developed in the web, nay be seen from the photograph of the final state of the girder, Fig. 47; these are y similar to the buckles in Fig. 40 for girder A.4. Deflexions were almost linear to a load of 5·65 tons, when some larger increments of deflexion were observed. ereafter, two increments of 0·05 ton were added, the deflexions increasing markedly, a final increment of 0·01 ton brought about sudden collapse.

der A.2

lthough the span was longer than for girder A.1 the shear force on the web was l high; theoretically, however, the full plastic moment should have developed at a

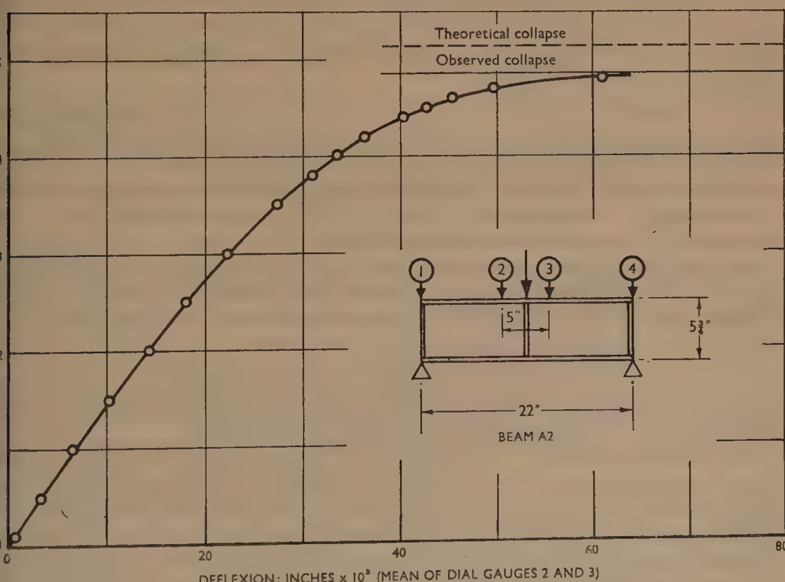


FIG. 48.—LOAD/DEFLEXION CURVE FOR GIRDER A.2

nt near B in portion AB of the curve in Fig. 45 (see Fig. 46). In this test only, observed collapse load was less than that calculated, the discrepancy being just than 6%. The load deflexion curve is plotted in Fig. 48, where a mean reading is orded for dial gauges 2 and 3 relative to the readings of gauges 1 and 4. This trumentation was adopted for each of these three tests, no record being taken of nor-axis deflexions. In fact, collapse was promoted in this test by a lateral lexion of half of the beam, as may be seen from the photographs, Figs 49a and 49b

Girder A.3

In this test, there was little sign of web buckling, but the girder again collapsed, the collapse being finally accompanied by lateral deflexion of the unsupported top flange.

Discussion

In the first series of tests, failure of the three deeper girders occurred elastically, for girder C.4 by web buckling, and for girders D and E by top-flange instability. It is therefore not appropriate to adjust the load factors in Table 12 for these three girders to allow for the yield stress being greater than 15·25 tons/sq. in. The lateral stability failures effectively masked observation of failure in bending or shear.

The various design clauses may be discussed in the light of the tests.

Failure did not occur by bending in any of the four girders.

For the girder with $d/t = 85$, the full yield stress in shear was imposed on the web. At this state collapse occurred, with the flanges below the yield stress. The same sort of web buckling was experienced as for girders A.1 and A.2 in the second series of tests, deflexions becoming large for the girder as a whole before the buckles became apparent. Simple plastic theory predicted collapse when the whole of the web was subjected to the full yield stress in shear, leading to a collapse load of 5·13 tons, compared with 5·2 tons observed.

With the possible exception of girder C.4 ($d/t = 240$), the large initial imperfections of the webs did not seem to contribute to the promotion of collapse.

For the second series of tests, the test results agree remarkably well with the simple plastic analysis, despite the discrepancy for girder A.2 due to lateral failure of this specimen. This analysis does not allow for lateral instability, and a plastic design should be checked for stability under the working loads. This question will not arise, of course, if the girder forms part of a structure which provides adequate restraint for the compression flange; a concrete deck or floor, for example, cast over the flanges, will effectively prevent any lateral movement. The tests were carried out on girders with $d/t = 94$, and will apply to the unstiffened girders up to $d/t = 85$ of B.S. 153. Exploratory tests on girders with unstiffened webs of $d/t = 120$ showed that plastic analysis is no longer applicable, the webs buckling and promoting failure of the girder as a whole at a load lower than that predicted.

If the plastic analysis were adopted for girders with d/t less than 90, the value of the load factor must be settled. The lowest load factor in the tests was 1·49 (failure in shear), and the highest 1·91 (failure in bending), comparing with the corresponding theoretical range 1·47 to 1·85. It is felt that a uniform load factor of 1·75 would be appropriate; plastic designs of simply supported girders would gain slightly against B.S. girders designed to a bending stress of 9·5 tons/sq. in., and would lose rather more in the relatively uncommon instances when the full B.S. shear stress of 6 tons/sq. in. was used. Nevertheless, a uniform load factor seems more rational. If the girder is redundant, then, of course, plastic design will show considerable economies; in addition, design is simpler than the corresponding elastic method, especially if a number of load combinations has to be considered.

Part II.—Tests on "full-size" plate girders

Details of test girders

Number and sizes selected

The proposed new rules for the design of webs indicated that four different combinations of web stiffening should be investigated, as was done for the scale-model girders described in Part I of this Paper. Thus, the four test girders were of the following type, each as near as possible to the maximum depth allowed for the web stiffening provided:

Girder No. 1

Web unstiffened.

Ratio of depth of web between flanges to web thickness. $\frac{d}{t} \simeq 85$

Girder No. 2

Web stiffened with vertical stiffeners only. $\frac{d}{t} \simeq 200$

Girder No. 3

Web stiffened with vertical stiffeners, and one horizontal stiffener located one-fifth of the depth from the top (compression) flange. $\frac{d}{t} \simeq 300$

Girder No. 4

Web stiffened with vertical stiffeners, and two horizontal stiffeners—one on the neutral axis and one at a distance one-fifth of the depth from the compression flange. $\frac{d}{t} \simeq 400$

For the greatest d/t-ratio (400) the maximum size of web for which failure in buckling could be ensured with the available test rig was 75 in. deep by $\frac{3}{16}$-in. thick. All webs were accordingly of this thickness in order to compare the effects of the stiffening provided for each limiting condition.

In addition to testing the different combinations of web stiffening, opportunities were sought to carry out tests to check criteria for flange stability and to investigate the effects of inducing maximum working stress in the flange and web simultaneously. For a predetermined size of web, there remained the following variables to be settled for the girder:

(i) Thickness and width of flanges.
(ii) Span (assuming the girders to be simply supported in each case).
(iii) Position and method of loading.
(iv) Conditions of end and/or intermediate support to the top flange.

In order to obtain the maximum information from each girder, it was envisaged that the following tests could be carried out in succession—only the last test causing complete failure:

First, tests for buckling of flange only, both with ends free and with ends restrained against lateral rotation.

Secondly, tests on flanges and web at maximum working stresses simultaneously.

Finally, tests to destruction on web and stiffeners only, supporting the flange to prevent failure by flange buckling. Provision to be made for varying the size and spacing of vertical stiffeners if practicable.

With these objects in view, the variables mentioned above were fixed by a process of trial and error, and the full details selected for the four test girders are shown in Fig. 50. Special web stiffeners were provided at load points and supports, and steps were taken to prevent over-stressing the web in bearing at these points.

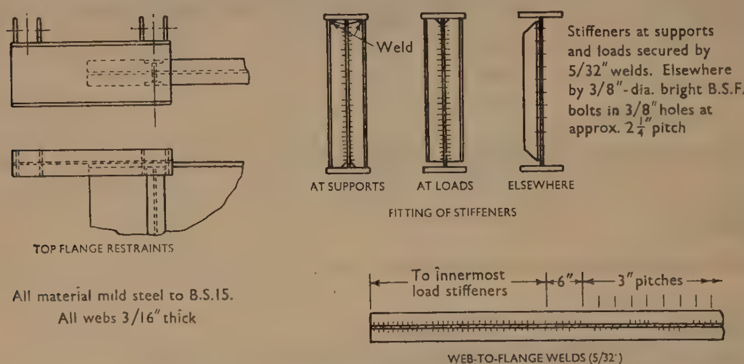


FIG. 50a.—DETAILS COMMON TO ALL GIRDERS

Physical Properties of the Girder Materials

The test girder details were designed on the assumption that all materials would conform to B.S. 15—i.e., the minimum yield point would be 15·25 tons/sq. in. for material $\frac{1}{4}$- to $\frac{3}{4}$-in. in thickness and 14·75 tons/sq. in. for material more than $\frac{3}{4}$-in. thick.

The properties of the materials actually used are given in Table 14. Particularly noteworthy are the low limits of proportionality observed in some cases. The effect of these low values was reflected in the actual tests by the earlier development of flange and web buckling than would have been otherwise expected.

Fabrication

Every effort was made during manufacture of the girders to minimize distortion. The plates forming the flanges and webs were obtained with their edges planed to tolerances of $\pm$ 0·01 and $\pm \frac{1}{32}$ in. respectively. The tolerance in planing the flange plates controlled their width and not their degree of straightness. Special packing and transport were arranged to avoid damage, in view of the awkwardness of the load (the $\frac{3}{16}$-in.-thick web plate for the largest girder was obtained in one piece measuring 6 ft 3 in. deep by 32 ft 9 in. long).

Two main adjustable rotating welding jigs were made to cater for all four girders in succession. In addition, seven pairs of subsidiary jigs were constructed to hold the flanges and web together during welding and to minimize buckling of the web. These subsidiary jigs effectively maintained the web in the centre of, and at right angles to, the flanges.

Welding proceeded on all four longitudinal runs in succession, working in about

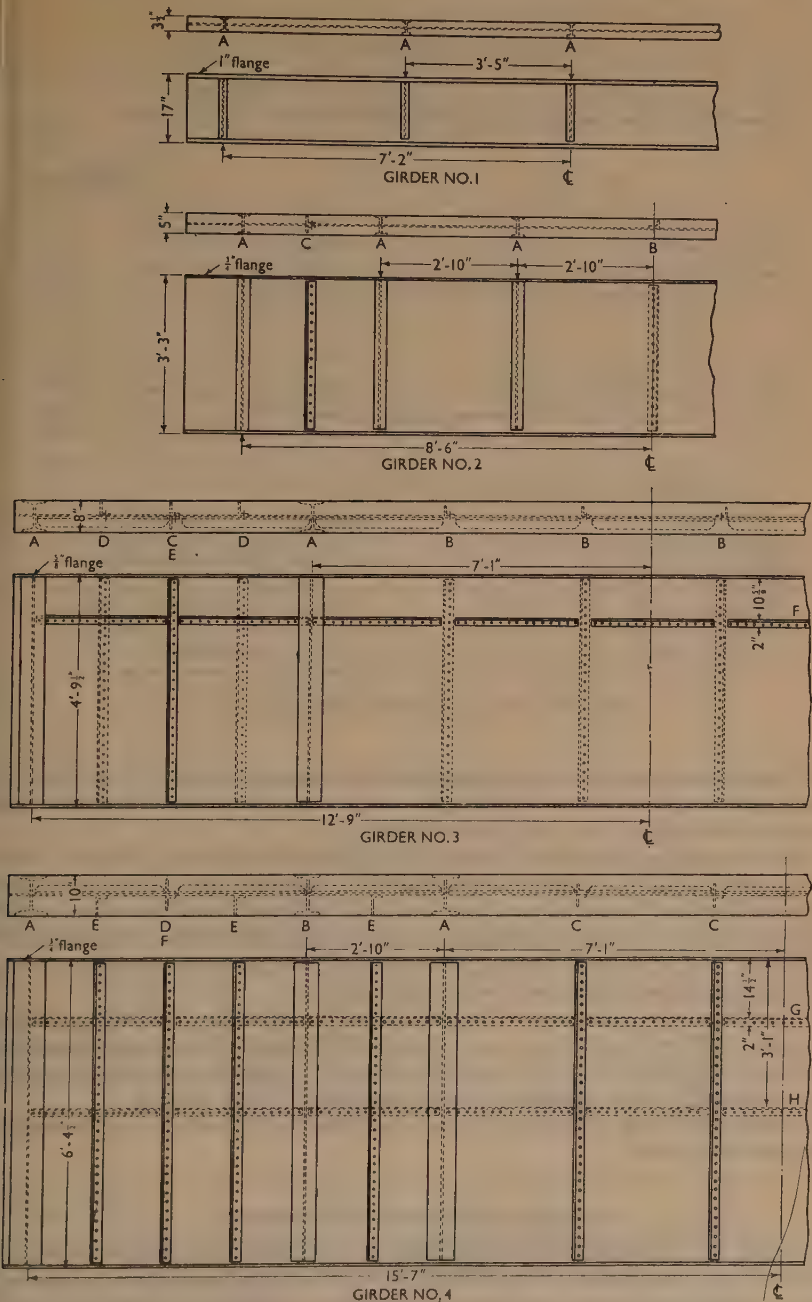


FIG. 50b.—DETAILS OF GIRDERS NOS 1, 2, 3, 4.

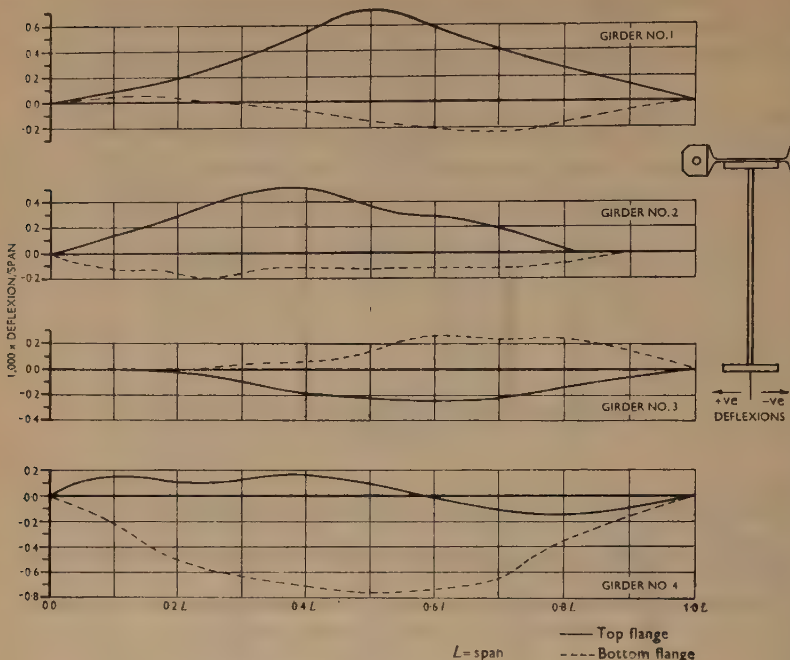


FIG. 50C.—FLANGE PROFILES BEFORE TESTS

3-ft lengths at a time from one end progressively towards the other. Thus, all four welds were completed for one 3-ft length, closely controlled by the subsidiary jigs before commencing the next 3-ft length, and so on.

After completing the first 3-ft length, the end bearing stiffeners at this end were tack-welded in place, and the load-point stiffeners similarly tack-welded after the flange welding had proceeded past their locations. All welds were down-hand, with 8-gauge mild-steel electrodes, giving $\frac{5}{32}$-in. nominal-size fillet welds. The holes for the fitted bolts in the web securing the stiffeners were jig-drilled to ensure interchangeability between different sizes of stiffeners.

It was found that, on release of the girder from the subsidiary welding jigs, slight buckling of the web had taken place. However, this largely disappeared when the stiffeners were bolted on. Some lateral bowing of the flange plates was apparent before assembly, and no significant change was observed after welding.

Profiles of the flange of the four girders were plotted before testing, and the results are given in Fig. 50c.

LOADING AND RESTRAINT REQUIREMENTS

In order to test the girders under the most severe conditions for flange buckling, it was decided to suspend two point loads from the top flange, symmetrically disposed about mid-span and spaced as far apart as possible consistent with obtaining maximum bending moment in the flanges without causing failure of the web. Thus, a

FIG. 50d.—WEB-STIFFENER DETAILS

Girder No.	Stiffener details			When used
	Type	Size	No. off	
1	A	2″ × 1⅝″ × ⅜″ tee	10	All tests
2	A	3″ × 2⅜″ × ½″ tee	12	All tests
	B	2½″ × 2″ × ⅜″ angle	1	All tests
	C	2¼″ × 2¼″ × ¼″ angle	4	Test S
3	A	6″ × 4″ tee ex 10″ × 6″ × 40 lb. R.S.J.	8	All tests
	B	2½″ × 2½″ × $\frac{5}{16}$″ angle	4	All tests
	C	2½″ × 2½″ × $\frac{5}{16}$″ angle	2	Tests H, L, and M
	D	4″ × 2½″ × $\frac{5}{16}$″ angle	4	Tests J and K
	E	3″ × 2½″ × $\frac{5}{16}$″ angle	4	Tests J and K
	F	2½″ × 2″ × $\frac{3}{16}$″ angle	9	All tests
4	A	7″ × 4½″ tee ex 9″ × 7″ × 50 lb. R.S.J.	8	All tests
	B	6″ × 4″ × ½″ tee	4	All tests
	C	2½″ × 2½″ × $\frac{5}{16}$″ angle	8	All tests
	D	2½″ × 2½″ × $\frac{5}{16}$″ angle	4	Tests A, B, and C
	E	5″ × 3″ × ⅜″ angle	6	Tests D, E, F, and G
	F	4″ × 2½″ × $\frac{5}{16}$″ angle	4	Tests D, E, F, and G
	G	2½″ × 2″ × $\frac{3}{16}$″ angle	11	All tests
	H	2″ × 2″ × $\frac{3}{16}$″ angle	11	All tests

maximum length of the top flange would be under maximum (and approximately uniform) stress.

It was required for some tests that the load points should suffer minimum lateral restraint. To prevent premature collapse, however, it would be necessary to limit the amount of lateral deflexion.

It was also required that certain tests should be carried out with the top flanges encastré at each end against lateral rotation.

TABLE 14.—PHYSICAL PROPERTIES OF MATERIALS

Girder No.	Size of plate: in. × in.	Limit of proportionality: tons/sq. in.				Yield point: tons/sq. in.				Ultimate tensile strength: tons/sq. in.				Percentage elongation				E: tons/sq. in.
		No. of specimens	Max.	Min.	Average	No. of specimens	Max.	Min.	Average	No. of specimens	Max.	Min.	Average	No. of specimens	Max.	Min.	Average	
Top flanges																		
1	$3\frac{1}{2} \times 1$	2	9·9	9·9	9·9	3	15·5	14·0	14·6	2	29·1	28·3	28·7	2	35·0	27·5	31·2	13,400
		(2)	(14·35)	(14·15)	(14·25)	(2)	(15·6)	(15·6)	(15·6)	—	—	—	—	—	—	—	—	
2	$5 \times \frac{3}{4}$	2	8·9	8·2	8·55	3	17·7	16·5	16·9	2	33·2	33·0	33·1	2	32·0	30·5	31·25	13,400
		(2)	(16·4)	(16·1)	(16·25)	(2)	(17·8)	(17·8)	(17·8)	—	—	—	—	—	—	—	—	
3	$8 \times \frac{5}{8}$	6	15·4	12·6	13·8	6	17·4	15·9	16·8	6	35·3	32·8	33·7	5*	37·0*	33·0*	33·8*	13,250
4	$10 \times \frac{3}{4}$	6	14·9	11·1	13·0	6	16·9	15·9	16·3	6	33·8	31·7	32·9	6*	36·0*	33·0*	33·7*	13,000
Bottom flanges																		
1	$3\frac{1}{2} \times 1$	2	9·3	3·3	6·3	2	14·9	13·3	14·1	2	28·9	28·8	28·85	2	36·0	29·0	32·5	13,400
2	$5 \times \frac{3}{4}$	2	5·3	4·1	4·7	2	16·6	16·5	16·55	2	31·8	31·4	31·6	2	33·0	32·5	32·75	13,400
3	$8 \times \frac{5}{8}$	6	14·5	12·0	13·0	7	16·9	15·4	16·0	6	33 5	30·4	31·2	6*	38·0*	34·0*	36·1*	13,250
		(1)			(15·5)	(1)			(16·83)	—	—	—	—	—	—	—	—	
4	$10 \times \frac{3}{4}$	5	15·3	3·7	10·50	6	17·20	16·10	16·83	6	35·0	33·6	34·53	6*	33·0*	29·0*	31·5*	13,250
		(1)			(16·0)	(1)			(16·32)	—	—	—	—	—	—	—	—	

Webs

Girder No.	Size of plate: in. × in.	Limit of proportionality: tons/sq. in.				Yield point: tons/sq. in.				Ultimate tensile strength: tons/sq. in.				Percentage elongation			
		No. of specimens	Max.	Min.	Average	No. of specimens	Max.	Min.	Average	No. of specimens	Max.	Min.	Average	No. of specimens	Max.	Min.	Average
1	$15 \times \frac{3}{16}$	2 [—]	5·7	5·3	5·5	2 [—]	18·7	18·6	18·65	2 [—]	29·1	28·9	29·0	2 [—]	35·0	29·5	32·25
		2 [|]	11·3	11·0	11·15	2 [|]	19·4	18·3	18·85	2 [|]	28·9	27·2	28·0	2 [|]	29·0	24·0	26·5
2	$37\frac{1}{2} \times \frac{3}{16}$	2 [—]	15·5	14·3	14·9	2 [—]	21·1	20·8	20·95	2 [—]	31·9	31·6	31·75	2 [—]	34·0	28·0	31·0
		2 [|]	12·1	11·3	11·7	2 [|]	20·6	20·6	20·6	2 [|]	33·4	31·8	32·6	2 [|]	29·0	24·0	26·5
3	$56\frac{1}{4} \times \frac{3}{16}$	2 [—]	17·6	11·3	14·5	2 [—]	21·8	20·0	20·9	2 [—]	31·6	30·3	30·95	2 [—]	31·0	29·0	30·0
		2 [|]	12·8	8·9	10·85	2 [|]	21·8	20·5	21·15	2 [|]	31·7	30·6	31·15	2 [|]	31·0	28·0	29·5
4	$75 \times \frac{3}{16}$	2 [—]	21·0	15·1	17·65	2 [—]	29·6	29·1	29·35	2 [—]	36·1	36·0	36·05	2 [—]	22·0	21·0	21·5
		2 [|]	21·6	21·1	21·35	2 [|]	29·3	29·1	29·2	2 [|]	36·6	35·8	36·20	2 [|]	17·5	14·0	15·75

Notes:

(i) For web specimens rules in brackets [—] and [|] indicate direction of length of specimens in relation to length of web.

(ii) Figures in parentheses relate to compression tests. All others relate to tensile tests.

(iii) Sizes of specimens ranged from $\frac{3}{4}$-in. gauge length for miniature specimens to 8-in. gauge length for standard test pieces.

(iv) Percentage elongation was measured on a 2-in. gauge length except for specimens marked with an asterisk which were of $\frac{3}{4}$-in. gauge length.

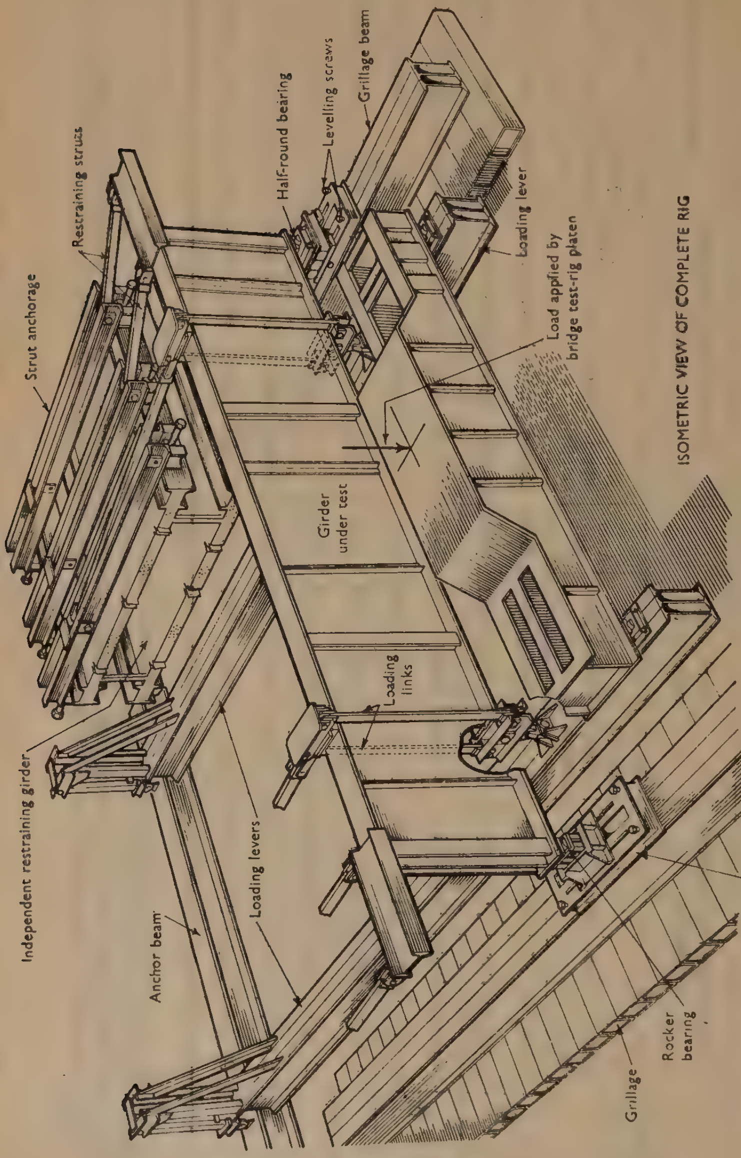


FIG. 51a.—ISOMETRIC VIEW OF COMPLETE RIG

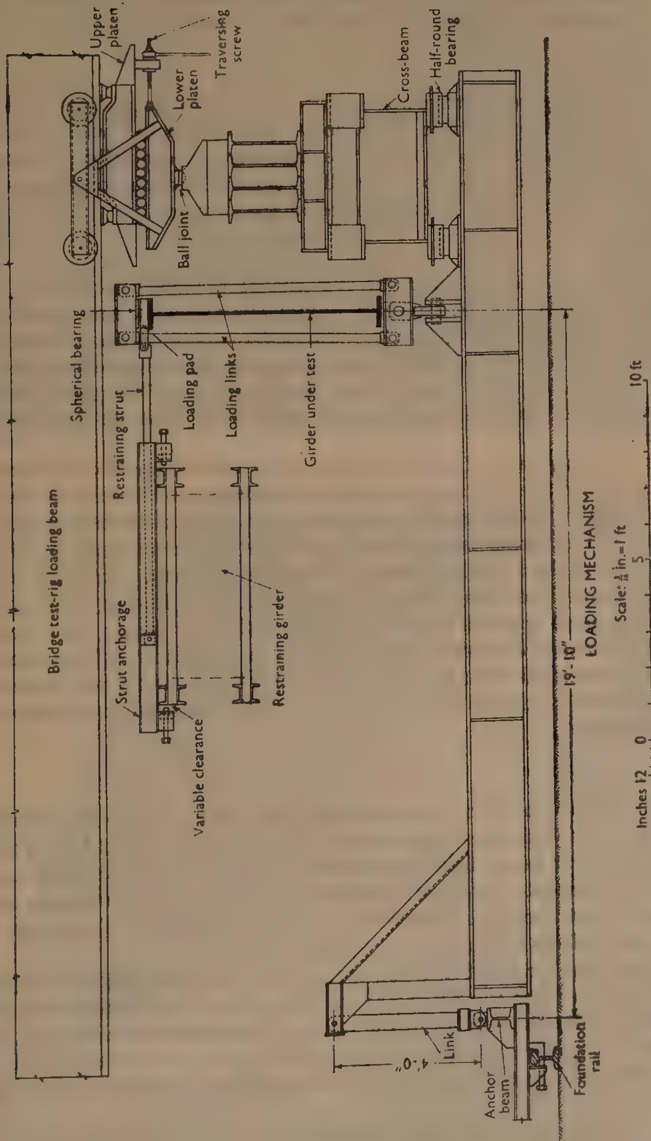


FIG. 51b.—LOADING MECHANISM

Description of the test rig

The test rig used is shown in Figs 51 and 53. In designing it, maximum use had to be made of existing equipment. Its main features were as follows:

Girder supports

The two ends of the test girder were carried on half-round and rocker bearings respectively, so that the girder was free to rotate at each end in a vertical plane and to move at one end under load. These bearings were on centrally pivoted tilting frames mounted on grillage beams. Levelling screws at each corner of a tilting frame enabled the girder webs to be plumbed at the supports.

Method of loading

The test girder was loaded indirectly by one of the 250-ton-capacity loading gantries of the M.E.X.E. Bridge Test Rig. Since this rig may form the subject of a separate Paper, it is sufficient to say here that each gantry carries a loading beam which applies load through the ball joint of a loading platen suspended from it by four wheels. Rollers between the upper and lower portions of the platen enable the lower portion carrying the ball joint to move transversely with the deflexion of the structure under test.

In these tests the platen load was distributed to two loading pads on the top flange of the test girder by a cross-beam assembly connecting two loading levers, which were suspended from the test girder at its load points by pairs of loading links.

The far ends of the loading levers were connected to an anchor beam by pin-ended links, free to pivot in a vertical plane. This arrangement, combined with the transverse freedom of the lower loading platen, allowed the loading levers and their loading links to move over with the top flange of the test girder when horizontal deflexions of the flange occurred. Since friction in the platen rollers would have prevented this movement from taking place to its full extent, the two halves of the loading platen were connected by a screwed rod, with which the lower platen could be moved as required.

Top-flange restraints

In order to test the criteria for flange buckling when fixed at the ends against lateral rotation, each girder had an extension piece in the form of a short length of rolled-steel joist welded on to its top flange at each end. Each extension piece was connected laterally by two horizontally-hinged restraining struts to a strut anchorage on top of an independent restraining girder. This girder was packed up from, and rigidly attached to, the grillage beams.

With the adjusting screws at each end of the strut anchorages tightened up against the restraining girder, the top flanges of the test girder were effectively encastré in plan. With the adjusting screws of the outer anchorages slackened back, the top flange of the test girder was merely held laterally at points vertically above the supports. With the screws on all four anchorages eased back, the top flange was unsupported.

A similar arrangement with a strut pinned to the top flange loading pad enabled the load points to be fixed or to be given whatever degree of freedom was thought desirable.

In the tests on the smaller girders, rollers were placed under the strut anchorage

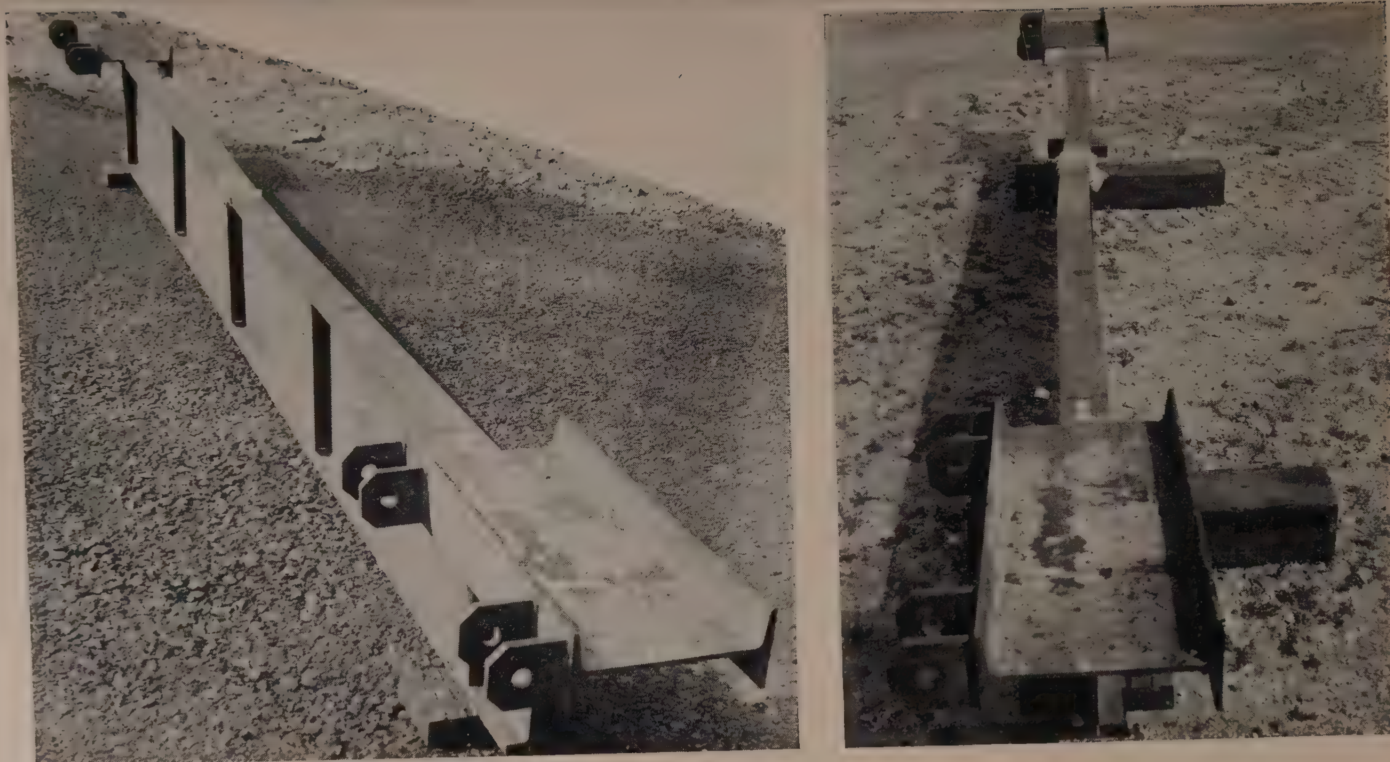


Fig. 61.—Girder No. 1 after failure under 19·1 tons at each load point

FIG. 62.—GIRDER NO. 2 AFTER FAILURE UNDER 66·7 TONS AT EACH LOAD POINT

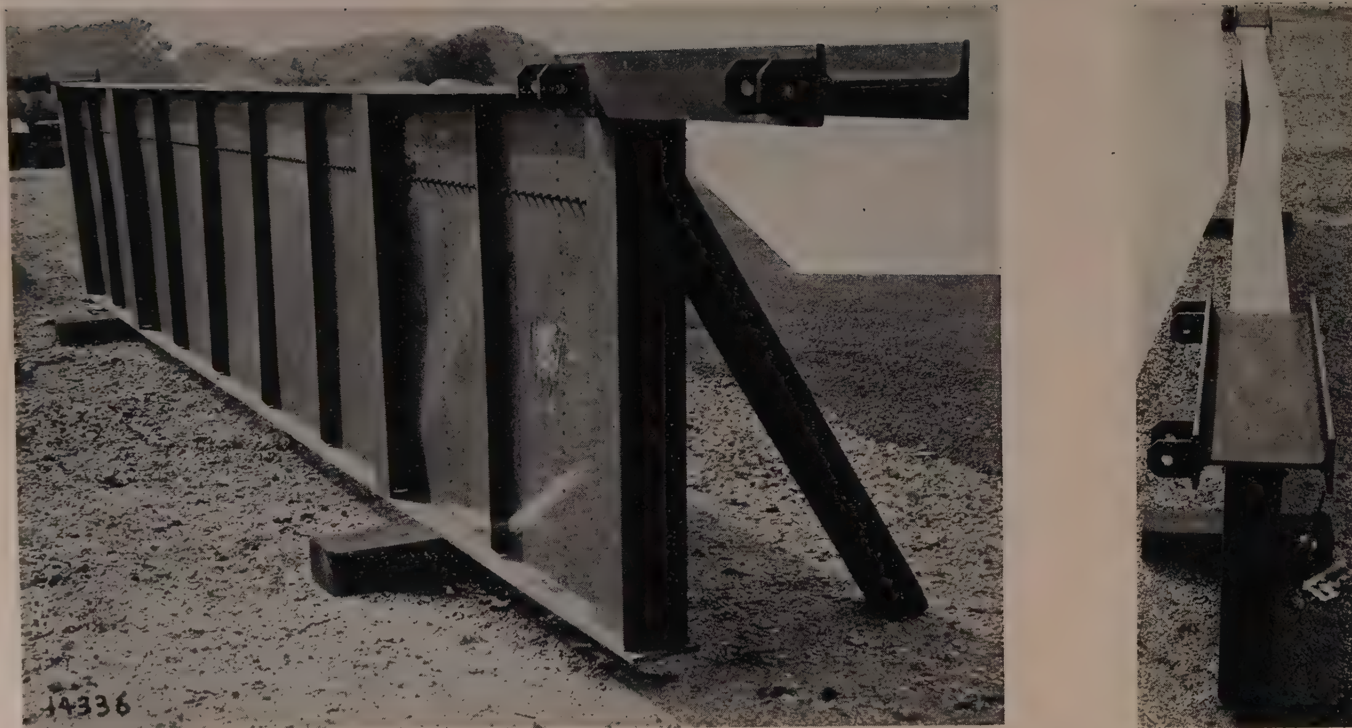


Fig. 63.—Girder No. 3 after failure under 86·7 tons at each load point

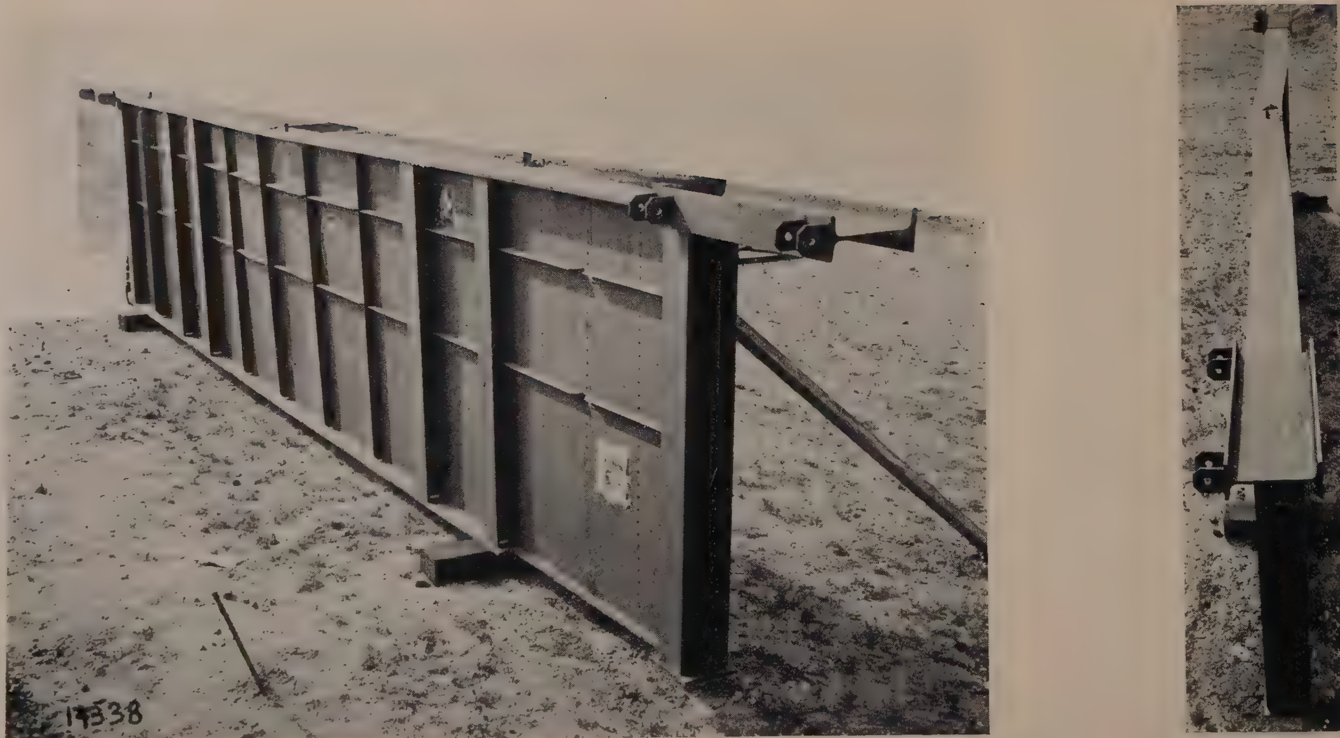


Fig. 64.—Girder No. 4 after failure under 106 tons at each load point

so that they would move freely when tests with an unrestrained top flange were in progress.

The restraining girder was of relatively large proportions, so that no significant lateral deflexion should be caused by the restraining strut loads.

The pins connecting the restraining struts to the strut anchorages, the top-flange extension pieces and the loading pads were 0·001 in. smaller than the holes through which they passed.

In all cases, the struts permitted free vertical deflexion of the test girders, and they were long enough to render lateral movement due to changes of slope negligible.

Modifications to suit girder being tested

Fig. 51 shows the test arrangement for girder No. 4—the first to be tested. For the next girder (No. 3), which was 19 in. shallower, the restraining girder was lowered 19 in. and the loading links were reduced in length from 7 ft $4\frac{1}{2}$ in. to 5 ft $9\frac{1}{2}$ in. In the remaining tests this loading-link length and restraining-girder height were maintained, the grillage beams being raised (and the packing between them and the restraining girder being correspondingly reduced) to suit the depth of the girder being tested. In these latter tests also, the cross-beam assembly distributing the platen load to the loading levers took different forms.

To provide a single central load for some of the tests on the last girder (No. 1), one loading-link system was connected to the middle of a special beam pinned between the normal linkage attachment points of the loading levers.

Test sequence adopted

The girders were tested in the order in which they were made—4, 3, 2, and 1. The individual tests on each girder were carried out in the alphabetical sequence given in Table 15, facing p. 486, with the object of determining the loads required to produce the limiting conditions shown. In some cases, the order in which the tests took place had to be changed from that originally envisaged because of the earlier development of unstable conditions caused by the low limits of proportionality of the girder materials.

Table 15 also shows the number of vertical stiffeners fitted, the top-flange restraints applied, the position of the loads, and the results of each test.

Measurements taken

Platen load

The load applied by the loading platen of the Bridge Test Rig is measured by a strain-gauged load cell immediately above the ball joint of the lower platen. The signal from this cell, which measures vertical components only, was passed to a self-balancing Wheatstone Bridge circuit and exhibited directly as a load in tons on an indicator dial.

Load on test girder

The load in each of the four loading links was measured by strain gauges and/or mechanical extensometers. Two sets of strain gauges were used on those links carrying them, the compensating gauges being at right angles to the main gauges and wired up to eliminate bending effects. The gauges were mounted indoors with "Durofix," and were dried for 72 hours by infra-red lamps before being waterproofed with "Di-jell."

The form of the mechanical extensometers used is evident from Fig. 52, the only point worthy of mention being the stable end conditions given by the spring-wire connexion between the rods and the adjusting screws on the lower mountings.

In the tests on girder No. 4, two links carried strain gauges and two carried 50-in. mechanical extensometers. The same arrangement held in the tests on girder No. 3, except that 25-in. extensometers were used on the shorter links. In girders Nos 1 and 2 tests, all loading links carried strain gauges, whilst 25-in. extensometers were

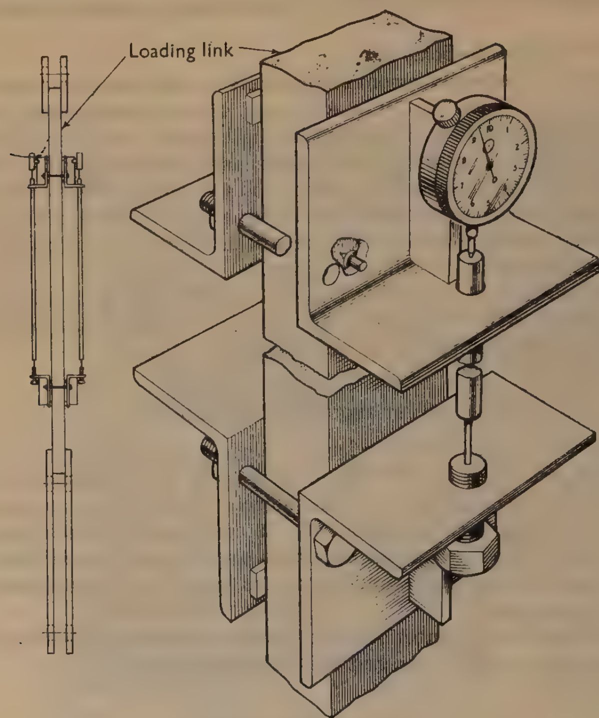


FIG. 52.—MECHANICAL EXTENSOMETER

also used on two of the loading links in the girder No. 2 tests. Extensometer extensions were read to 0·0001 in. by means of dial gauges, whilst the strains given by the strain gauges were estimated to the nearest 0·000005 by means of a standard Tinsley Bridge Box.

Young's modulus for one of the loading links was obtained from the deflexions of the link as a beam, and that value was adopted for all the links, as they were made from the same stock, except those for the girder No. 1 tests.

The strain-gauged links were initially calibrated by tensile tests, in which they were in series with links carrying mechanical extensometers. These calibrations were subsequently confirmed by tensile tests in which the links were in series with a 40-ton proving ring.

Flange and web strains

Using the same mounting technique as that for the loading-link gauges, strain gauges were fixed to the test girders in the positions shown in Fig. 54. Gauge factors were determined by the constant curvature beam method.

Girder deflexions

The horizontal deflexions of the top and the bottom flanges were obtained by deflectometers mounted either on the restraining girder or on an independent beam. Each deflectometer consisted of a vertically mounted pulley carrying a pointer to magnify its rotation. Two wires were attached to the lowest point of its periphery—one, of invar, had a half-turn anticlockwise around the pulley and was secured to the girder flange; the other had a three-quarter-turn clockwise around the pulley and carried a hanging weight, thus maintaining a constant tension in the invar wire. Flange deflexions caused the pulley to rotate and its pointer to move past a graduated scale. Readings were taken from a distance by theodolite to avoid disturbing other instrument supports. Vertical deflexions were taken in the tests on girder No. 4 by means of dial gauges.

Web profiles

Web profiles in the shear zone were obtained from measurements taken with a millimetre scale from a wire stretched between the load and support stiffeners to the ntersection points of a grid marked on the web. Whilst measurements were being aken, the wire with its tensioning weights was conveniently held at the required evels by means of magnets on the vertical stiffeners.

Test procedure

With the ball joint of the lower loading platen well clear of the cross-beam assembly, he free ends of the loading levers were jacked up until the loading links became slack nd the only load on the test girder was the dead-weight of the upper portion of the oading-link system and half the weight of the restraining struts.

The strut anchorage screws were then adjusted to give fixity or the desired degree f freedom to the top flange at its ends and load points. At this stage, all strain auges and deflectometers were set to zero and extensometers set to some convenient atum.

The jacks under the loading levers were then removed to transfer the dead-load of he rig to the test girder, and all strain gauge and instrument readings were taken.

The loading beam of the Bridge Test Rig was then brought down on to the cross-eam assembly until the ball joint of the lower platen was seated. Further lowering f the loading beam produced successively higher loads on the test girder proportional o the measured platen load.

After each increment of load and before any readings were taken, the loading levers ere moved together, as necessary, to replumb the loading-link assembly over the iddle of the deflected top flange of the test girder. This involved traversing the wer platen one way or another to maintain its initial distance from the top flange f the test girder. In some tests (itemized in Table 15) the loading levers were not oved, and the girder deflexions caused the loading-link assembly to incline and roduce a small lateral restraint at the load points.

To avoid the possibility of overloading when the loads on the girder were approach-g critical values, and so rendering the girder unserviceable for further tests, suitable

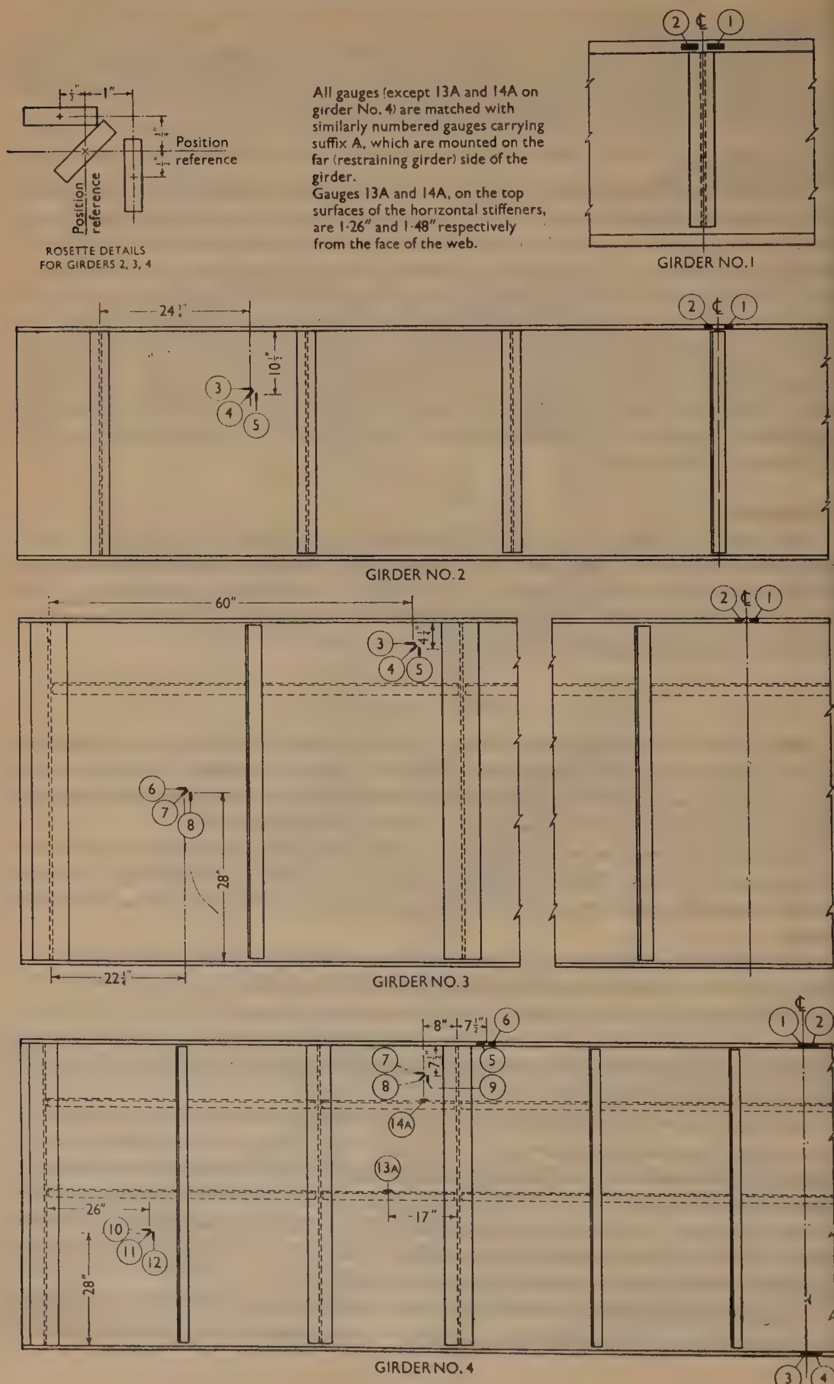


FIG. 54.—LOCATION OF STRAIN GAUGES ON GIRDERS

ncrements of load were either forecast from plots of the mid-span gauge readings or vere obtained by stopping the loading when a particular gauge reading had reached pre-selected value. In addition, the strut anchorages opposite the load points vere set to limit the amount of flange buckling that could occur.

In the web buckling tests, the web profiles of girders Nos 2, 3, and 4 were taken prior to loading, and then again with the girders carrying load.

In the final tests, the girders were taken up to their collapse loads. Figs 61 to 64 how the girders after these tests together with the loads applied.

Most of the tests took place during hot sunny weather, with passing clouds—a potential source of differential temperature effects. The loading links were screened, n an effort to minimize the effect of this on the strain gauges and extensometers carried by them.

TEST RESULTS

The test results are presented in the form listed below. Discussion of them forms part of Structural Paper No. 48 (see p. 3).

(*a*) Figs 55, Plate 1, and 56, Plate 2, show the observed mid-span strains on each side of the top flange of the girders which failed by flange buckling. Each graph is characterized by a rapid divergence of the strains on either side of the flange from the observed mean strain as lateral bending of the flange develops.

In some instances, an initial bowing of the flange in one direction is reversed as the loading increases, and shows itself in the crossing-over of the individual flange strain curves.

The calculated stresses against which the strains are plotted were obtained from the measured loads on the girder and its section modulus.

(*b*) In Fig. 57, Plate 2, are plotted horizontal and diagonal strains measured in the webs during web buckling tests on girders Nos 2, 3, and 4.

(*c*) Fig. 58, Plate 2, shows the observed stress in the horizontal stiffeners of girder No. 4 during a buckling test.

(*d*) The changes in web profile under load for girders Nos 2, 3, and 4 are shown in Fig. 59, Plate 2. The web form under no load was taken as datum, measurements at this stage being taken before the girder had been loaded, except in the case of girder No. 4, which had previously suffered loadings A and B. The web form was then re-determined with the girders carrying the following loads at each load point:

Girder No. 2	36·3 tons
,, ,, 3	33·8 ,,
,, ,, 4	130·4 ,,

Fig. 60, Plate 2, shows the horizontal deflexions of the top and bottom flanges in girder No. 1 (Test T) and girder No. 2 (Test N).

Figs 61 to 64 (pp. 480, 481) show the four girders after the collapse under the application of the following loads at each load point—viz.,

Girder No. 1	19·1 tons
,, ,, 2	66·7 ,,
,, ,, 3	86·7 ,,
,, ,, 4	106·0 ,,

Acknowledgements

The first group of tests was carried out at Cambridge. The Authors wish to thank Mr E. W. Morris, a Post-Graduate student in the course on Structures and Strength of Materials at Cambridge University, who made the tests and helped with the design of the miniature girders. Grateful thanks are also due to Mr O. A. Halliday for his patience in welding up the specimens.

The Authors would also like to acknowledge with thanks the assistance given by Mr A. E. Long, B.Sc., A.M.I.C.E., and Mr J. N. Barnikel, M.Sc., of M.E.X.E. for their help in preparing Part II of this Paper, and the assistance of the British Constructional Steelwork Association in obtaining materials for the test girders.

The Paper, which was received on the 5th December, 1955, is accompanied by seventeen photographs and eleven sheets of drawings and diagrams, from which the half-tone page plates, folding Plates 1 and 2, and the Figures in the text have been prepared.

Discussion

Mr G. A. Gardner, said that as Chairman of the British Standards Committee wrestling with the problems with which the Papers dealt, it was perhaps fitting that he should open the discussion—a discussion which the members of the Committee, including the Committee revising B.S. 449, hoped would settle those "oscillations between hope and despair, resolution and discouragement", with which they had been beset. It was an epoch-making Paper which should dispel much past ignorance.

It was regrettable that engineers did not habitually keep data bearing on their practical work, because such data were invaluable in work of that sort and would have materially helped the committee.

As a consequence, the Committee had had to instigate practical research in order to indicate a probable line of advance. He implored members not to be discouraged by the seeming complexity of the basic formulae. It was merely a case of history repeating itself; the same thing had occurred 50 years before, when rational strut theory had been evolved. In the present instance, it would be found that certain types of structures could be brought into line with the proposed rules by simplifications—for instance, in the ordinary multi-storey building, where the number of differing cases would be very much less than those covered in the Papers, as compared with bridge work and general steel frame construction.

One of the difficulties in aiming at simplification had been the exceedingly great range of sectional proportions of girders and flange-curtailment variations, depth to flange thickness, and so forth, ratios which came within a practical purview. It was worth noting that when the Committee in the past had tried to over-simplify, the B.S. Institution had been bombarded with innumerable queries from designers about cases which were not covered. The subject of the stability of compression flanges was more difficult than strut theory, for British Standard strut theory had perhaps somewhat lightly glossed over the form factor effect, whereby a stout tubular strut designed from the same formulae as an I-section with a thin web and heavily-plated flanges was supposed to embody the same load factor.

That brought to the fore the much-debated question of the factor of safety. The ethical desideratum should be, he supposed, to approach unity in any part of the structure under its most unfavourable condition of loading, an approach which was not infrequently made in the very foundations of a structure. But then there came the complexity of deciding what *was* the loading.

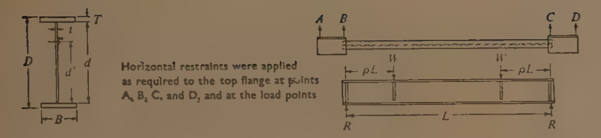


TABLE 15.—SUMMARY OF RESULTS

Details of girder							Test conditions							Results								Mode of failure		Remarks
Girder No.	Test	Flanges: $B \times T$ inches	Web: $d \times t$ inches	D/T	d/t	d'/t	L inches	l/r_y	ρL inches	Vert. stiffs in shear zone, $\rho(L)$: No.	Vert. stiffs in shear zone, $\rho(L)$: Spacing	Top flange restraints: At ends	Top flange restraints: At loads	Max. W: tons: Estimated critical†	Max. W: tons: Test	Shear stress: tons/sq. in.: Actual $= W/dt$	Shear stress: tons/sq. in.: Proposed permissible	Shear stress: tons/sq. in.: Load factor	Flange stress: tons/sq. in.: Actual $= M/Z$	Flange stress: tons/sq. in.: Proposed allowable	Flange stress: tons/sq. in.: Load factor	Expected	Actual	
1	T	$3\frac{1}{2} \times 1$	$15 \times \frac{3}{16}$	17	80	—	172	206	86	3	$3d$	B & C	—	19·5	11·5	—	—	—	8·3	4·8	1·74	FB	FB & FY	W = Single load at mid-span (T, U, V)
	U							206	86	3	$3d$	—	—		11·6	—	—	—	8·4	4·8	1·76	FB	FB	
	V							169	86	3	$3d$	A, B, C, & D	—		12·3	—	—	—	9·9	5·3	1·7	FY	FB	
	W							206	45	2	$3d$	B & C	—	15·6	12·9	—	—	—	9·9	4·8	2·07	FB or WY	FB & FY	
r_y= 0·86 in.	X							206	45	2	$3d$	—	—		12·4	—	—	—	9·5	4·8	1·99	FB or WY	FB & FY	
	Y							169	45	2	$3d$	A, B, C, & D	—		14·8	—	—	—	11·3	5·4	2·1	FY or WY	FB & FY	
	Z							82	45	2	$3d$	A, B, C, & D	Both		16·1	—	—	—	12·3	8·2	1·5	FY or WY	FB & FY	
	Ω							103	45	2	$3d$	B & C	—		19·1	—	—	—	14·6	7·5	1·95	FB or WY	FB collapse	Top flange restrained laterally at mid-span
2	N	$5 \times \frac{3}{4}$	$37\frac{1}{2} \times \frac{3}{16}$	52	200	—	204	200	68	3	$0{\cdot}9d$	B & C	—	18·3	17·2	—	—	—	5·8	2·6	2·24	FB	FB	
	O							200	68	3	$0{\cdot}9d$	—	—		14·2	—	—	—	5·0	2·6	1·92	FB	FB	
	P							165	68	3	$0{\cdot}9d$	A, B, C, & D	—	33·4	19·2	—	—	—	6·2	3·5	1·78	FB	FB	
r_y= 1·04 in.	Q							—	34	2	$0{\cdot}9d$	A, B, C, & D	Both	50·0	36·3	5·2	4·7	1·1	—	—	—	WY	WB & WY	Top flanges at points ρL = 68" prevented from moving towards restraining girder
	R							—	34	2	$0{\cdot}9d$	A, B, C, & D	Both	50·0	58·8	8·4	4·7	1·8	—	—	—	WY	WB collapse	Web heavily deformed
	S							111	34	3	$0{\cdot}45d$	A, B, C, & D	Both	50·0	66·7	9·5	5·8	1·6	11·6	5·4	2·04	WY	WB & FB collapse	Stiffeners bolted to deformed web over lower three-quarters of its depth
3	H	$8 \times \frac{5}{8}$	$56\frac{1}{4} \times \frac{3}{16}$	92	300	240	306	—	68	3	$0{\cdot}75d'$	B & C	—	35·0	31·2	3·0	4·4	0·7	—	—	—	FB	WY	Local yield near load point
	J							140	68	5	$0{\cdot}375d'$	B & C	—	35·0	59·4	—	—	—	10·7	3·9	2·74	FB	FB & WY	
	K							140	68	5	$0{\cdot}375d'$	—	—		52·4	—	—	—	9·4	3·9	2·40	FB	FB	
r_y= 1·61 in.	L							116	68	3	$0{\cdot}75d'$	A, B, C, & D	—	65·0	66·8	—	—	—	12·1	4·9	2·46	FB or FY	WB & WY	
	M							74	68	3	$0{\cdot}75d'$	A, B, C, & D	Both	65·0	75·7*	7·2	4·4	1·6	13·6	7·9	1·73	FY or WY	FB & WY collapse at 86·7	
4	A	$10 \times \frac{3}{4}$	$75 \times \frac{3}{16}$	102	400	200	374	131	68	3	$0{\cdot}9d'$	B & C	—	71·0	92·4	—	—	—	7·8	4·2	1·86	FB	FB	Stress observed at W = 84·8, corresponding to a calculated stress of 7·85
	B							131	68	3	$0{\cdot}9d'$	—	—	71·0	63·7	—	—	—	5·9	4·2	1·42	FB	FB	
	C							—	68	3	$0{\cdot}9d'$	A, B, C, & D	Both		130·4	9·3	4·7	1·9	—	—	—	WB	WY	
r_y= 2·07 in.	D							108	102	7	$0{\cdot}45d'$	A, B, C, & D	—	100·0	101·2	—	—	—	14·1	5·3	2·66	FB or FY	FY & WY	
	E							131	102	7	$0{\cdot}45d'$	B & C	—	82·0	65·5	—	—	—	9·1	4·2	2·16	FB or FY	FB	
	F							131	102	7	$0{\cdot}45d'$	—	—		52·8	—	—	—	7·3	4·2	1·74	FB or FY	FB	
	G							70	102	7	$0{\cdot}45d'$	A, B, C, & D	Both		106·0	—	—	—	14·8	8·1	1·83	FY	FB collapse	

Notes:—* After effective failure at W = 75·7 tons, girder No. 3 was taken up to W = 86·7 tons for collapse.
† Based on early rules.
In all tests on girders Nos 3 and 4, the loading levers were fixed so that horizontal loads were unavoidably induced at the loading points as the top flange deflected laterally. In all other tests the loading levers moved over with the top flange, the loads on which remained vertical.
FY denotes flange yielding, FB flange buckling, WY web yielding, and WB web buckling.

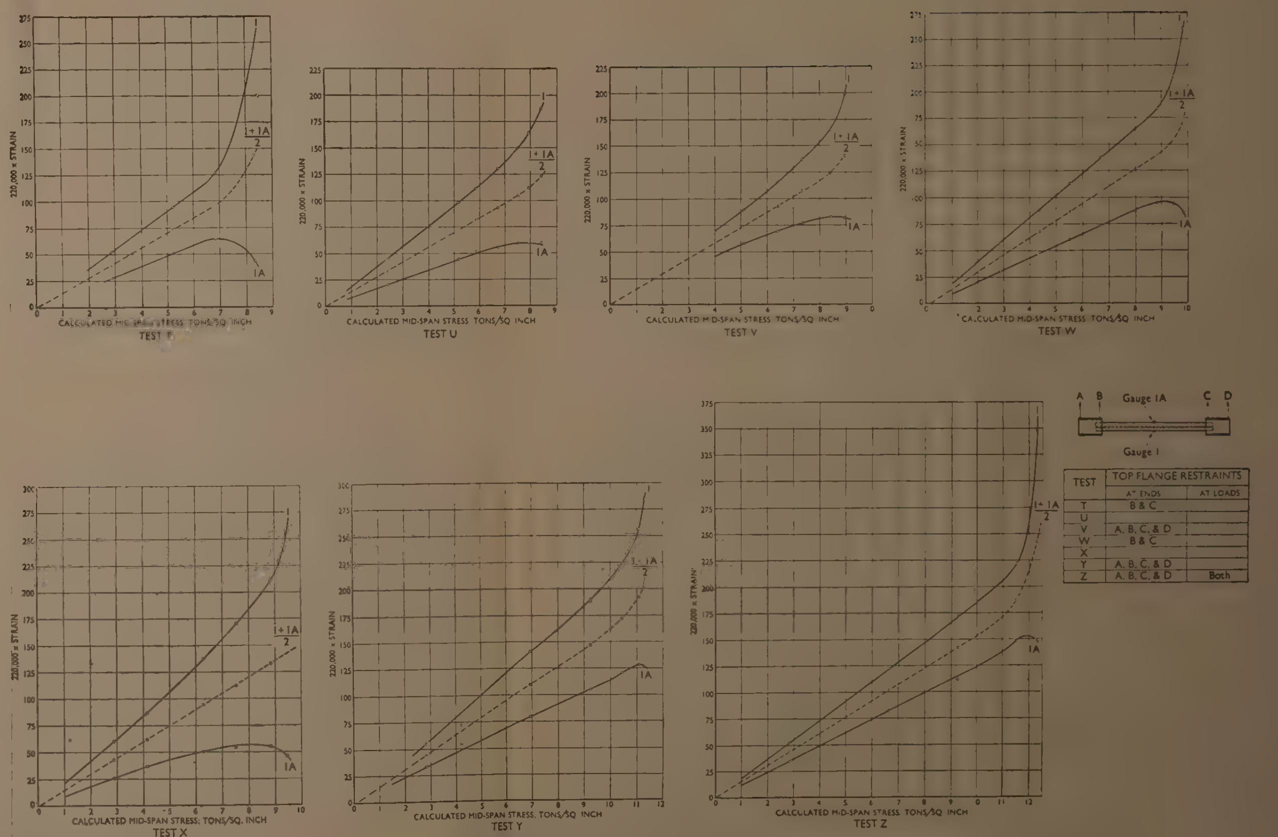
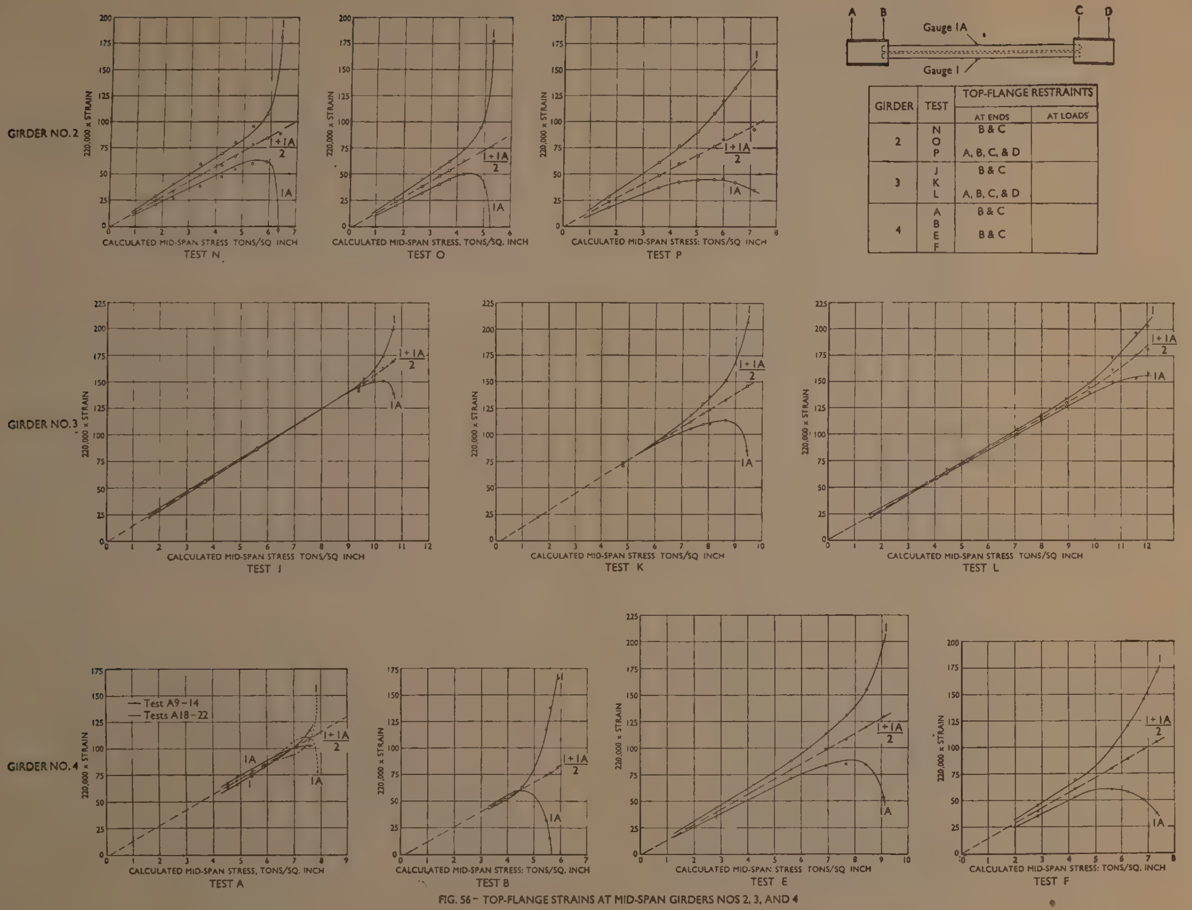


FIG. 55- TOP-FLANGE STRAINS AT MID-SPAN GIRDER NO. 1

ERIC LONGBOTTOM and JACQUES HEYMAN

EXPERIMENTAL VERIFICATION OF THE STRENGTHS OF PLATE GIRDERS DESIGNED IN ACCORDANCE WITH THE REVISED B.S. 153



GIRDER	TEST	TOP-FLANGE RESTRAINTS AT ENDS	TOP-FLANGE RESTRAINTS AT LOADS
2	N	B & C	
	O		
	P	A, B, C, & D	
3	J	B & C	
	K		
	L	A, B, C, & D	
4	A	B & C	
	B		
	E	B & C	
	F		

FIG. 56 – TOP-FLANGE STRAINS AT MID-SPAN GIRDERS NOS 2, 3, AND 4

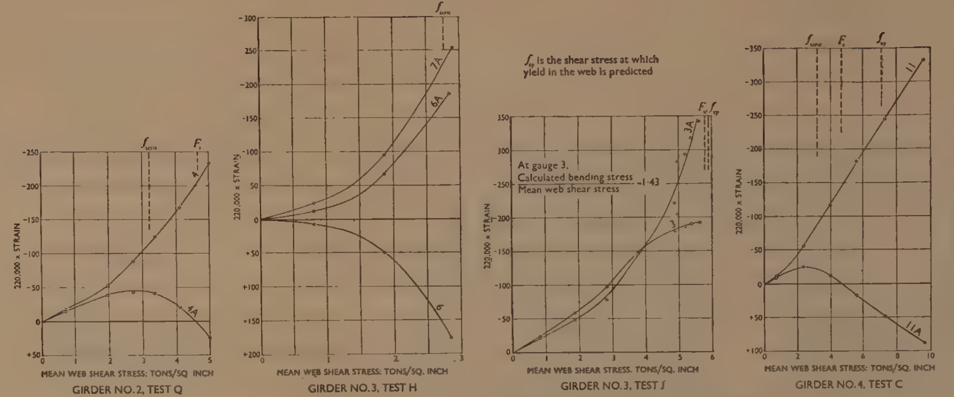


FIG. 57 – HORIZONTAL AND DIAGONAL STRAINS IN WEBS
(Fig. 54 shows the locations of strain gauges on the girders)

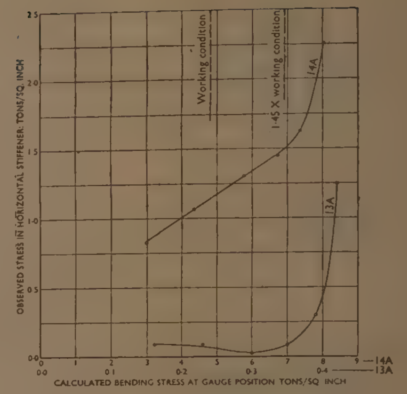


FIG. 58 – STRESSES IN HORIZONTAL STIFFENERS

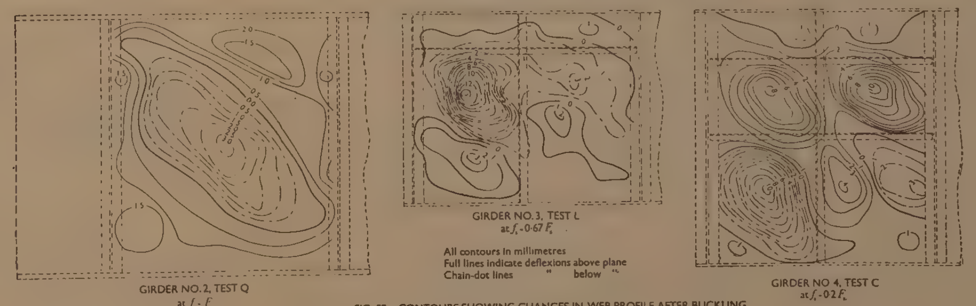


FIG. 59 – CONTOURS SHOWING CHANGES IN WEB PROFILE AFTER BUCKLING

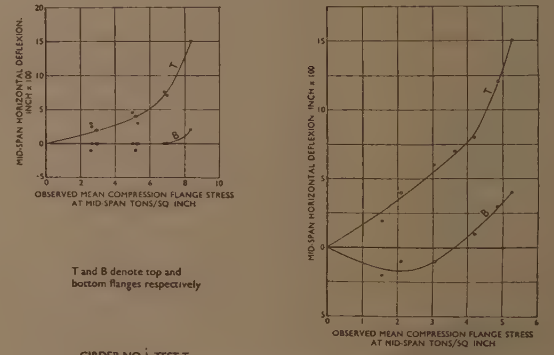


FIG. 60 – HORIZONTAL DEFLEXIONS OF TOP AND BOTTOM FLANGES

That argument had been fought out in that same hall in respect of highway bridges, nd to a certain extent the fight was still going on. Although the layman was technically norant, he was nevertheless very astute. Under his strictures the engineer, especially he highway-bridge engineer, felt much more comfortable if he could design for a general very-day loading with a substantial stress margin which his profession had sanctioned, that the juggernaut could be carried on occasion without collapse.

Nevertheless, a rational decision on what the factor should be was entangled with the omplexities of probability and again hampered by the lack of accumulated data. He oped the evening would not pass, however, without the members giving the B.S. Com-ittees their *fiat* for bringing the work of a number of years to an end for the time being nd publishing it as a Standard.

Dr Flint had said that the lack of data on imperfections in girder flanges had caused him use the Perry-Robertson strut approach. That was an approach of the right order but was not known whether those imperfection values were truly applicable. Again, in eciding on a factor of safety it was reasonable to differentiate between local yield in a nall part of the structure and the more devastating condition of large volumes of high ress.

Many structures could not be designed economically without facing the condition of a rincipal stress approaching yield; as, for instance, in steel bunker construction, and it ould have been noticed that in the draft regulations of B.S.153 it was proposed to anction a principal stress of 14 tons/sq. in. for a combination of flexural, shear, and bearing resses.

One of the troubles in deciding on a minimum factor was a subscription to relatively ough work-a-day methods in design, whereby approximations were made which them-elves carried an error of an appreciable percentage—for instance, the subscription to uclidian lines and points and gross areas, which were known not to be true.

The other great difficulty was an erring judgement in deciding degrees of end restraint members subject to flexure and compression and yet, nevertheless engineering judgement ust be relied on; it was the perquisite of a good engineer. The task in devising such a tandard as that being considered was to limit that judgement within certain prescribed mits and to frame relatively simple rules for use. Those rules could be further simplified y Tables prepared by engineers-in-charge for the use of their staff.

It was impossible to pay too high a tribute to the Authors for their enthusiastic work in rashing out the complex problems which the common beam and plate girder embodied. s the Chairman had hinted, there seemed to be more in it than many practising designers alized. The common place girder was a structure to which Fidler's phrase aptly applied -one with a simplicity of structure but an unknown complexity of function: whereas warren girder had a relatively complex structure and a known simplicity of function.

Mr Kerensky had been particularly helpful in all that: not only had he done a vast nount of work himself but had inspired Dr Flint and others to prosecute their work to a ractical conclusion; moreover, he had tried things out in his own office. That had been the greatest value.

It was perhaps a pity that the graphs in Figs 22 to 27, which illustrated where the ommittee stood in relationship to their previous selves and others, for various typical mmetrical beams and girders, given for yield stress, did not include a plot of the pro-osed working values which could be obtained from Figs 18 to 22. Members could, owever, easily do that for themselves.

It would be seen from the Papers that it was proposed to replace the ideal curve by a orizontal cut-off where the value of l/r_y was about 60, otherwise there would be the ques-on of a curve conforming to a strut graph in which the mean stress fell away from the asic stress value the instant the flange had any effective unsupported length. It would em unreasonable if that were not done, because all engineers felt and tests seemed to ow that up to a slenderness ratio somewhat short of 100 many beams and girders could designed for the basic maximum stress. Otherwise the only cases which would permit

the full basic stress would be where there was continual support along the flange, such as that provided by a slab.

As Chairman, he would again say that the Committee hoped the meeting would confirm what was proposed. He had not mentioned shear in detail, but it should be noted that it was proposed now to recognize buckle in stable webs. One of the questions, which Dr Heyman had mentioned, was the difference in the load factor for flexure and so-called shear, but he did not know whether there was in fact a practical case for making them the same.

Mr Gardner added that in considering the factor of safety question in relation to the full-size test results it should be noted that the girders themselves embodied those imperfections of materials and workmanship for which the usual inclusive factor of safety was supposed to allow. That should therefore be taken into account in a philosophical approach to the safety problem.

Mr T. J. Upstone (Chief Designer, Bridge and Contracting Department of Dorman Long (Bridge and Engineering) Ltd) said that engineers and designers would find the Papers a valuable companion to the new Standard when in use, for it might help them to understand some of the many additional clauses which would appear in it. He recommended the production of a Paper giving similar explanations of the derivation of the revised loading now given in part 3A of the specification.

Naturally, there had not been much time to gain experience with the use of the revisions as applied to practical design, but he gathered from the Papers that the design was produced by a reference to many Tables given in the specification. So far as he could see—and he had tried it out—it was a relatively simple matter. It tended to be a mechanical process, however, and there was a certain danger that designers could carry out those mechanical processes without realizing what the figures produced really meant.

It appeared that when questions of stability were involved, designing beams and plate girders resolved itself into a "trial and error" method. Sometimes the convergence of successive trials towards the final design was not very rapid and considerably more time was required for design than was required formerly when designing to the old British Standard. If that led to a more economic design, the additional time spent by the designer might be more than compensated by the saving in weight.

By far the greater proportion of girders would still be designed at maximum basic stresses and the special clauses would be applicable only to a small number of designs.

Turning to the proposed clauses, he asked if the implication was that the addition of stiffeners to the web of a plate girder would reduce its carrying capacity. That was the interpretation which could be placed on clause V, where the proposed basic permissible stresses for tension as well as compression were given as 9·5 tons/sq. in. for plate girders with unstiffened webs and only 9·0 tons/sq. in. for plate girders with stiffened webs. That did not seem rational and further explanation was required.

He agreed with the Authors' assumption that normal holing of the tension flange of a girder did not influence the position of the neutral axis as derived from the gross section of the girder and that the maximum stress in the tension flange might be obtained by multiplying the stress calculated from the properties of the gross section by a suitable factor.

He thought, however, that that factor should be the largest value of the reciprocal of the efficiency of any of the elements of the flange taken separately and not as a whole, as was proposed. Taking, for example, a riveted girder with $\frac{3}{8}$-in. web, 4-in. × 4-in. × $\frac{1}{2}$-in. flange angles and 10-in. × $\frac{1}{2}$-in. flange plate, the efficiencies of the various elements when holed with $\frac{15}{16}$-in. holes were 76·6% for the portion of web between the vertical legs of the angles, 75% for the angles themselves and 81·2% for the flange plate, giving reciprocals of 1·31, 1·33, and 1·23 respectively. That seemed to indicate that the maximum stress would occur in the flange angles and would be 1·33 times the value calculated on the gross section. The ratio of the gross section of the flange to the net section, as defined in the

proposed clause, was 1 : 1·29, and using that as the appropriate factor would appear to underestimate the maximum stress in the flange angles.

Turning to the clauses dealing with the stiffeners on a stiffened web plate, he said the required stiffness proposed for vertical stiffeners was that which would allow the full allowable shear strength of the panel to be developed. Was such complete stiffening really necessary when large sections of plate girders were not very highly stressed in shear?

The introduction of rules governing the design of horizontal stiffeners was a welcome step forward, but he thought the specification should not be so rigid as to the location of those stiffeners. A horizontal stiffener two-fifths of the distance from the compression flange to the neutral axis might not fit in with other details. Some specifications dealt with the stability of web panels in the form of curves giving working stresses for different ratios of width to depth of panel and for different stress patterns in the depth of the panel. Such an arrangement was more adaptable and gave the designer greater choice with the spacing of both vertical and horizontal stiffeners.

Some of the new clauses were concerned with the stability of compression flanges of half-through bridges. Formulae were given enabling the designer to calculate the effective length of such flanges based on the restraining influences of the U-shaped frames formed by stiffeners and cross-girders. Those formulae, however, appeared only to cover the case of two girder bridges or perhaps the outer girders of three-girder bridges.

With the modernization programme of railways in mind, there were likely to be large numbers of three-girder half-through bridges built in the near future. The top flanges of the centre girders of such bridges were restrained by the U-shaped frames on both sides and the extension of the formulae to take account of that fact would be a valuable addition. The specification called for those U-shaped frames to resist a load calculated as $1\frac{1}{4}\%$ of the flange load applied at the top flange of the girder. The requirement that each frame should carry that load, was difficult to understand since the forces should bear some relation to the spacing of the frames.

Mr E. K. Bridge (Assistant Engineer (Bridges), British Railways, Southern Region) said that the two Papers provided a valuable basis for the design of plate girders which eliminated many of the uncertainties encountered in the application of old standards, particularly those concerning effective length of compression flanges and stability of members with unusual sections.

He welcomed the proposal to do away with the anomaly of using, for the design of compression flanges, a critical-stress formula which did not take account of the imperfections which were bound to occur, and which had been considered in assessing permissible stresses in struts for some time.

A point to bear in mind was that most structural sections were straightened cold; if they had a kink they were straightened in a press. Part of the flange yielded and part did not, and the inevitable result was locked-up stresses. That might account for one of the illustrations, showing a girder in which an initial flange-buckle in one direction had reversed under load.

The riveting of a girder produced a slight increase in the length of the sections, apparently as a result of the transverse compression set up around the rivets, and the resulting longitudinal stress might have some effect on the buckling of the top flange. Mr Bridge's attention had been drawn to the need for considering imperfections when assessing girders supporting some roof trusses, where it was evident that they were not perfectly straight and that the loads were unlikely to be applied exactly on the centre-line of the web.

Permissible stresses were derived from equations containing terms for both initial curvature and twist. The equations were of the same type as those used in the Paper. Their solution was somewhat laborious, but the answer could be shown as a family of curves giving permissible stress. Such curves were generally easier to use than Tables and it might help if they could be provided in the new standard.

A point which might require attention in some applications of the standard was a small

eccentricity of the applied load. The Steel Structures Research Committee had reported stresses due to torsion of 3·9 tons/sq. in. for ¼-in. eccentricity of a 7-ton point load on a 10-in. × 4½-in. beam 13 ft long. That was a high stress for a small eccentricity and there might be other factors, but it seemed an aspect which had to be watched.

For bridges of moderate size, the new method of calculation might call for larger sections in some unrestrained beams, but in general, most girders and beams in a bridge were restrained by the floor or the cross-girder and stiffener U-frames. It was not always advisable to cut down sections in railway bridge work, where corrosion was a serious factor, and therefore the same economy could not always be obtained as in a frame building protected against the weather.

Dr K. C. Rockey (Lecturer in Engineering, University College, Swansea) said that the design proposals contained in the draft copy of B.S.153 represented a considerable change in the outlook of engineers with regard to the significance of web-plate buckling.

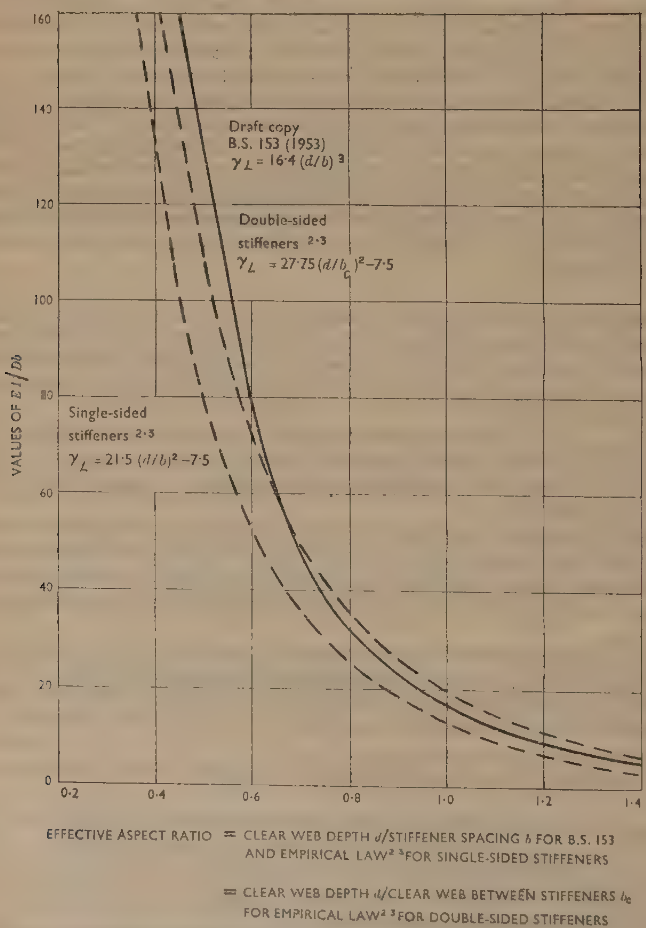


FIG. 65.—RELATION BETWEEN EI/Db AND EFFECTIVE ASPECT RATIO

He supported the proposal to permit the use of web plates which were loaded beyond their buckling loads, because as the Authors had clearly stated, web-plate buckling was not dangerous but resulted merely in changes in the manner in which any additional load was carried by the web plate.

Although the proposed formula for the design of intermediate vertical stiffeners represented a considerable improvement on the present empirical law which had been in use since the beginning of the century, recent theoretical [39] and experimental [40, 41] work had shown that the proposed relationship was not the most suitable. That law which was based on the work of Moore in the United States, would result in the use of excessively strong stiffeners when the stiffeners were closely spaced and relatively weak stiffeners when the stiffener spacing approached or was greater than the clear depth of the web plate (see Fig. 65). The reason was that according to Moore's formula the ratio of the required flexural rigidity of the stiffener to the flexural rigidity of a strip of web plate equal in width to the stiffener spacing varied as the third power of the ratio d/b, whereas according to the work referred to above, it varied as the second power of the ratio d/b.

Tests to destruction which Dr Rockey had conducted on web plates reinforced by stiffeners designed in accordance with his empirical formulae had shown that such stiffeners would operate effectively up to at least double the buckling load of the web plate. As the result of the experience gained from those tests he supported the Authors' proposal to limit the outstand of a stiffener to 12 times its thickness unless it was reinforced by some form of lip; that was especially important with single sided stiffeners.

It was therefore suggested that further consideration should be given to stiffener design, because it was highly probable that once a design formula was recommended and accepted, it would remain in use for some considerable time.

The use of horizontal stiffeners was another innovation so far as British specifications were concerned. Since the Authors of Structural Paper No. 48 had stated that little experimental evidence was available regarding the behaviour of horizontal stiffeners, they would no doubt be glad to learn that the results obtained from about eighty tests conducted at Swansea on web plates reinforced by horizontal stiffeners indicated that the proposed design formula would result in stiffeners of ample rigidity.

With regard to the design of the flanges, in Dr Rockey's opinion, there was one serious weakness in the proposed design procedure. The present specification permitted the design of plate girders with flanges of a low flexural rigidity about their xx axis (see Fig. 66). That was well illustrated by examining the properties of the experimental girders described in Structural Paper No. 49.

When a web plate subjected to shear operated beyond its buckling load, the membrane stresses set up in the web plate exerted a lateral load on the flange. If the flange had insufficient rigidity it would deflect badly under that lateral load and in doing so influenced the behaviour of the web plate. That had been observed a few years previously by Dr Rockey when testing a welded steel girder, the flanges of which had consisted of a simple flat at right angles to the web plate. He had noticed that as the girder was loaded beyond its buckling load the direction of the buckle changed until finally a single large buckle running diagonally across the panel had been formed. As a result it was decided to investigate the matter in detail and tests had been conducted on aluminium shear panels and plate girders to determine the effects of flange rigidity upon the post buckled behaviour of web plates.

More than thirty girders had been tested, the general procedure being to determine the buckle pattern of the web at various multiples of the buckling load. A full report dealing with the work would be published shortly. Fig. 66 showed how the ratio, maximum depth of the buckled plate (Δ) to the thickness of the plate t varied with the non-dimensional parameter I/b^3t; I being the second moment of inertia of the flange about its xx axis, b the stiffener spacing, and t the thickness of the web plate.

For a given value of the ratio, applied load W/web plate buckling load W_{cr}, if the

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[39] References 39 *et seq.* are given on p. 520.

stiffness of the flange was reduced below a given value, the magnitude of the maximum deflexion increased considerably. It was therefore desirable that plate girders should be designed so that the stiffness of the flange, as represented by the parameter I/b^3t, did not fall below a given value. Fig. 66 showed that as the ratio applied load W/web plate

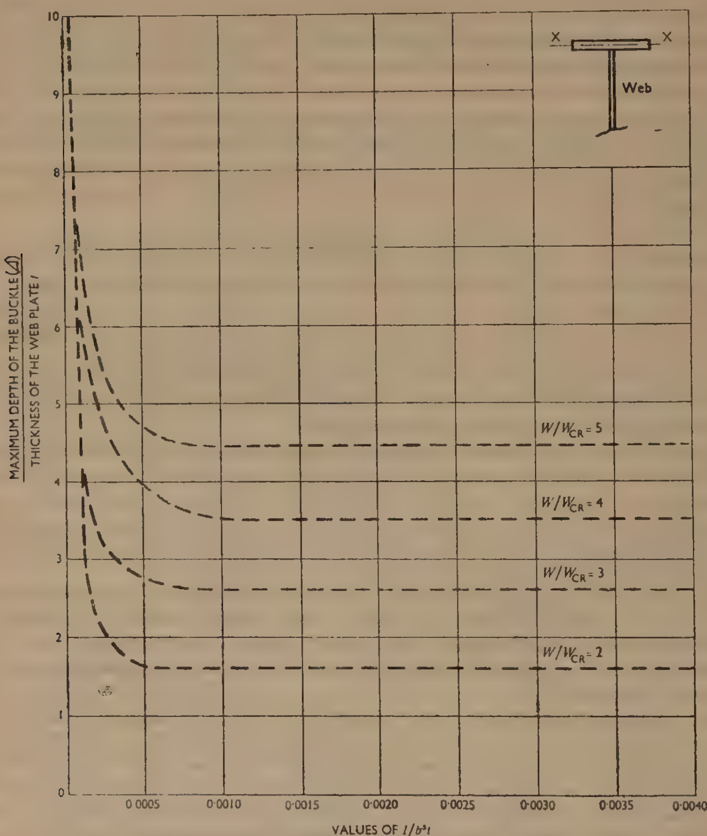

FIG. 66.—RELATION BETWEEN Δ/t AND I/b^3t

buckling load W_{cr} increased so the minimum desirable value of the flange stiffness (I/b^3t) increased.

The effects of employing flanges with low flexural rigidity as discussed above, was probably the reason why low load factors were obtained in certain of the tests described in Structural Paper No. 49. For example, considering girder C4, the I/b^3t value of the flange stiffener assembly at the ends of the girder where failure occurred was 0·000038. As would be seen from Fig. 66, that was a low value and large lateral deflexions of the web plate would be expected when the web plate was loaded beyond its buckling load. As a result of the test, the Authors decided to restrict the maximum d/t ratio for web plates reinforced by vertical stiffeners only, to 200, instead of the previous value of 240. That alone, however, would not have the desired effect, as indicated by the results obtained from girder 2. That girder had a d/t ratio of 200, but the flanges were quite flexible, the

I/b^3t value for the critical panels being 0·000024. Even if it was assumed that the panels were only simply supported, then the $W_{collapse}/W_{critical}$ value for the web plate of that girder was only 1·67.

The curves shown in Fig. 66 were derived from elastic tests on plates which were initially plane. The Authors had referred to the fact that the web plates of several of their girders had large initial deformations, for example, girders C4 and 2. It would be appreciated that if there were initial imperfections in a web plate, then the membrane stresses would occur earlier, with the result that the lateral loading was imposed on the flanges at a lower load than would be the case for a plane plate and the effects of flange flexibility would therefore be more pronounced.

Mr A. E. Long (Assistant Director, Military Engineering Experimental Establishment, Christchurch) said that the examples given by the Authors were very helpful in following the build-up of the theory.

Referring to the full-size tests, there had been several remarks about the imperfections and the degree of deflexion of the top flanges, but he wished to make it clear that those imperfections were relatively small compared with what was normally experienced in practice. For example, the lateral deflexion of the top flange of the largest girder was about $\frac{1}{16}$-in. in 31 ft, which was very much smaller than would normally be expected, and even the greatest deflexion of the smallest girder was only $\frac{1}{8}$-in.

Those imperfections had had a significant effect on the buckling loads, and one wondered what was likely to occur in practice and what effect the imperfections would have on the actual safety factor. Any information which was forthcoming on those points would be interesting.

He was particularly glad to see the rules proposed for horizontal stiffeners, which would be widely welcomed and should result in considerable economies for the deeper girders. One question on stability which he did not follow, and on which he had had conversations with Dr Flint, was that of torsional stability. The effect of torsional fixity was mentioned on p. 412, for the case of a girder with no lateral support to the top flange. The question was how deep a girder could be designed without that lateral support. It was suggested, arbitrarily, that there should be a limiting d/t-ratio of 200. The ratio d/t was the ratio of the depth of the web to the thickness of the web, and yet in the formula for flange buckling the four parameters were l/r and D/T; the thickness of the web was apparently not pertinent. Why was the thickness of the web introduced in the clause covering the depth of the girder needing no support. He emphasized that in that case, the girder had no lateral support whatever, except at its bearings. It was very unusual, and he wondered whether it could not better be dealt with by increasing the effective length to the span plus twice the depth of the girder.

The suggestion of increasing the effective length had been followed in dealing with the question of load which was free to move with the deflecting top flange; there the effective length was increased by 20%.

Dr M. R. Horne (Assistant Director of Research, Engineering Department, University of Cambridge) said that the behaviour of deep plate girders was certainly complicated, and the Authors had done well in achieving a manageable procedure which, nevertheless, enabled the main factors to be taken into account. There were, however, certain minor details in their procedure which were open to criticism.

He asked for some clarification over the allowance for flange curtailment. It was not clear from the text whether the factors α and $\alpha\sqrt{\beta}$ allowed for the beneficial effect of the more favourable bending-moment distribution which necessarily accompanied flange curtailment. Equation (24) of Appendix V seemed to indicate that advantage of the more favourable bending-moment distribution was not allowed for, since the factor α applied a modification to the solution for a central load. On the other hand, the curves in Figs 8–11 all rose to a value of about 1·37 for a uniform section, thus indicating that advantage was being taken of the favourable distribution. In that case, equation (24)

was in error, and the factor 17·2 should be replaced by 4π or 12·6, corresponding to the solution for uniform bending moment. That equation would then agree with equation (12) in the text.

It thus appeared that advantage of the more favourable bending-moment distribution had in fact been taken when dealing with flange curtailment. It should, however, be noted that a similar advantage had been assumed when considering the effect of top-flange loading. It thus seemed that the Authors were allowing for the same favourable factor twice, and might thus be obtaining unsafe results. Incidentally, in connexion with top-flange loading, it was a pity that the allowance for the vertical position of the load had not been made to depend on the length-to-depth ratio of the girder, and also that advantage had not been taken of the comparatively beneficial effect of loads applied on the bottom flange.

Turning to a consideration of the safety factors implied in the proposed shear stresses, Dr Horne noted that Mr Kerensky had mentioned them in his introductory remarks, in which he had admitted that the permissible mean shear stress in unstiffened webs of 6 tons/sq. in. was probably excessive. Whereas the safety factor for a beam under pure bending, taking a longitudinal stress of 9·5 tons/sq. in. and a shape factor of 1·15, was about 1·85, that for a web with a mean shear stress at working loads of 6 tons/sq. in. was only 1·47. Under combined shear force and bending moment, the safety factor against local failure might fall to 1·10. Whilst that was mentioned in the first Paper, it was assumed that the safety factor against ultimate failure would not fall below about 1·45. Actual load factors for combined loads might be derived from the plastic analysis for thick-web girders given in the second Paper as followed.

A girder of depth D in., single flange area A_f sq. in. and web area A_w sq. in. was assumed to be designed to withstand a bending moment M tons-inches and a shear force F tons, so that under such loading, the longitudinal stress was f_b tons/sq. in. and the mean web shear stress f_s tons/sq. in. Then, if the depth D was moderately large in relation to the flange thickness, it followed that:

$$M \doteq D(A_f + A_w/6)f_b \qquad (1)$$

$$\text{and } F = A_w f_s \qquad (2)$$

If the load factor at collapse was λ, and if failure occurred with combined bending and shear deformation, then the mean web stress would be λf_s and the equation for the moment of resistance became:

$$\lambda M = D\left\{A_f f_y + \frac{A_w}{4}\sqrt{f_y^2 - 3\lambda^2 f_s^2}\right\} \qquad (3)$$

where f_y was the yield stress. If the value of M from equation (1) was substituted in (3), and the ratio $\frac{A_f}{A_w}$ was denoted by r, it followed that:

$$\lambda = \frac{3f_y\{8r(1 + 6r)f_b + \sqrt{[4(1 + 6r)^2 f_b^2 + 27(1 - 16r^2)f_s^2]}\}}{4(1 + 6r)^2 f_b^2 + 27 f_s^2} \qquad (4)$$

When the flange area was sufficiently large in relation to the web area, the section might fail in shear deformation without bending, in which case $\lambda = \frac{1}{\sqrt{3}}\left(\frac{f_y}{f_s}\right)$. The correct value of the load factor was the lower of the two estimates.

It had been assumed in the foregoing analysis that the design of the web was governed by the allowable mean shear stress f_s. If there was a limitation on the maximum shear stress to some value f_s', and if that was the governing criterion, then equation (2) was replaced by:

$$F = \frac{2}{3}\left(\frac{1 + 6r}{1 + 4r}\right)A_w f_s' \qquad (5)$$

At a load factor of λ, the mean shear stress in the web became $\frac{2}{3}\left(\frac{1 + 6r}{1 + 4r}\right)\lambda f_s'$, and that

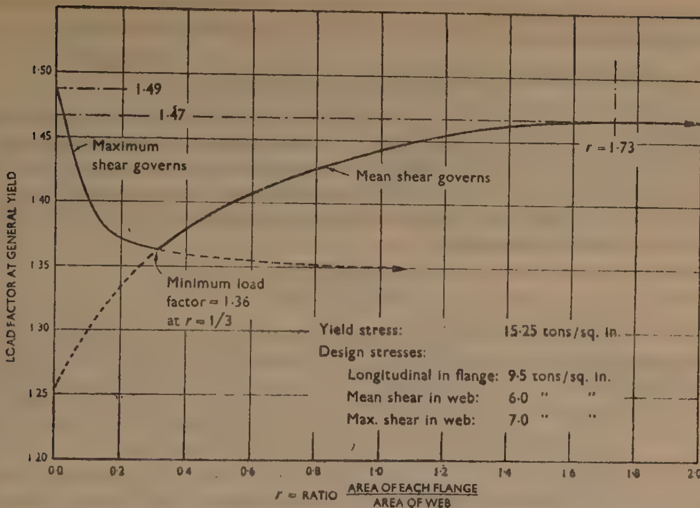

FIG. 67.—LOAD FACTORS FOR GENERAL YIELD IN SYMMETRICAL PLATED GIRDERS

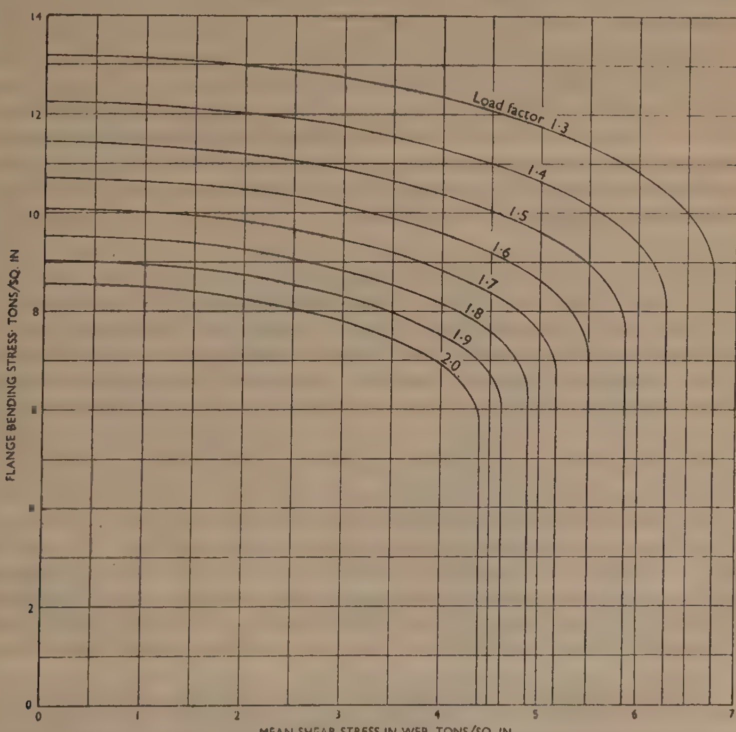

FIG. 68.—LOAD FACTORS UNDER COMBINED SHEAR FORCE AND BENDING MOMENT

quantity had to be substituted for λf_s in equation (3). The substitution of the value of M from equation (1) finally led to the following expression for the load factor λ:

$$\lambda = \frac{3(1+4r)f_y\{4r(1+4r)f_b + \sqrt{[(1+4r)^2 f_b^2 + 3(1-16r^2)(f_s')^2]}\}}{2(1+6r)\{(1+4r)^2 f_b^2 + 3(f_s')^2\}} \quad . \quad . \quad (6)$$

The values of λ calculated from equations (4) and (6) when $f_b = 9{\cdot}5$ tons/sq. in., $f_s = 6{\cdot}0$ tons/sq. in., $f_s' = 7{\cdot}0$ tons/sq. in., and $f_y = 15{\cdot}25$ tons/sq. in. were shown by the two curves in Fig. 67. The *mean* shear stress ruled the design when $r > 1/3$, and the *maximum* shear stress governed when $r < 1/3$. The minimum load factor occurred when $r = 1/3$, and had a value of 1·36. It might be noted that in general the minimum load factor occurred at a value of r given by:

$$r = \frac{3f_s - 2f_s'}{12(f_s' - f_s)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Whilst $r = 1/3$ was a somewhat low value, ratios of $r = 1/2$ were quite common, and Fig. 67 showed that load factors below 1·4 were possible with practical sections. Load factors obtained with $r = 1/2$ and any combination of bending stress f_b and shear stress f_s were shown in Fig. 68. That chart could be used to find the minimum practical load factor obtained under combined loading, since the maximum shear stress would not usually be so limited that it controlled the design of normal sections.

It was now possible to enter into a fuller discussion of design shear stresses. The acceptance of a lower load factor against shear failure than against bending could be argued, in view of the less serious nature of failure under shear. Even with that consideration, however, a difference as between 1·85 for pure bending failure and 1·47 for pure shear failure was surely unjustifiable. It was maintained that the mean web shear stress at sections not subjected to simultaneous maximum shear force and bending moment ought to be limited to 5·5 tons/sq. in. giving a safety factor against failure in shear of 1·60. Turning to combined failure at sections where maximum shear and bending occurred together, it was difficult to justify a safety factor below 1·60. Fig. 68 showed that, if it was desired to retain, at such a section, a longitudinal working stress of 9·5 tons/sq. in. in the flanges, then the mean web shear stress ought to be limited to 4·1 tons/sq. in. If, on the other hand, the mean web shear stress of 5·5 tons/sq. in. was retained, then the longitudinal flange stress should be limited to about 7·5 tons/sq. in. If it was contended that those stress reductions were unnecessary because higher stresses had been used for many years, then the conclusion would have to be drawn that safety factors higher than about 1·5 were nowhere required, and the whole code ought to be redrafted on that basis.

Finally, it was unfortunate that the Authors had eliminated explicit mention of the essential properties of the section from the basic equation for critical stress, thus transforming equation (2) into equation (9). Whilst appreciating the advantages which accrued from expressing all properties empirically in terms of T/D, that limited the solution strictly to I-section girders of more or less orthodox design, and also gave results which were, for some cases, unnecessarily conservative. Dr Horne thought that it should at least be made permissive to use the more accurate formula for symmetrical members of homogeneous section, thus enabling the method to be applied to girders in which special measures had been taken to increase the torsional rigidity. Besides extending the scope of the code, such a step would have a high value in the encouragement it would give to a more widespread understanding of the problem of lateral stability. The best course would be to permit the more accurate treatment as an explicit alternative. Failing that, the next best solution was to have the full basis of the code readily available, and that was fortunately available in the two Papers under discussion. Too many British Standards were produced without the publication of the steps which led to their formulation.

Mr William Henderson (Ministry of Transport, and Civil Aviation) said that the Papers presented the results of an immense amount of work and study, which had successfully resolved the highly complicated theoretical problems and the equally awkward or even more awkward question of imperfections into a system of design rules which were

realistic. From the examples shown, it was clear that they were perfectly usable provided they were approached with the respect which was their due and were studied thoroughly in the first place.

It was possible, and even probable, that modifications would be produced in course of time, but he felt that the Paper approached as near as was practically desirable or possible to the theoretical truth on the subject. The modifications which he envisaged as possible were those which could be brought about by improvements in materials and fabrication, which would permit a reduction to be made in the allowances for imperfections.

In examining the range of imperfections, which might occur, the Authors and the B.S. Committee found little positive evidence that deviations from perfection could be accurately assessed. He recommended for future committees the gains which could be achieved by specifying precisely the maximum deviations from shape, etc., which should be permitted. Such a restriction might prove unpopular initially, but he was confident that it would not long be so, and in the long run it would not only benefit design but would reflect to the credit of the British bridge-building industry.

Attention had been drawn to the very wide range of the limit of proportionality shown in Table 14. Several of the committee, including himself, had been quite shocked when they had first become aware that it was not an exceptional state of affairs; there again it would be very satisfactory if an improvement could be brought about.

It was perhaps fortunate that the M.E.X.E. tests provided some considerable imperfections unintentionally, in spite of the fact that the girders were exceptionally well made. The serious effects of imperfections having been established by those tests, the Authors had produced a method of dealing with them in design which was very ingenious, taking account, as it did, at one extreme of the shape factor of shallow girders and at the other of the identity with strut conditions in the flanges of deep girders. Designers would no doubt feel that their work had been considerably increased, and that was undoubtedly true. On the other hand, they should bear in mind that actual loads and stresses were being increased and load factors reduced. The hidden margins for ignorance and approximation were being gradually pared away and the consequence was that design methods would have to become much more precise.

Moreover, a large part of the increased complexity of the proposed new design rules lay in their much wider scope. Thus the field of asymmetrical sections and sections with unequal flanges was very competently covered, whilst the information on cantilevers was invaluable.

Similarly, the provision for taking into account various degrees of restraint was an invaluable and essential refinement. Little account had been taken of them in the past and it seemed probable that failure to do so had to some considerable extent compensated for the optimistic values given by former design rules in the range of slender members. The degree of restraint required was such that judgement on the qualitative basis given was on the safe side and covered the vast majority of practical cases. Personally, he had an inherent objection to assessments which were not measured numerically, but there was no great drawback there.

In the note given on clause Z, however, it was said that restraint against torsion was adequately provided by bearing stiffeners in the case of plate girders on a bearing. That might go too far and it might be desirable to provide something more substantial than simply bearing stiffeners for girders with high d/t ratios yet not in the range requiring horizontal stiffeners. He thought that more support should be provided where d/t exceeded 120.

The maximum permissible shear stress suggested in clause V of 6 tons/sq. in. seemed to present a problem which was at least philosophically serious since it reduced the factor on yield to 1·45, as compared with the higher value used elsewhere. There had been a very strong feeling that in the ordinary bridge girder fairly high shear stresses were justified. Partly that might be due to a misconception that shear yield was 10 tons/sq. in. instead of 8·8 tons/sq. in.; and partly to a conviction that other circumstances might exist which modified the apparently low load factor, such as, for instance, non-uniform shear distribution. He was afraid, however, that there was not much substance

in either case, and it seemed to him illogical to work to a lower load factor in shear unless some other justification could be produced. On the face of it, there seemed no such justification. He therefore reluctantly suggested that maximum shear stress be reduced to 5·5 tons/sq. in.

The proposed new design rules gave a flexibility and scope which far outweighed the additional trouble to which the designer might be put. The results of their adoption would be to produce far more precise designs than had hitherto been possible and would at the same time bring about a useful saving in steel.

For those who had little time to follow the rules to the logical conclusion by using unequal flange sizes, the procedure was little more complicated than formerly for girders having large values of l/r_y, although the savings would not be so great.

Similarly, the technique of dealing with curtailment of flanges was, as the Authors rightly pointed out, rather clumsy. Where flanges were to be curtailed, however, the laborious assessment of the effects could quite easily be dodged in most cases by taking T from the minimum flange section. In very many cases the effect of doing so would be unimportant and would always be on the safe side.

Mr G. B. Godfrey observed that since 200 miles of bridges had been built in Western Germany since the war, it might be of some interest to compare their standard specifications with the proposed B.S.153.

The permissible bending stresses σ in mild steel for German railway and highway bridges, contained in DV 804 and DIN 1073 respectively, were in course of revision to 10·16 tons/sq. in., compared with 9·0 tons/sq. in. in B.S.153. In shear the German permissible stresses τ were both being raised to 5·86 tons/sq. in. ($\tau = 0{\cdot}577\sigma$).

In welcoming the introduction of deep girders, he suggested that the adjectives "transverse" and "longitudinal" should be employed to describe the two types of stiffener envisaged in such girders.

With regard to the stiffening of web plates, the Germans, in DIN 4114, gave Tables for fifteen different types of loading and stiffener arrangement. Massonnet had proposed the use of eighteen types in Belgium. Whilst Mr Godfrey felt that such Tables reduced stability problems almost to absurdity, there was a danger that the Authors had over-simplified the arrangement of, and calculations for, longitudinal stiffeners.

For a certain range of web d/t-ratios, the Authors had specified that there should be one longitudinal stiffener placed two-fifths of the distance from the compression flange to the neutral axis. In addition, it was specified that the stiffener should have a single minimum I-value. More slender girders were to have a further stiffener placed on the neutral axis.

There were anomalies in such rules. Consider, for example, a simple span with a web d/t-ratio of 240. It had been demonstrated by Dubas, the Swiss investigator, that where bending stresses were high and shear stresses very low, such as at mid-span, the best position for a longitudinal stiffener was at one-fifth of the depth of the web. Towards the ends of the girder the bending stresses diminished whilst the shear stresses increased. In such cases it was quite certain that single longitudinal stiffeners could be placed in more effective positions than that specified by the Authors.

Furthermore, the I-values specified for longitudinal stiffeners varied widely in DIN 4114. Were the Authors sure that the single values given in the proposed B.S.153 embraced the worst conditions of loading anywhere in the girder?

Massonnet had made an interesting contribution to the discussion on the Paper by Young and Landau.[42] In Table 6 the Authors had shown that the I-values which they were proposing were nearly twice as great as those specified by the Germans. Massonnet had suggested that the various German coefficients should be multiplied by figures varying from 3 to 7.

Mr Godfrey stated that the Authors appeared to be considering plate girders as independent parts of a bridge. There was considerable economical advantage to be gained by considering a bridge superstructure as a single integral member. In the "guns before butter" era before the war the Germans wasted no steel in the design methods then

known to them. Nevertheless, they had achieved startling economies in the reconstruction of their large bridges, the saving in rolled steel being as much as 60% in some cases. These economies had been made in a number of ways, as follows:—

(1) By using wide box girders in which the decking served as the top flange to the webs and which, when stiffened, could be surfaced with a thin carpet of asphalt to form a carriageway.
(2) By making a reinforced or prestressed concrete deck composite with the girders.
(3) By employing high-tensile steel or by mixing high-tensile and mild steel in the same parts of girders.
(4) By using prestressed steelwork.

Any of those methods could be employed in the British Commonwealth. The rules for web plates specified by the Authors would be of direct application, whilst in (1) and (2) the complications inherent in unsupported compression flanges would be largely avoided.

During the war, members of the armed forces were exhorted to know their enemies. In peace time, it was desirable to know one's competitors.

Mr C. J. S. Branscombe (Technical Assistant, British Railways, L.M. Region) thought that the most important part of the Papers was that dealing with the reduction of critical stresses arising from initial imperfections in the girders. B.S.153 was a standard for girder bridges and he felt that the more normal types of girders had been very severely treated in order to cover the more exceptional cases. The imperfections of initial bow and twist as given in Appendix VI, p. 459, seemed large. If his reckoning were correct, the initial bow amounted to 1/700 of the span and the initial twist to 1/350 of the span-to-depth ratio. Hence, in a 50-ft girder having a span-to-depth ratio of 13, the initial bow and twist amounted to $\frac{7}{8}$-in. and 2° respectively. He felt that in the case of bridges with normal types of floor, even if the fabrication were passed by the steelworks inspector, the floor members would, to a large extent, remove the bow and twist. Hence the reductions in the permissible stresses as given in Table 10 were excessive when applied to girder bridges of normal type.

Referring to the test girders, it appeared that, with the possible exception of girder No. 1, the span-to-depth ratios were unrealistic for bridge girders, No. 1 having a span-to-depth ratio of 10 and each of Nos 2, 3, and 4 a ratio of approximately 5. Similarly, the flange width was small compared with girder depth, the depth-to-breadth ratio being 2·9 for girder No. 1 and about $7\frac{1}{2}$ for Nos 2, 3, and 4. Those factors, combined with severe initial imperfections, and top-flange loading, seemed to make for low critical stresses —not representative of girders used in the majority of bridges. He also noticed that the web-to-flange welds were intermittent, which would have a worsening effect on the girders, and internal stresses appeared to have been set up by the use of jigs.

Turning to the example in Appendix II, he said the actual value of r_y had been used in the determination of critical stresses. It appeared that in the case of symmetrical girders of normal proportions it would be more accurate to use the value of r_y given by breadth of flange divided by 4·2, since that approximation had been used in the evaluation of the constants in equation (9). In the example, the critical stress would be increased by about 1 tons/sq. in., with corresponding increase of permissible stress.

Fig. 13 appeared to have been plotted using different constants in the basic equation and Table 10 thus seemed to give somewhat low values.

Further information about the curtailment of flange plates under fatigue conditions would be appreciated. The Authors' opinion as to the permissible stress in the smaller section would be interesting. Some tests done in America[43] obtained fatigue strengths of only 4·2–4·6 tons/sq. in. at 2,000,000 cycles/sec, when the plate extended to a point where the bending stress in the smaller section equalled that at the centre. Would the Authors comment on that?

Dealing with the use of U-frames to effect a reduction in effective span, he asked if the Authors could clarify the conditions necessary at the end of the span. Would a single U-frame at mid-span, together with normal end beams and bearing stiffeners at the ends

of the span, halve the effective span? Similarly, would the use of two U-frames, one at each quarter point, together with the same end beams and bearing stiffeners, give an effective span of $L/2$ or $L/4$?

The basic equation (9) was derived from the general equation for a symmetrical I-beam under uniform bending moment.

Timoshenko had shown (in Chapter 5 of reference 1) that for a beam under uniform bending moment the critical value of the moment could be written:

$$M_{\text{crit}} = \frac{m_1\sqrt{B_1C}}{L} \text{ where } B_1 = EI_y$$

and for a beam under uniformly distributed load w per unit length:

$$M_{\text{crit}} = \frac{m_2\sqrt{B_1C}}{L}$$

For various values of the factor $\frac{2CL^2}{EI_ch^2}$ the values of m_1 and m_2 were compared in Table 16, from which it was seen that the critical stress for a beam under uniformly distributed load was 1·13 times that for the beam under uniform moment.

TABLE 16

$\frac{2CL^2}{EI_ch^2}$	4	8	32	400
m_1	5·85	4·70	3·59	3·18
m_2	6·63	5·33	4·08	3·58
$\frac{m_2}{m_1}$	1·13	1·13	1·13	1·13

Furthermore the foregoing values of m_2 were applicable for loads at the neutral axis of the beam.

For loads at the bottom flange there was an increase in critical stress and for loads at the top flange a decrease, as shown in Table 17.

TABLE 17

$\frac{2CL^2}{EI_ch^2}$	4	8	32	400
Increase for bottom-flange loading: % . .	46	39·8	24·3	7·5
Decrease for top-flange loading: % . .	31·4	28·6	20·2	6·1

Many railway bridges were of the through and half-through type, the design loads were uniformly distributed, and some provision should be made to enable advantage to be taken of the increase in critical stresses due to loading being below the neutral axis and uniformly distributed. Since many normal types of girder had a value for $\frac{2CL^2}{EI_ch^2}$ in the range 4 to 20, an appreciable increase in permissible stresses should result.

Mr David Allen (Assistant Engineer, Messrs Freeman Fox & Partners, Consulting Engineers) commented on some tests carried out at the University of Washington on a series of rolled steel joists to test the accuracy of the A.I.S.C. beam formula. See reference 37 on p. 444. The first series was carried out on four rolled steel joists with simple supports and Table 18 summarized the results. (The first beam was a light beam.) The approximate English equivalents of the others were 10 in. × 4½ in., 12 in. × 5 in., and 18 in. × 6 in. In all the tests the loading had been on the top flange at the quarter points and

Table 18.—Flange stress (tons/sq. in.) at failure by lateral instability for four American beams, simply supported, compared with permissible stresses from B.S.449 (1948) and proposed B.S.153

Beam		B.S. 449		Test	B.S.449		Proposed B.S.153					
							$l = L$			$l = 1{\cdot}2\,L$		
	D/T	K_1	L/r_y	f_b	F_{BC}	L.F.	C	F_{BC}	L.F.	C	F_{BC}	L.F.
10″ × 4″ I 15 lb. (10 LB 15)	37	1·02	75	15·0	10·0	1·5	33·2	8·0	1·9	23·9	7·0	2·1
			112	12·8	9·1	1·4	16·4	5·6	2·3	12·2	4·6	2·8
			150	9·4	6·8	1·4	10·3	4·2	2·2	7·8	3·4	2·8
10″ × 4⅝″ I 25·4 lb. (10 I 25·4)	20	1·20	135	11·3	8·9	1·3	16·9	5·7	2·0	13·4	5·0	2·3
			165	9·7	7·3	1·3	13·1	5·3	1·8	10·5	4·2	2·3
			264	8·0	4·6	1·7	7·6	3·4	2·3	6·3	2·9	2·8
			408	5·2	2·9	1·8	4·8	2·3	2·3	4·0	2·0	2·6
12″ × 6½″ I 27 lb. (12 WF 27)	30	1·38	88	12·7	10·0	1·3	26·3	7·3	1·7	19·7	6·3	2·0
			107	9·1	10·0	·9	19·1	6·1	1·5	14·4	5·2	1·8
			175	7·3	7·9	·9	9·1	3·8	1·9	7·2	3·2	2·4
			255	5·1	5·4	·9	5·6	2·6	2·0	4·5	2·2	2·3
18″ × 7½″ I 50 lb. (18 WF 50)	32	1·09	79	11·6	10·0	1·2	31·2	7·8	1·5	22·7	6·9	1·7
			100	11·7	10·0	1·2	20·8	6·5	1·8	15·5	5·4	2·2
			160	9·6	6·8	1·4	10·0	4·1	2·3	7·8	3·4	2·8
			239	5·5	4·6	1·2	5·8	2·7	2·0	4·6	2·2	2·5
			398	3·1	2·7	1·2	3·2	1·6	1·9	2·7	1·4	2·2

special precautions had been taken to centre the loads over the centre-line of the web. In some cases the loads had been adjusted laterally to give no twist of the beam when the applied load was about a quarter of the predicted collapse load. That procedure had eliminated eccentricity of loading, and also in some cases the effects of imperfections of the beams had been reduced. The top flange was held at the supports to give the end conditions assumed in the theory.

The steel had been to a specified yield of 33,000 lb/sq. in., and was in all cases higher than 15·25 tons/sq. in., the specified yield for mild steel to B.S.15. In most cases it was much higher—e.g., 18·2 tons/sq. in. for the first beam.

Mr Allen drew attention to the low load factors on B.S.449. The average was 1·3 and the lowest 0·9. That was to say, those beams would fail below their design load. The proposed B.S.153 stresses were given first for effective length equal to the span; then the load factor dropped to 1·5. When the effective length equalled 1·2 times the span, which the proposed rules specified for top-flange loading, the lowest load factor was 1·7 and the highest 2·8. As stated in the Paper, the correction for top-flange loading for joists was probably over-conservative by 5 to 10%.

TABLE 19.—FLANGE STRESS AT FAILURE BY LATERAL INSTABILITY FOR A 10-IN. × 4-IN. × 15-LB. R.S.J. (10 LB. 15) WITH VARIOUS END CONNEXIONS (SEE REFERENCE 37, P. 444), COMPARED WITH PERMISSIBLE STRESSES FROM B.S.449 (1948) AND PROPOSED B.S.153

End conditions	D/T	B.S. 449 K_1	L/r_y	Test f_b	B.S.449 F_{BC}	B.S.449 L.F.	Proposed B.S.153 l	C	F_{BC}	L.F.	l	C	F_{BC}	L.F.
Simply supported	37	1·02	75	15·0	10·0	1·5		33·2	8·0	1·9		23·9	7·0	2·1
			112	12·8	9·1	1·4	L	16·4	5·6	2·3	$1{\cdot}2 \times L$	12·2	4·6	2·8
			150	9·4	6·8	1·4		10·3	4·2	2·2		7·8	3·4	2·8
Web cleats	37	1·02	111	14·8	9·0	1·6		21·4	6·5	2·3		15·8	5·4	2·7
			150	11·4	6·8	1·7		13·4	5·0	2·3		10·0	4·1	2·8
			226	8·0	4·6	1·7	$0{\cdot}85 \times L$	7·3	3·3	2·4	$1{\cdot}2 \times 0{\cdot}85 \times L$	5·6	2·6	3·1
			298	6·1	3·3	1·8		4·7	2·3	2·7		3·8	1·9	3·2
			451	5·2	2·3	2·3		3·0	1·5	3·5		2·4	1·2	4·3
Top and bottom cleats . . .	37	1·02	113	15·9	9·2	1·7		32·1	7·9	2·0		23·0	6·9	2·3
			150	13·3	6·8	2·0		18·4	6·0	2·2		13·5	5·0	2·7
			221	9·7	4·5	2·2	$0{\cdot}70 \times L$	9·5	4·0	2·4	$1{\cdot}2 \times 0{\cdot}70 \times L$	7·2	3·3	2·9
			304	8·8	3·4	2·6		6·4	3·0	2·9		5·0	2·4	3·7
			448	6·7	2·3	2·9		3·7	1·9	3·5		3·0	1·5	4·4

The eccentricity assumed in the proposed rules was arbitrary and the only guide was he Robertson strut value. Further tests might prove it over-conservative.

Table 19 showed the next series of tests, carried out on the 10-in. light beam with onventional end supports:—one web cleats and the other top and bottom cleats only. 'he load factors had risen considerably. The connexions had been made with $\frac{5}{8}$-in. bolts 1 $\frac{11}{16}$-in. holes and the faces to which the cleats had been fixed were relatively rigid ut free to move longitudinally. The "test" bending stresses shown had been worked ut from the failing load, assuming that the beams were simply supported. No account ad been taken of end fixity in the vertical plane. Those were the usual design assumpions, which in the case in point had led to an increase in load factor of 10–30%. Effecive lengths had been taken of 0·85 times the span for the web-cleated beams and 0·70 imes the span for the top- and bottom-cleated beams. Those effective lengths for similar onnexions might not be always justified in practice. The same constant lateral end estraint was clearly shown to be more effective as the l/r_y increased.

The conclusion was that beams designed to B.S.449 would have low load factors which vere not acceptable, according to the arguments on pp. 416–418. The reason that a arge number had not collapsed was probably fortuitous and because web cleats in fact rovided vertical end restraint which increased load factors, although if the 12-in. beam ad been tested with web cleats the load factor would probably not have been much bove unity. The A.I.S.C. rules—the latest American beam rules—also gave low load actors, and alterations had been suggested by means of transition curves.

It was interesting to note that the general approximate equation for asymmetric girders—quation (10)—might be obtained from the Winter equation—equation (6)—by only two najor approximations, apart from $k_2 = 2\lambda - 1$. They were $Z_x = 0{\cdot}37AD$ and $K =$ ·$30AT^2$, where A denoted the total area. For the case of symmetrical girders, putting $A = 3BT$ in other words, the area of web equalled the area of one flange—the result vas $Z_x = 1{\cdot}1BTD$ and $K = 0{\cdot}90BT^3$, which were the values given on p. 402.

The Perry-Robertson formula might be written entirely in terms of the Euler buckling tress and the yield stress. Table 10 was, in fact, merely the relationship between the ritical Euler stress and the Perry-Robertson limiting stress, with a cut-off and a reducion factor of 1·7.

The idea of using the critical stress as a basis for the allowable stress might possibly find pplication in other forms of elastic-plastic stability problems where instability caused ollapse.

He had looked at the case of simple plates under compression, first for a long plate with wo simply supported sides, for instance, the flange of a box girder; the proposed cut-off t $d/t = 50$ corresponded to a critical stress of about 20 tons/sq. in. from the Timoshenko ormula. For a long plate with one end free and one simply supported—the flange of an -girder—the proposed cut-off at $d/t = 16$ corresponded to a critical stress of about 24 ons/sq. in. The proposed beam formula had a cut-off at $l/r = 60$ for deep girders, when he critical stress was 48 tons/sq. in. At that cut-off point the theoretical load factor was bout 1·6. On that basis it appeared that the load factors for the plates would be 1·4 or ·5. Would the Authors comment?

Full allowable stress (9·0 tons/sq. in.) could be used in a girder whose critical stress C was 8 tons/sq. in. or more. The suggestion for girders with elastic supports on p. 411 was that he stiffness of the supports needed only to be sufficient to give a critical stress of 48 ons/sq. in. That condition was given approximately by $\delta \leqslant 1{\cdot}2\ B/TS$ in/ton.

There was penalty for top flange loading. Why should there not be a compensation or bottom flange loading by a decrease in the effective length?

Dr J. C. Chapman (Research Engineer, British Shipbuilding Research Association) onfined his comments to the question of web deflexion. He was not convinced that suffiient attention had been paid to the influence of initial distortion of the web. In welded irders particularly that deflexion might easily be of the order of the plate thickness, and n that event the surface bending stresses might be very appreciable. It was stated in

the first Paper that "At loads in excess of the critical value, the buckled form is similar to that predicted by the theory of initially flat plates and the theoretical analysis in this range is therefore directly applicable to practical cases", and that seemed to be the guiding principle for the proposed basis of web design. It was true that when the applied shear was considerably in excess of the critical value, the web deflexion was largely independent of the initial distortion, provided that the initial form of the plate was similar to the preferred mode of the plate after buckling; but what happened if the applied shear was, say, about 0·9 of the critical? Fig. 69 showed approximately the growth of deflexion with applied

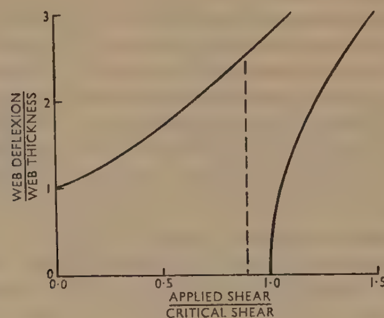

Fig. 69.—Growth of deflexion with applied shear

shear for a plate which was initially flat and for a plate which had an initial deflexion equal to the plate thickness. It could be seen that in the latter case the deflexion was very appreciable even for values of shear much less than the critical. It followed that the surface bending stresses and the membrane stresses in the plate would also be appreciable, and he would quote a number of cases where failure of a web had occurred through local bending long before the critical shear or moment had been reached.

It seemed then that the initial web deflexion should be taken into account in the design rules, and perhaps the first thing to do would be to establish what would be a reasonable value of initial deflexion to accept (say, for example, a distance equal to the plate thickness). Such a deflexion could then be incorporated in the theoretical treatment on the assumption that it occurred in the same mode as the buckled plate (any other mode tended to reduce the amplitude of buckling). It should then be stipulated in the Code that the initial plate deflexion should not exceed that chosen amount.

It sometimes happened that the initial mode differed so much from the preferred mode that it was not possible for the plate to "tune in" under load to the initial form. Then when the load reached a certain value there might be a sudden transition from the initial mode to the preferred mode of the plating. That transition might be accompanied by a very audible bang, and whilst it might not result in yielding of the plating, it would nevertheless tend to damage the morale of bridge users. That was an additional reason why the initial deflexion should not be too large.

The proposed basis for theoretical flange design was expounded very thoroughly in the Appendices to the first Paper. Was it possible to add a similar Appendix giving the proposed theoretical basis for web design?

It seemed that horizontal stiffeners were the latest fashion in plate girder design. Two or three years ago Dr Chapman had noticed a disused plate-girder bridge spanning a canal in Essex. Although about 40 years old and nearly rusted away that bridge had had horizontal stiffeners.

Mr F. M. Easton (Assistant, Steelwork Design, British Railways, Western Region) ked for more information on the subject of the fatigue allowances prescribed in the aft British Standard. He asked for it because Mr Gardner had said that if the meeting proved the proposals, earlier publication might be possible.

The reduction factors in permissible stresses given in Table 2 of the draft British tandard implied rather onerous increases in section, particularly for welded construction, nd although Mr Easton was certainly not in a position to say that they were unacceptable in any way wrong, he was sure that many others who had to make use of the reductions design would appreciate some statement explaining the basis upon which the reductions ad been deduced.

*** **Mr J. S. Terrington** (Plant Engineering Division, British Iron and Steel Re-arch Association) remarked that in the proposed clauses only the simplified equation r permissible flange stress was specified to be used, and that was essentially applicable the ordinary "I" or rolled-steel-joist type of section. If the theoretical equations had so been made mandatory it would have enabled calculations to be made to cover a wider nge of sections such as plate girders with edge-stiffened flanges. The simplified equa-on was obviously not strictly applicable to sections with heavily stiffened flanges and yet at equation appeared to have been used for sections with stiffened flanges in Table 2. would seem that whilst the simplified equation gave a true idea of the permissible stress r a section which had simple "plate" type flanges it offered an inadequate conception of e reserve of strength afforded by other forms of transverse section, e.g., those ith stiffened flanges.

Turning to webs, Mr Terrington said that reference had been made to the limiting mem-rane stress approaching the yield stress. Did the proposals allow for direct loads such as wheel load also being imposed directly on a web-panel so loaded? Would not the per-issible stresses in such a case, in all probability, be exceeded?

Professor G. G. Meyerhof (Head, Department of Civil Engineering, Nova Scotia echnical College, Halifax, Nova Scotia, Canada) observed that the proposed methods of esign were based on the theory of elasticity with a limiting stress equal to the yield ress. It should be noted, however, that critical flange and web stresses were in the astic buckling range only if the stresses were below the elastic limit (or limit of pro-ortionality) in bending or shear. If the stresses were above the elastic limit, buckling ccurred in the inelastic range and the corresponding critical stresses were smaller than e yield stresses which were approached only for stocky beams and thick webs.

Thus, in the case of flange stresses of beams and girders subject to lateral instability, eoretical analysis[44] showed that for mild steel to B.S. 15 with an elastic limit of the rder of two-thirds of the yield stress the critical stresses, in the absence of strain-hardening nd imperfections, might be up to about 20% less than the yield stress. Professor Meyer-of felt therefore that that reduction of critical stresses due to inelastic buckling was the ain reason why the observed collapse load of short laterally unsupported beams was equently lower than the load corresponding to the yield stress. In order to retain e simple approximate approach of a constant permissible flange stress for stocky beams ithout unduly reducing the margin of safety in the inelastic buckling range, Professor leyerhof fully agreed with the Authors that the various buckling load factors of safety the draft B.S. 153 should be increased to those proposed for tension and compression. or the same reason it was also desirable to increase the proposed load factors of safety n the yield stress in shear governing the permissible web stresses. Moreover, in that ay approximately the same factors of safety were provided for bending, direct, and earing stresses, which led to a more rational basis for design.

*** This and the following contributions were submitted in writing after the closure f the oral discussion.—Sec.

Professor George Winter (Head of the Department of Structural Engineering, Cornell University) was pleased that a paper on lateral buckling of girders [4] which he wrote in 1941 had been utilized for providing the basis for a small part of the pertinent revised B.S. 153 provisions, and that another [18] of 1944 had proved to be of some heuristic value inasmuch as it seemed to have introduced for the first time the devices of initial imperfection and of effective length into the analysis of lateral buckling of unbraced slender beams. He was equally pleased to note that the proposed provisions for designing deep thin webs represented a systematic utilization of the post-buckling strength of thin steel plates. The Specification and Manual for the Design of Light-Gage Steel Structural Members, American Iron and Steel Institute, for whose formulation Professor Winter had had a measure of responsibility, made extensive use of that post-buckling strength. Indeed, it could be said that the economical use of light cold-formed sections would be severely limited if their design were based on theoretical buckling stresses. There was no basic difference between a thin cold-formed section and the beam and girder sections of conventional construction, since what mattered in plate buckling was the ratio of width to thickness rather than the absolute value of either. It was for that reason that Professor Winter had suggested that post-buckling strength also be utilized for the usual type of steel structures wherever appropriate, and for deep stiffened webs in particular.[45]

The treatment of web buckling in Structural Paper No. 48 was extremely condensed and the omission of cross-referencing with the pertinent appended bibliography made it difficult to follow the development in detail. The determination of design stresses based on relating the maximum stresses in the slightly buckled web to the Hüber–von Mises–Hencky yield condition was very ingenious. Since those maximum stresses depended on the details of the configuration of the buckled plate, which in turn depended on the ratio of nominal to critical shear stress and to some extent on the configuration of initial deviation from flatness, it might be desirable for the Authors to elaborate on that feature, either by supplying details not given in the Paper, or by appropriate reference to the source material.

Since the experimental evidence presented in support of the Authors' approach (which was novel to Professor Winter) appeared to be somewhat limited, it would be desirable if correlation were established with the results of the extensive and important experimental and analytical investigations on strength of deep girder webs published during the past 5 years in a series of papers by Professor Charles Massonnet of the University of Liege, Belgium.[46]

With regard to lateral buckling, Professor Winter was pleased to note the basic superiority of the proposed provisions as compared with any of the current stipulations, which were all based on one single non-dimensional parameter, such as L/B, L/r_y, or LD/BT. The lateral buckling strength of flanged beams consisted of two fundamentally different and additive components supplied by the resistance of the flanges to bending in their own plane on the one hand, and to the contribution of the shear stresses to the torsional resistance on the other. As indicated by the Authors, the second term under the square root of equation (2) represented the latter contribution; the first term referred to the flange bending. In a previous discussion[47] Professor Winter had emphasized the fairly obvious fact that there was no conceivable way of combining those two additive components into a single non-dimensional parameter and that, therefore, all single parameter formulae could be reliable only within certain limits of relative cross-sectional dimensions. Thus, formulae of the L/B or L/r_y type neglected the contribution of the torsional rigidity, whilst those of the LD/BT type neglected that of the flange rigidities.

The formulae of the proposed revision of B.S. 153, all of which contained the required two additive terms under the square root, were therefore basically superior to any of the previously used requirements. With that significant progress achieved, Professor Winter regretted to see part of the gained accuracy sacrificed by the introduction of the simplifications leading to equation (9). They had undoubtedly been made for the sake of curtailing numerical work in design. They resulted in formulae which depended on two non-dimensional parameters, L/r_y and T/D. Inspection of equation (2) revealed that the accu

ate expression also depended on two non-dimensional parameters, namely $KL^2/I_y h^2$ and $_y h/Z_x L^2$. In those parameters, all pertinent properties were tabulated for rolled sections, r in any event had to be computed for fabricated sections, except for the torsional constant . Since that constant was useful in many other connexions, it should be listed in hand-ooks along with other sectional properties. Until that was done, simple approximate rmulae could be given for K for rolled shapes as well as for fabricated girders. In that anner, if the accurate formulae were used, numerical work in design would hardly be creased whilst accuracy would be much improved, for the normal shapes as well as for pecial ones. As an example of the latter, a girder made of flat web welded to two flanges, ach of which consisted of a channel whose web was at right angles to the girder web, was articularly favourable in regard to lateral strength. The approximate formulae of the ype of equation (9) would greatly underestimate the buckling strength of such a girder, hilst those of the type of equation (2) would give a reliable answer.

It was gratifying to see that in the proposed Clause Z the conditions were specified which ade a bracing system fully effective. It was Professor Winter's impression that, at least the United States, large quantities of steel were wasted by applying a stress reduction r lateral buckling in cases in which other parts of the structure, such as the floor or roof pported by the particular girder, provided fully adequate lateral bracing without the ecessity for either special bracing members or for reduced design stresses. In fact he elieved that, with the exception of certain through bridges, crane runway girders, or eams supporting unusually light roofing not capable of developing diaphragm action, ost beams and girders in finished structures were laterally effectively braced and could e designed to the full unreduced permissible stress. (That appeared to be true at least elastic design; limit loads as defined in plastic design seemed to be very sensitive to lastic lateral buckling even over very short lengths.) With the stated exceptions, the rimary danger of lateral buckling seemed to occur in the erection stage when members ere temporarily unbraced. If that opinion were shared, a clearer formulation might sult if, to the heading of Clause W, the phrase "completely or partially unbraced" ere added. Lacking such phraseology Professor Winter would assume that reduced resses according to Clause W would be widely applied in cases where the relatively mplex niceties of Clause Z would show that actually no stress reduction was necessary.

With regard to lateral bracing it might be added that specifying the required rigidity bracing might not be quite sufficient. It was desirable, in addition, to specify the neces-ry strength of the bracing members and their connexions. That could be done in a asonably rational manner by assuming realistic degrees of imperfection and, based on em, by calculating the actual displacements and corresponding forces in the bracing at cipient failure of the girder. Such an approach, for struts and stanchions, had been ven by Professor Winter [48] and had been incorporated in the previously quoted American sign methods for light-gauge (cold formed) steel structures. The experimental part of at paper described a number of cases where a few cardboard strips $\frac{3}{4}$ in. wide and of all thickness (10-ply) were sufficient in rigidity and strength to provide full lateral pport for the cold-formed steel studs being tested. That might serve merely as an ustration of the small amount of lateral support actually required for effective bracing. r Wm Zuk, one of Professor Winter's former students, in his doctoral dissertation had panded the analytical approach of the previous paper and had applied it not only to ruts and stanchions but also to beams and girders.[49] His general conclusion as to the all magnitude of the required bracing strengths would tend to support the suggestion at stress reductions should be made only when bracing was non-existent or demonstrably adequate.

Mr J. McHardy Young (Project Designer, Cleveland Bridge and Engineering Co. d), referring to the section of the Paper dealing with permissible web stresses, was sur-ised to find that no reference was made to the work of Dr Rockey (University College Swansea) or to that of Professor Massonnet of the University of Liège.

The proposed web stresses appeared to be reasonable. In the case of combined stresses the Hüber–von Mises–Hencky theory was correctly applied and the relation between shear and bending stresses shown in Fig. 33 could be accepted.

The recommendation to base the flexural rigidity of intermediate stiffeners upon that of the panels which they supported was a distinct advance but the formula $I = 1{\cdot}5\ b^3t^3/d^2$ appeared to give rather high values. Professor Massonnet [50] had given a value of $0{\cdot}9\ b^3t^3/d^2$ and it had been shown from Dr Rockey's work that a reduction in the value of γ to 50% of γ_L caused a reduction of 10% only in the permissible web stresses.[51] Whilst the Authors argued that stiffeners were a very small proportion of the total weight of the girder, it was doubtful whether that was so in the case of deep girders.

The proposals for horizontal stiffeners required amplification to cover the case of stiffeners spaced at various depths. It could be agreed that the values given in DIN 4114 were low and Professor Massonnet had suggested that the theoretical optimum rigidity be increased by from 3 to 7 for varying positions.[52] One point which the Paper had failed to bring out was that a horizontally stiffened panel was equivalent to a horizontally unstiffened panel of a greater (equivalent) thickness, and therefore the rigidity of the vertical stiffeners should be increased accordingly. It was to be hoped that that omission would be rectified before the draft proposals were put into their final form for publication.

Mr J. E. Spindel (a Senior Engineering Assistant (Bridges), British Railways, Southern Region) observed that the Paper showed clearly how soundly and elegantly the proposed draft standard dealt with the stability of the wholly unsupported girder and the effective length of girders supported by U-frames, but it dismissed the design of those frames very lightly.

If the support given to the compression flange were the only consideration, the load on the frames should be determined by the deflexion of the compression flange carrying its permissible load multiplied by the load factor and the stiffness of the frame $(1/\delta)$.

Unfortunately, however, U-frames were a mixed blessing unless symmetrically placed and loaded either side of the girder whose flange they were to support. The load on the cross-girders which formed part of those frames caused their ends to rotate. That movement—or rather the forces tending to restrain it—bent the top flange into a shape which might even be similar to that to which it tended to buckle. The extent of that distortion and the stresses due to it depended on the geometry of the bridge, the restraint imposed by the bearings against top-flange movement, and the relative stiffness of flange and U-frame in the calculation of which account had to be taken of the almost inevitable flexibility of connexions.

The magnitude of those stresses and the extent to which they could be relieved by changes in the construction of the bridge were being investigated. The results of that and other work on the behaviour of structures could be applied to design quite easily as soon as they became available if only standards would lay down clearly such fundamental data as load factors and initial imperfection allowances.

Another aspect of the forces and movements at the ends of cross-girders was their effect on the design of stiffeners. Given conventional connexions and main girder bearings, restraining forces produced appreciable moments on the stiffeners near the bearings and some guidance on allowances for that would be helpful. In the past, a number of cross-girders had been connected to webs between stiffeners without apparent ill effect. Since webs were comparatively flexible they would presumably deflect out of their plane sufficiently to allow the cross-girder ends to rotate as though they were simply supported. Those deflexions were of the same order as the allowable buckles discussed in the Paper and it might be worth investigating whether they were equally harmless. If they were, it would be possible to economize in the construction of through-type bridges with shallow decks where cross-girders were at very close centres by omitting stiffeners at some of the connexions.

Mr F. A. Partridge (Design Consultant, Braithwaite & Co., Engineers, Ltd) emarked that the Authors' brilliant treatment of the many complexities of the girder roblem was unfortunately slightly marred by the surprisingly irregular—and in some ases seriously large—disagreements between theory and fact as exemplified by the figures the "Max. W" columns of Table 15. To mention the worst cases in test No. 2P the stimated critical load was given as nearly twice the actual failure load, and in test No. J as little more than half. The Authors could hardly claim from those figures that heory had been experimentally verified.

But how should those critical loads have been estimated—from equation (2) or equation)? Equation (2) was an "exact" equation for the "perfect" girder, and one would herefore expect it to give estimated critical loads greater in all cases than the actual ilure loads. Equation (9), the basis of the proposed design clauses in the new B.S. 153, pplied also to the "perfect" girder. It was based on equation (2) but introduced "cerin approximate geometric properties" which provided "a lower limit to the critical ress". Compared with actual failure loads, the critical loads estimated by equation (9) hould therefore be lower to a degree dependent on the approximations and higher to the xtent of the imperfections in the girders tested. In fact, equation (9) gave a set of gures for critical load quite different from those given in the Table but much more reasonble in comparison with the test loads. There was obviously something wrong with the bulated figures, and no doubt the Authors would be able to put it right. In passing it ight be noted that the errors, whatever they were, were not carried forward into the olumn for "Flange stress—Proposed allowable", all figures of which, with the exception those for Test 2N, were substantially correct. Perhaps, after all, it might be more seful to give the estimated limiting loads for the imperfect girders, since it was those ads divided by the load factor which gave the permissible working loads. Comparison ith actual failure loads would then demonstrate the adequacy of the design proposals.

The Authors' argument for the adoption of a load factor of 1·7 for combination (a) was tional. They had made it clear that if all the deficiencies such as poor quality material, accurate calculation, and faulty fabrication and erection were to be present together in a ridge structure, and if that structure were to be overloaded by as much as 20%, there ould still be—with a factor of safety of 1·7—a real margin of about 15% against failure. n overload of only 10% would leave the much more favourable margin of 30%. For ose bridge members whose limiting load was based on yield stress, i.e., for tension embers and short struts which were deemed to have failed when the point of permanent stortion had been reached, the margins against disaster were even greater. That might true also of the compression flanges of slender girders as treated in the Papers for which e limiting loads had been somewhat conservatively estimated. In combinations (b) d (c) the real margins of safety seemed to have been reduced to uncomfortably low nits, particularly in the case of battened struts.

A noteworthy feature of Paper No. 48, in both theoretical analysis and the design oposals, was the consideration given to unsymmetrical girders and curtailed flange ates and to channels, tees, and angles. The consideration of such details hitherto had ten been a matter of doubtful judgement, if not guesswork. The case for the unsymetrical girder was convincing. It was pleasing to note that, though the proposed rules design might be more complicated than the old, they did appear to give what the thors had set out to find—a means of designing better and lighter girders. In that spect the Authors had been more successful than the Steel Structures Research Comttee who had attempted to improve the design of multi-storey building frames. There s no doubt that the application of the proposed rules in practice would involve a little ore effort than was indicated by the nine examples set out so simply in the Paper, but at should be no stumbling block to the capable designer.

The Authors, in reply, thanked all the contributors, whose comments would unubtedly be of great assistance to the committee concerned with revising the B.S. 153.

They would also like to express their appreciation of Mr Gardner's enthusiasm, out o which had arisen the proposals outlined in the Papers. There could be few precedent for research being carried out with the prospects of such immediate application and it wa hoped that the co-operation between designer and research engineer would produce revised design basis both easy to use and accurate.

Mr Gardner had emphasized the aims and difficulties of the Committee and the Author agreed with him that whatever the scope of Standard rules there would always remain th need for the exercises of the engineer's judgement—in the present problems, more particu larly relating to degrees of restraint and imperfection. The Authors agreed that if a factors affecting design and construction could be evaluated the factor of safety shoul approach unity, but they believed that some provision for human error and imperfection of fabrication and material should always be made. That problem was discussed o p. 416 *et seq.*

The Authors reminded Mr Upstone that the derivation of highway loading had bee described by Henderson[53] and the railway loadings had been fully dealt with in Bridge Stress Committee report published in 1928, so that only the wind loadings migh need further explaining and presumably that was what Mr Upstone had in mind.

The Authors agreed with Mr Upstone that the design of symmetrical girders woul not be much more difficult than at present, but they could not agree that it would becom a mechanical process and were sure that no one need have any fears on that account.

It was not, of course, intended to imply that the addition of stiffeners to the web of plate girder would reduce its carrying capacity; the contrary was the case. The increas of permissible bending stresses in unstiffened girders was on account of the better shap factor of shallow sections and in the final text of the relevant clauses that would be mad clear by introducing limiting d/t-ratios.

There were various methods of allowing for local increase of stress caused by the intro duction of holes. The method proposed in Clause Y was simple and ensured that th average fibre stress in the horizontal portion of the flange would not exceed the appropriat permissible stress. It was doubtful if a more complicated procedure would be justifie for, in any case, the average stress in the element calculated on its net area was only conventional measure of the maximum intensity of stress near the holes. In arriving a the area of the flange, the vertical legs of the angles and the portion of web contained b them had been added to make sure that the stress in the angle leg was also kept within th maximum permissible. (That was particularly necessary in riveted girders without flang plates.)

With regard to the stiffeners, Mr Upstone was correct in saying that the required inerti of the stiffeners was such as to ensure the maximum allowable shear strength of the we plate for any given shape of the panel. It would be relatively easy to relate the require stiffness of the stiffener to the actual shear stress in the web, as well as to the dimension of the panel, but on the whole the development of the full strength of the web for a give stiffener spacing was thought to be preferable. It should be permissible, however, not t increase the inertia of the stiffeners from the minimum necessary if, for some reason, the were spaced closer together than was necessary for strength reasons alone, or if the we was made thicker than the minimum required.

The expression for the stability of the compression flange supported laterally by th U-frames was quite general and would cover the case of any number of girders provided δ the unit deflexion of the frame, was calculated correctly for the given system of U-frame and girders. The particular case of rectangular U-frames between two girders wa included in the British Standard as an illustration only. It should be appreciated tha the stability of the compression flange depended on the stiffness of the U-frames and tha the forces exerted by the flange on each individual frame were not known. They depende not only on the relative stiffness of the flange and frames, but also on the deformation o the frame itself under the action of external loading. The $1\frac{1}{4}\%$ cross-shear was intro duced only for designing the stiff corners of the U-frames and was derived empiricall as was the $2\frac{1}{2}\%$ cross-shear used for the design of lacing, laterals, etc. It might be possibl

o make some allowances for the spacing of the U-frames, but further research was required before any simple rule could be formulated with safety.

Mr Bridge had agreed that imperfections should be allowed for in the design of compression flanges of girders, but the Authors considered that the locked-up stresses resulting rom cold straightening of bent flanges and from riveting should in no way affect overall lastic stability, for those stress systems were self-equilibrating. Such initial treatment vould reduce the limiting load for a crooked girder, however, and might cause premature ollapse if the imperfection allowances were inadequate. The direction of buckling was lependent purely on the loading conditions and imperfections in geometry of a girder and vas not influenced by residual stresses.

The Authors agreed that eccentricities of loading might give rise to high flange stresses ut intended that small accidental torques should be covered by the proposed rules.

Mr Bridge's suggestion that curves giving critical stresses for different values of l/r_y nd D/T should be given instead of Tables could be adopted in every design office by lotting the values given in the Tables. It was generally more satisfactory in the specification to give the exact values rather than graphs which had to be scaled.

Whilst the Authors agreed that it might not always be advisable or possible to cut ections down to the minimum, they believed that any additional material must be added y the designer as circumstances demanded and should not affect the basic allowable tresses in a British Standard. Increasing thicknesses of parts in order to provide against ad maintenance was a doubtful remedy, but if adopted it should depend on the required ninimum thickness of the part, its vulnerability to corrosion, and its importance to the tructure.

In reply to Dr Rockey, it was true that for certain ratios of panel depth to breadth, the roposed rule for web stiffener design gave greater values for stiffener inertia than that uggested by him. However, both rules were based on experiment, and the agreement s indicated on Fig. 65 seemed to the Authors to be reasonably close. It should be borne n mind that the effective aspect ratios of most practical plate girders were between the imits of 0·75–1·5, i.e., in the region where the proposed rule gave a mean value of the wo proposed by Dr Rockey. Also the Authors understood that Dr Rockey's curves had een determined from tests on riveted aluminium girders with angle stiffeners. In that ase, double stiffeners would usually be used and the proposed rules and Dr Rockey's ormula would produce almost identical designs. The single-sided stiffeners were recommended only in welded construction with the toe of a section or the edge of a plate welded o the web. It would appear that such stiffeners had not been included in Dr Rockey's ests.

The main aim in deriving the stiffener rules had been to make them as simple as possible o use, whilst producing safe stiffeners within the limits specified for the plate depth/thickness ratios; the Authors thought that had been achieved and Dr Rockey's contribution went a long way in supporting it.

The Authors were grateful to Dr Rockey for discussing the influence of flange rigidity on the post-buckling behaviour of web plates. That problem, which would seldom exist with riveted forms of construction, might evidently become serious in welded girders as a result of the absence of flange angles. It would appear that the early onset of yield in he large test girders Nos 2 and 3 and in girder C4 was partly due to that effect. Both irders Nos 2 and 3 had I/b^3t of the order of 0·000025 whilst girder No. 4 had a ratio of bout twice that value; all of them were well within the range of increasing web distortion ndicated by Fig. 66. It was consequently curious that girder No. 4 behaved as predicted p to a shear load in excess of twice the critical value whereas girder No. 2 and, more specially, girder No. 3 showed much larger displacements and stresses than predicted t shear loads less than twice the critical. Whilst agreeing with Dr Rockey that flange lexibility was important, the Authors suggested that its influence depended to a considerable extent upon the initial web distortion, the results for girder No. 4 indicating that the urves of Fig. 66 gave excessive deflexions for webs initially flat. It was a pity that the urves had not been related to initial web shape.

It had been recently shown by Dr Rockey[54] from the results in Fig. 66 that the limits to be imposed on flange proportions might be empirically expressed by:

$$\frac{I}{b^3 t} \geqslant 0{\cdot}00035\left(\frac{W}{W_{cr}} - 1\right)$$

It had only to be decided what multiple of the critical loading was the maximum to be catered for. Taking $W/W_{cr} = 2$, that limit would require the stiffener spacing to be reduced to the order of a half that used in the early tests on the large girders. However, the value of flange I used ignored any contribution from the web and might be too small in consequence. It would seem reasonable to include some portion of the web in evaluating that flange stiffness. A simple rule would probably be introduced into the Standard either to restrict flange dimensions or to reduce the stiffener spacing.

The Authors confirmed Mr Long's remarks that the fabrication imperfections of the test girders made at Christchurch were relatively small. The girders had been excellently made and the M.E.X.E. craftsmen and engineers should be congratulated on their achievement. However, the plates from which the girders had been made were relatively thin and the quality of steel unfortunately had been far from perfect. Table 14 (p. 476) showed the rather considerable variation in the limit of proportionality and, as Mr Henderson had said, in girders Nos 1 and 2 that was greater than most people would probably expect.

Mr Long had implied that the allowances for imperfections proposed by the Authors were not big enough, whilst Mr Branscombe thought that they were too big. Perhaps after all, the proposed allowances were a fair mean.

The assumption of an effective imperfection in a girder or a column, had been misconstrued by several contributors who were clearly thinking in terms of geometrical imperfections alone, generally as a lack of straightness. As pointed out by Professor Meyerhof, material deficiences were of greater significance in the stockier beams and girders and the assumed value of η was required to cover the non-linear behaviour beyond the elastic limit. That had been made clear by Robertson in his original proposals for struts and more recently it had been shown that the Perry-Robertson relation provided a satisfactory empirical approximation to the tangent modulus curves for aluminium-alloy columns. The use of a tangent modulus approach was not feasible with steel to B.S. 15, where no elastic limit was specified and the stress/strain relation was unknown, and in any case it would not provide an explicit solution. In the test girders at Christchurch the materials had suffered from very low proportionality limits and tests had in most cases been stopped when those limits had been attained, collapse then being imminent. Professor Meyerhof's statement that that type of imperfection might reduce the critical stress by as much as 20% was in agreement with the Authors' estimates.

The specification of the torsional support required to girders both with and without load-bearing stiffeners had proved difficult to introduce and both Mr Long and Mr Henderson had commented on that question. It was agreed that the arbitrary limit to girder proportions requiring lateral support to the top flanges at bearings was unsatisfactory and that the simple parameter, d/t was not a sufficient guide. Theoretical work, backed by test data, had shown that the critical loading of a beam was reduced if the supports had inadequate rigidity against the torsional loading about the longitudinal beam axis. The solutions provided a basis for rational design of reaction bearing stiffeners. In addition to such stiffness requirements the supports had to be of adequate strength to carry the torque arising from loads displaced laterally, and an additional criterion might be deduced from the theoretical results for an imperfect beam under mid-span point loading.

A few tests carried out in America[55] had shown that the load-carrying capacity of unstiffened beams might be considerably impaired if top-flange restraint was not provided at the supports. It was considered that Mr Long's suggestion of increasing the effective lengths might be applied to that case, although the increase suggested appeared to be too arbitrary to apply over the entire range of sections.

Dr Horne had pointed out an error in equation (24) of Appendix IV and was corre

n assuming that the numerical coefficient should in fact be 4π when α and $\alpha\sqrt{\beta}$ were plotted as in Figs 8 and 9. The advantage of the triangular bending-moment distribution had been taken over the range of low values of ν, as seen from those figures. In practice n will seldom be lower than 1·5, however, and in consequence the basis for treating curtailed-flange girders loaded through their shear centres should prove conservative. Where the point load was applied to the top flange at mid-span, the critical value was, at worst, reduced by 40% in contrast to 31% obtained from the rules. In the case of a uniform girder that discrepancy was offset by the increase afforded by the triangular distribution and it was hoped that the curtailed-flange basis was conservative enough to cover what at most, amounted to a 10% drop in critical stress in deep girders. The commoner forms of curtailed-flange girders had low ratios of D/T which the reductions due to top flange loading were negligible. The use of an increased slenderness ratio to deal with top-flange loading did inherently cover variations in the L/D ratio for large D/T ratios and was otherwise conservative.

Both Dr Horne and Mr Allen had suggested that allowances be made for bottom-flange loading. Where unrestrained bottom-flange loading occurred it was agreed that in some cases the carrying capacity of a girder might be greatly improved, as illustrated by the curves plotted in Fig. 70. Those examples, plotted on the basis of Fig. 6 in the Paper,

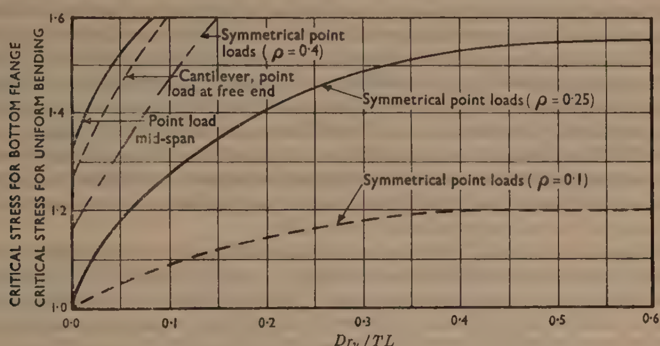

FIG. 70.—INFLUENCE OF BOTTOM-FLANGE LOADING ON STABILITY

showed that point loads, at lower flange level near mid-span or on cantilevers, had far higher critical values than expected from the uniform-bending basis. On the other hand with two symmetrical point loads approaching the supports ($\rho \rightarrow 0$) the increase in stability was slight. There was no envelope to those curves to enable a simple modification to be made to effective L/r_y as for top-flange loading, and with a view to treating the worst case it was felt that no simple allowance was possible in a Standard. In any case it was considered that that loading condition was extremely rare. In such instances as the through-bridge girders, the loading system introduced restraints necessitating the use of a modified basis of design.

Mr Henderson had rightly pointed out that symmetrical girders with curtailed flanges could be simply, if somewhat inefficiently, designed by taking the most pessimistic view of the effect of curtailment. It was only when one attempted to design the lightest possible girder with unequal flanges, unequally curtailed, that the process became rather laborious. The increase in labour was the price one had to pay for the wider scope and greater precision of the method.

The Authors would also underline Mr Henderson's remarks on end restraints by pointing out that the reduction of allowable bending stresses in slender girders as compared with B.S. 449 (1948) might often be offset by making proper allowances for the end conditions.

Mr Godfrey's statement that the permissible bending and shear stresses in mild steel for German railway and highway bridges were about to be increased from the present 8·9 and 5·3 tons/sq. in. respectively, to 10·2 and 5·86 tons/sq. in. respectively, was interesting. It showed that the German engineers were thinking along the same lines as the British, using a load factor of approximately 1·5 instead of 1·7. It should be noted however, that those permissible stresses were for pure bending and pure shear and that when they were combined, the German specification called for a check of maximum critical stress according to the Mises–Hencky equation. Mr Godfrey had since informed the Authors that that stress was limited to 11·5 tons/sq. in. so that, in fact, the actual allowable bending and shear stresses in German designs might, in many cases, be smaller than the British.

Mr Godfrey's suggestion that vertical and horizontal stiffeners should be described as transverse and longitudinal ones was a good one and should be noted by the British Standards Institution. His and other speakers' statements that there were more effective positions for the longitudinal stiffeners than the ones given in the Paper might sometimes be true, but the proposed locations were quite efficient and adequate in most normal cases. The labour of search for absolute efficiency would hardly be justified every time a girder was designed. Even when bending stresses were small and the shear stresses large, a slender web still tended to buckle in the compression zone. In the example given by Mr Godfrey a stiffener placed at one-fifth of the depth of a girder from the compression flange would make the web stable. It could not be denied that a longitudinal stiffener in the end panels placed at the neutral axis would also be satisfactory and slightly lighter, but it would be difficult to decide at which point one should change from one location to the other. The German and Belgian methods gave data for a large number of different types of stiffener arrangements and web stresses which Mr Godfrey called an absurdity, yet that seemed to be the only alternative to the less efficient and more limited rules now proposed, if the safety of the structure was not to be sacrified to the whim of the designer. In any case the required stiffness of the stiffener placed at about one-fifth of the depth of web from compression flange was greater than that for any stiffeners lower down and therefore could be adopted for all such stiffeners with safety.

The proposed values of I for the longitudinal stiffeners appeared from tests to be adequate for most adverse conditions of loading and Dr Rockey had confirmed that in the tests carried out at Swansea University. It should also be remembered that in the proposed method the longitudinal stiffeners were intended to act between the transverse ones and in all the tests they had, in fact, been discontinued at those stiffeners.

Mr Godfrey's description of methods of saving of steel adopted by the Germans in the designs of their post-war bridges was very valuable, but the Authors could assure Mr Godfrey that all those methods were well known to the British designers and had been adopted whenever it produced true economy. Battle-deck construction had been developed for the proposed Severn Bridge in 1947[35] before the new Rhine bridges using that type of deck had been built (it had also been extensively used in the U.S.A.); composite reinforced concrete and steel girder decks had been adopted in many bridges, such as Surat and Rama VI bridges in Thailand, Ganga Bridge in India, Auckland Harbour Bridge in New Zealand, and many others. High-tensile steel had been used in most large bridges since about 1930, and the Sydney Harbour Bridge had been designed in silicon steel in 1923. Of them, the great Howrah Bridge in Calcutta, the Storstrøm Bridge in Denmark, and the military Bailey bridges were perhaps the most striking examples. Although Robert Stephenson adopted wide box girder construction for his Britannia Bridge in 1850, that type of construction was seldom practicable, for it required either very powerful cranes for handling (as had been the case with the new Rhine Bridges) or a very considerable quantity of field jointing. Prestressed steelwork was a new development which might prove economic in certain special cases.

Mr Branscombe was quite correct in saying that the test girders had been unrealistic. They had been specially designed to produce most unfavourable conditions of web and flange instability combined with coexistent maximum bending and shear stresses, but the Authors could not agree that the tests had not been representative; span-to-depth ratios

from five upwards were quite possible; the physical imperfections due to fabrication had not been excessive, especially for welded work; the intermittent welds should not weaken statically loaded girders and the use of jigs was normal practice in many works and was not prohibited by any specification. It might perhaps be said that the girders represented a pessimistic view of what could be produced under proposed rules, but, after all, the tests had been devised to check those rules and the Authors submitted that for that purpose the pessimistic view was the correct one to take.

With regard to the examples given in Appendix II, the actual value of r_y had been used because those examples had been devised to illustrate the application of the rules. The use of actual width of the compression flange instead of r_y had been discussed on p. 402. Such use would be limited to fully symmetrical girders and would result in different formulae and tables for sections of different shapes.

Mr Branscombe and Mr Easton had asked for more information regarding the rules dealing with fatigue. That information was, of course, outside the scope of the Papers but the Authors could say that coefficients for reduction of stresses to allow for fatigue effects had been determined from the best available data (mostly German and American). They would remind the speakers that it was left to the engineer's judgement to estimate the number of critical stress cycles. If an optimistically realistic view was taken of that the proposed allowances for fatigue would be small. It would seem that provisions for fatigue were based on philosophical rather than factual arguments, for insufficient data were available on fatigue of bridges. It was, perhaps, fortunate for the designers that the fatigue effects did not become apparent until after very many years of service, so that few of them had an opportunity of seeing their own designs tested in that respect.

The treatment of through-bridge girders held by one or two intermediate U-frames was, of necessity, much simplified but conservative. In the case cited by Mr Branscombe of a single frame at mid-span the effective span would be halved if the frame stiffness was high but might be only slightly reduced if that stiffness was low. With rigid frames at quarter-span the proposed rules would give an effective span of $L/2$ which was slightly on the safe side.

Mr Branscombe had suggested that an increase should be allowed in working stress under uniformly distributed loading. It was agreed that a 13% increase in load-carrying capacity was obtained with such loading along the shear-centre axis and even greater increases might be expected with other distributions of load. When the loads were applied to the top flange, however, the critical-stress coefficient 1·13 dropped rapidly with Dr_y/TL, as seen from Fig. 6, and closely approximated to the lower envelope of all the curves over the practical range. Thus the proposed basis, with the increased effective length, provided very nearly the right solution for that distribution of load. The alternative, rejected by the Authors, had been to allow different increases in compressive stress for a number of load distributions and to provide for different reductions for top-flange loading in each case. The factor of 1·13 did not apply to through bridges but only where a girder was unrestrained between reactions. In the through bridge the discrepancy between the uniform bending solution and that for distributed loading was negligible when the half-wavelength of the buckled mode was much less than the span.

Mr Branscombe had also suggested that advantage might be taken of the lowered point of load application in cases of half-through bridges. It should be realized that the centre of rotation of the girder was at each cross-girder connexion and that in consequence the vertical eccentricity of the connexion relative to the girder shear-centre did not influence the overall stability in the same way as in the case of a load free to move with the girder. Lowering of the cross-girders might in fact be detrimental owing to the resulting drop in the U-frame stiffnesses.

Mr Allen had done a great deal of "backstage" work in connexion with the design proposals. He had been responsible for the majority of the numerical calculations and the Authors were very grateful to him.

Mr Allen's analysis of the American test results was very interesting and clearly showed that the existing B.S. rules were unsafe. When the conservative assumptions made in

assessing end restraints were considered it was satisfying to find such consistency among the load factors.

Mr Allen's deduction that the load factor against the initial buckling of plates in compression in the proposed B.S. 153 was about 1·5 was correct. That low factor had been adopted because buckled plates had further reserve of strength and also because the maximum allowable axial stress of 9 tons/sq. in. was hardly ever achieved in practical struts, which normally worked at stresses not exceeding about 8½ tons/sq. in., when the load factor would be about 1·6. The case of compression flanges of girders was significantly different because stresses of 9 and even 9·5 tons/sq. in. would be quite normal. The Authors therefore agreed with Mr Allen that the outstand of the compression flange plate should be kept small, say, not greater than 12 times the thickness of the plate, particularly in the case of welded girders which were liable to be initially distorted.

Dr Chapman had raised the question of initial deformation of the web plate, and had pointed out that, for web plates having an initial web deflexion of the order of the plate thickness, actual deflexion of the web was considerable, even below the critical value of shear stress and might extend up to twice its thickness around the critical stress. However, it was surely the increase in deflexion that mattered from the stress point of view (though not necessarily from an aesthetic one) and to obtain a true relationship concerning the stress increase, it was necessary to reduce the curve of actual web distortion shown on Fig. 69 by the plate thickness (see Fig. 71). The increases in combined stresses due to web buckling under shear for an initially deformed web (initial deflexion = web thickness) would, therefore, correspond to an initially plane plate stressed to somewhere in the region of 30–40% in excess of the critical value. Reference to Fig. 29 showed that to be of the order of 10% and, for plate with large initial buckle, Bergman[29] had shown that Fig. 29 became modified as shown in Fig. 72.

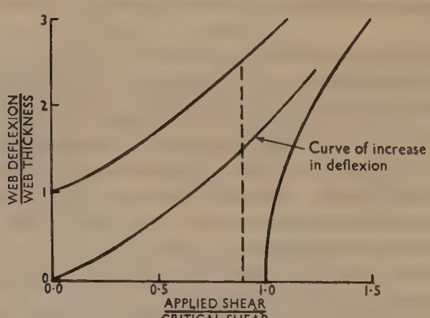

Fig. 71.—Growth of deflexion with applied shear

That would call for a small modification to Fig. 31, as shown in Fig. 73 in the region of d/t-ratios of 90 to 150 for all permitted stiffener spacings, with the result that the simplified straight-line relationship shown in Fig. 32 would be very close to the truth, for they lay below the theoretical values at the required d/t-ratios. It should be borne in mind, however, that although at the working stresses the overall deflexion for an initially badly buckled web plate might be considerable, and combined web stress higher than in an initially plane web subjected to the same shear stress, the increase in stress with increase in load would be small and therefore the required load factor would be maintained. It was interesting to note that in the region where influences of initial web deformations were greatest the proposed new rule gave stresses lower than those proposed by other specifications, e.g., B.S. 449 (1948), German DIN, and American A.R.E.A.

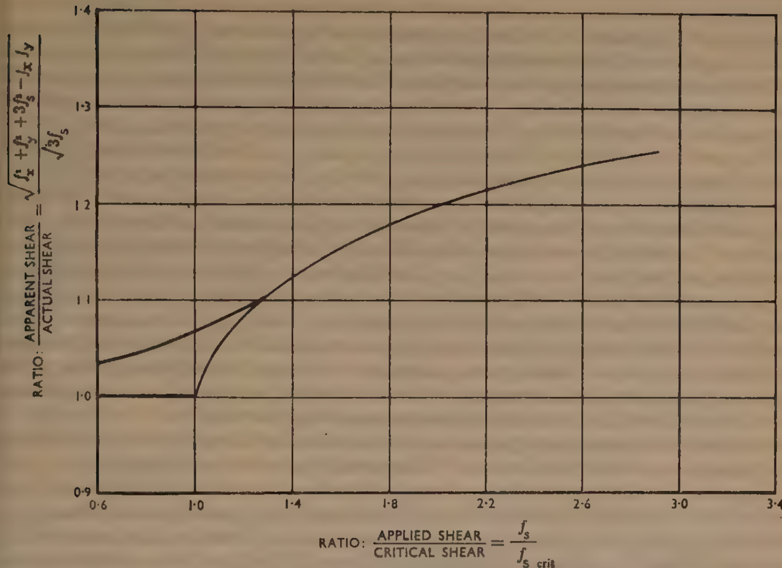

FIG. 72.—THE INCREASE OF APPARENT SHEAR STRESS (FAILURE CRITERION) WITH INCREASE OF RATIO OF APPLIED SHEAR TO CRITICAL SHEAR, MODIFIED FOR INITIALLY DEFELECTED WEB

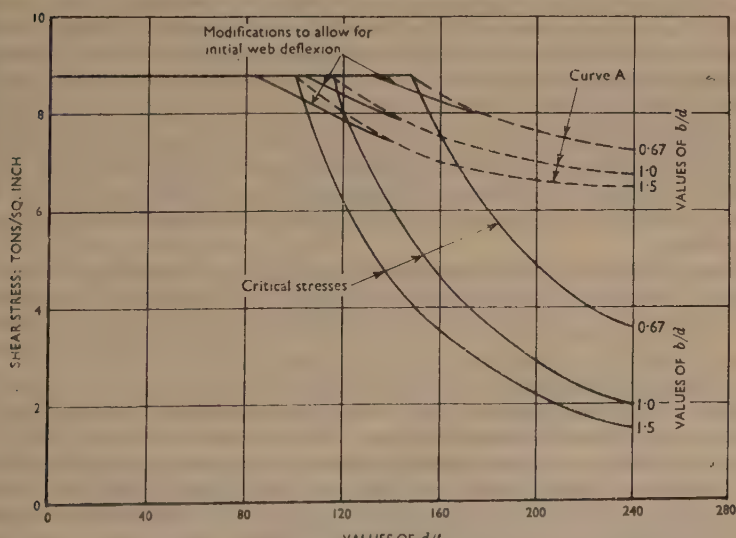

FIG. 73.—TIMOSHENKO'S CRITICAL SHEAR STRESSES AND THE CORRESPONDING SHEAR FAILURE STRESSES FOR DIFFERENT VALUES OF d/t AND b/d, MODIFIED FOR INITIALLY DEFLECTED WEB

Perhaps that added weight to the Authors' belief that the actual full-size girders contained proportionately smaller web irregularities than those found in the model girders.

In reply to Mr Terrington, although the simplified equation might not appear to be applicable to girders with flanges stiffened at the edges, in fact it was. That was explained in the Paper and illustrated by the examples in Table 2. Full advantage of the shape of the flange was taken in calculating r_y for the section, that being one of the reasons why r_y rather than the width of the flange was being used in calculating the values of C.

With regard to the design of webs subjected to loads, such as crane wheels, applied directly to the edge of the web, that had been provided for by introducing Clause X(c) given in Appendix I. If a high bearing stress existed in combination with maximum bending and shear stresses one or all of them had to be reduced to limit the resultant critical stress (calculated according to Mises–Hencky) immediately under the flange plate to 14 tons/sq. in. and that would also keep the maximum membrane stresses within safe limits. It should be remembered, however, that with rolling loads, maximum bending, bearing, and shear stresses would seldom occur together. In the case of continuous girders in which maximum bending and shear stresses were co-existent at the supports, the loads causing those stresses were usually some distance away from the supports, whilst in the case of simply supported beams maximum bearing stresses were usually co-existent with either maximum bending or maximum shear stresses, but not both.

The Authors were particularly pleased to receive a written contribution from Professor Winter upon whose earlier work some of their present proposals were based.

As requested by him, appropriate references had now been added in the text of the Paper (p. 429) to the works of Bergman and Wästlund which gave the whole theory and the experimental supporting data for the proposed method of web and stiffener design. They regretted that it had been found impossible in the time available to correlate the experimental results of tests carried out by Professor Massonnet at Liège University with the proposed design rules, but in general they believed that the allowable shear stresses were similar to those advocated by Professor Massonnet, whilst the proposed stiffness of the vertical stiffeners was somewhat higher than that suggested by him.

Professor Winter's remarks concerning lateral bracing to compression flanges agreed with the Author's views on the basis to be adopted for bracing design. Although difficult to express in a simple form, it was possible to specify the loading to be carried by restraining and supporting members by taking account of girder imperfections and that method was being adopted in the case of reaction bearing stiffeners. The simple loading of $2\frac{1}{2}\%$ of the compressive flange load served as a safe basis for the design of intermediate bracing.

The valuable Papers by Mr Young, Dr Rockey, and Professor Massonnet on the design of web plates and stiffeners had not been included in the bibliography because they had not actually been used by the Authors in the development of the proposed method. The appropriate references have now been added (on p. 521) for the benefit of future students of the problem.

Mr Young had given the value of the required inertia of the vertical stiffeners as derived by Professor Massonnet. It agreed closely with that derived by Moore[32] which, in the present proposals, had been arbitrarily increased by about 50% to provide a factor of safety. That somewhat conservative treatment had been considered necessary because of lack of practical experience with the new type of stiffener.

The Authors felt that there was insufficient evidence to corroborate Mr Young's hypothesis that a horizontally stiffened panel of web was equivalent to a horizontally unstiffened one of a greater equivalent thickness. In all the tests on the deep girders with horizontal stiffeners, the vertical stiffeners designed in accordance with the present proposals had shown no sign of distress until the predicted collapse load had been reached. Moreover Dr Rockey's tests also had not indicated any need for modification of the design of vertical stiffeners on account of the horizontal stiffeners.

In reply to Mr Spindel, it had been realized that the cross-girder loading in through bridges caused deformation of the compression chord. In the majority of cases, however, the half-wavelength of the buckled mode was small compared to the span and hence the

"initial" deflexions would have but slight influence on the collapse loading. Furthermore the "live" end moments at the cross-girder connexions existed only so long as the slope at the ends of those transverse members was less than for simply supported conditions. Thus once inward buckling of the compression chord took place, the chord would deflect laterally and the cross-members would first suffer reduced end moments and finally would again restrain the top flange. It would be seen, therefore that the effect of the deck loading could generally be ignored or treated on the basis of a strut in an elastic medium under lateral load.

The flexibility of connexions might seldom be calculated on any theoretical basis but had to be deduced from experimental results. Tests on typical connexions at present being carried out in conjunction with British Railways might indicate whether such flexibility was negligible compared to that of the remainder of a U-frame.

The design of cross-girder connexions in the vicinity of the bearings was not a matter that could be easily treated in a standard and it was felt that the common sense of the designer would cause him to base his design of the attainment of the full fixed-end moment in such a region, provided that did not imply overloading of the bearings. The omission of vertical web stiffeners by connecting cross-girders directly to the web might show economies in some instances. Although the web plate would be locally distorted it should be remembered that it would be subjected to both longitudinal and transverse tensile stresses around the junction and hence the flexural deformations were of slight importance. There might be danger of fatigue fractures in that region, however, unless some limit were placed on the peak principal tensile stress.

The comparison between the test results and the proposed design basis discussed by Mr Partridge had not perhaps been accorded sufficient space in the Paper. Referring to Table 15, facing p. 486, it should be made clear that the "estimated critical" values had been based on the proposed rules existing at the time of planning of the tests and were intended as a guide in the testing programme. Those estimates had ignored such items as the effect of heavy T-type end stiffeners, vertical eccentricity of loading, and the effects of restrained loading linkages, and the figures quoted, therefore, although based on equation (9), did not allow for the additional factors mentioned. Thus, for example, in the deeper girders made of high-quality material and almost free from imperfection the observed maximum loads were sometimes higher than the predicted critical.

The columns containing flange stresses, on the other hand, were based on the proposals set out in the Paper and showed the load factor that had been obtained in the test girders, which demonstrated the adequacy of the design rules, as requested by Mr Partridge. The fact that the rules included conservative assumptions concerning end fixity and the effect of stiffeners gave rise to high load factors in some instances. Moreover where imperfections of all kinds were small, as in girder No. 4, the design basis would underestimate the collapse load which would tend towards the critical value. The Authors were unable to locate an error in the case of Test 2N.

The effective values of L/r_y chosen in the instances of girders Nos 1 and 2 were those obtainable by direct application of the proposals contained in the Paper. For example, in girder No. 1, Test T, the effective length was given by L/r_y times 1·2 (top-flange loading) times $= 0{\cdot}85$ (welded end T stiffeners). In girder No. 2, Test S, $l/r_y = (1 - 2\rho)L/r_y$ times 0·85 (partial fixity at load points), no allowance being necessary for top-flange loading when the girder was held at the load points. When treating girders Nos 3 and 4 some account had to be taken of the effect of the restraint afforded by the load linkage and that had been allowed for by arbitrarily reducing the effective length based on top-flange loading. By using the basis of Appendix IV, however, and the curves of Fig. 70 it could be shown that in both those girders the effective vertical eccentricity was negative (downwards) and that the effective length for the given values of L should be multiplied by 0·85 as well as by the factors defining fixity. Using that factor for all cases where the flanges were unrestrained at the load points, Table 15 and the text had been revised to show a more realistic comparison between proposed and observed limiting stresses.

As a comparison between the theory on which the proposals were based and the test

results, the critical and limiting loads had been estimated for several cases by means of an energy solution. That analysis allowed for all the features of the loading system and included the effects of bearing stiffeners and end fixity in direction. The theoretical and test values of the mean flange stresses at collapse were shown in Table 20. In all cases the theoretical and actual limiting stresses were of the same order.

TABLE 20.—COMPARISON BETWEEN THEORETICAL LIMITING FLANGE STRESSES AND TEST RESULTS

Girder	Test	ρ	Mean flange stresses: tons/sq. in.		
			Estimated critical	Estimated limiting	Actual limiting
1	T	0·5	12·7	8·5	8·3
	W	0·26	11·3	7·8	9·9
	Z	0·26	15·6 yield	13·0	12·3
2	N	0·33	7·1	5·6	5·8
	S	0·17	17·8 yield	12·8	11·0
3	J	0·22	16·8	9·7	10·7
	M	0·22	16·8 yield	13·8	13·6
4	A	0·18	10·8	8·2	8·5
	E	0·27	8·8	7·0	9·1
	G	0·27	16·3 yield	14·1	14·8

With regard to the tests described in the second Paper, the Authors of that Paper were satisfied that the tests had been carried out as well as they might have been. Looking at the results, there were lots of things that they would like to have done had they known about them before they started. However, that was usually the case with original research.

Dr Horne had mentioned the load factor. In fact, there seemed to be a considerable measure of agreement among the contributors that the mean shear stress should be reduced from 6 tons/sq. in. to the value suggested by Dr Horne, 5½ tons/sq. in. That would lead to a load factor against collapse due to shear of 1·6, compared with 1·85 in bending.

Mr Henderson had asked whether a lower load factor could be justified because of failure by shear. It was thought it could, in that in all the tests that they had done collapse was not nearly so catastrophic if it occurred purely by lozenging of the web.

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Correspondence on this Paper is now closed.—SEC.

Paper No. 6108

FLOW OF FLUIDS IN CONDUITS AND OPEN CHANNELS

by

* Edwin Samuel Crump, C.I.E.

(Ordered by the Council to be published with written discussion)

SYNOPSIS

There is a fundamental difference between the approach of engineers and of physicists to the problem of flow of fluids. Originally, engineers were concerned only with flow of water in open channels. After d'Arcy and Bazin had demonstrated conclusively the effect of boundary roughness, engineers sought to express the mean velocity V in terms of the hydraulic radius $r = A/P$, the hydraulic gradient S, and some such factor as Kutter's n specifying the roughness of the boundary. Dimensional considerations were ignored completely and formulae of the general form $V = F(r, S, n)$ were purely empirical. Chezy's formula $V = C\sqrt{rS}$, derived from first principles, was accepted as a suitable framework into which experimental data could be fitted by expressing C as a function of some or all of the three variables r, S, and n. The formula of Ganguillet and Kutter so expressed long held the field until supplanted by the Manning or Strickler formula which claimed to give approximately the same results as that of Kutter. Later, when its limited application to what is now known as the stage of fully developed turbulence was realized, engineers advanced many so-called exponential formulae of the general form $V = Cr^{\alpha}S^{\beta}$ of which only those which satisfy the criterion $3\beta - \alpha = 1$ can claim to be dimensionally homogeneous.

In contrast to the engineers' empirical approach, the physicists regarded dimensional homogeneity as all-important. Reynolds first showed the physical significance of his dimensionless combination, now known as the Reynolds number, $R = VD/\nu$. The revolutionary experiments of Stanton and Pannell revealed that for a given roughness of the boundary, a unique relation connected the two dimensionless factors R and the so-called friction factor f (defined as $f = 2gsD/V^2$) which held good for all fluids whether liquid or gaseous. Working with smooth pipes, Prandtl and Von Karman established their smooth-pipe law $\frac{1}{\sqrt{f}} = 2 \log_{10}\frac{R\sqrt{f}}{2{\cdot}51}$. Later, Nikuradse, working with artificially roughened pipes, gave $\frac{1}{\sqrt{f}} = 2 \log_{10}\left(\frac{3{\cdot}7D}{K}\right)$ as the law for rough pipes. The gap that lay between smooth and rough conditions of flow was spanned by Colebrook and White with their universal transition formula:

$$\frac{1}{\sqrt{f}} = -2 \log_{10}\left[\frac{1}{3{\cdot}7} \cdot \frac{K}{D} + \frac{2{\cdot}51}{R\sqrt{f}}\right].$$

This formula is the last word in the physics of flow. Its validity in practice has been confirmed by much data collected by Lamont. By inserting the defined values of $f = 2gsD/V^2$ and $R = VD/\nu$ and generalizing by putting $r = D/4$ the transition formula can be expressed as:

$$V = -\sqrt{32rSg} \, . \log_{10}\left[\frac{K}{14{\cdot}8r} + \frac{1{\cdot}255\nu}{r\sqrt{32rSg}}\right]$$

which is acceptable to engineers and readily computable.

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* The Author is Senior Scientific Officer, Hydraulics Research Station, Department of Scientific and Industrial Research.

Empirical treatment

In devising formulae to give the flow of a fluid in closed conduits or open channels, engineers and physicists have approached the problem from basically different directions. In the early days of hydraulic science, engineers were concerned mainly with a single fluid—water—flowing in open channels. After d'Arcy and Bazin had conclusively demonstrated the effect of the roughness of the boundary on flow, engineers sought to express the mean velocity V, as a function of only three variables, namely, the hydraulic radius $r = A/P$, the hydraulic gradient S, and some factor such as Kutter's n specifying the roughness of the boundary. In presenting this relation in numerous forms, little or no attention was paid to dimensional considerations and, for the most part, flow formulae having the general form $V = F(r, S, n)$ were regarded as purely empirical. By hypothecating that the drag-force intensity (the τ of modern theory) was proportional to V^2 and equating the drag-force to the weight-component, Chezy deduced his famous formula:

$$V = C\sqrt{rS} \qquad \text{. . . . . (1)}$$

which was thereafter generally accepted as a suitable framework into which experimental data could be fitted by expressing the coefficient C in terms of some or all three variables R, S, and n, regardless of dimensional considerations. This procedure was adopted by Ganguillet and Kutter in presenting their rather cumbrous formula in which C was a function of all three variables. This formula continued in favour until superseded by the simpler Manning or Strickler formula:

$$V = \frac{1{\cdot}486}{n} r^{\frac{2}{3}} S^{\frac{1}{2}} \qquad \text{. . . . . (2)}$$

which, with n of the same value as in Kutter's formula, was claimed as giving results practically the same as the latter formula.

It is important to note that any formula, such as (2) above in which V is associated with $\sqrt{S}$, is thereby limited in its application to what is now known as the rough or fully developed turbulence stage of flow. Realizing or sensing this limitation, many engineers have advanced so-called exponential formulae of the more general type:

$$V = C\,.\,r^{\alpha}S^{\beta} \qquad \text{. . . . . (3)}$$

which may be subdivided into two distinct classes. It will be shown later that for (3) to be dimensionally homogeneous the necessary and sufficient condition is that the two indices α and β are interconnected by the criterion:

$$3\beta - \alpha = 1 \qquad \text{. . . . . (4)}$$

and it should be realized that a formula which fails to satisfy this criterion can only be regarded as purely empirical. In the light of modern theory this distinction between the two classes is, therefore, of great importance.

Generalized formulae: $R=A/P$ independent of A

Exponential formulae of type (3) have almost invariably been based on data obtained from pipes of circular section running full-bore. Further research might well be undertaken to ascertain whether formulae so derived hold good for a circular pipe running part-bore, or for conduits other than circular whether running full- or part-bore. Pending such investigation it is generally accepted that for the same values of S and $r = A/P$, V will have the same value regardless of the shape and magnitude of the separate factors A and P. Acceptance of this implies that any

formula expressed in terms of the diameter D of a circular pipe running full-bore can be converted for general use merely by substituting $4r$ for D wherever the latter occurs.

Scientific treatment: laws for smooth and rough boundaries

Turning now to the more scientific approach of the physicists, the laws evolved by them are based exclusively on full-bore flow in circular pipes of diameter D. In sharp contrast to the engineers' approach, the physicists regarded dimensional considerations as all-important. Reynolds was the first to show the vital significance of his dimensionless number, now known as the Reynolds number, $R = VD\rho/\mu = VD/\nu$. Experiments by Stanton and Pannell showed that, for a given roughness of the boundary, there was a unique relation between R and the so-called friction factor, $f = 2gSD/V^2$ which was common to water and air and—by inference—to all fluids whether liquid or gaseous. The factor f as so defined is clearly dimensionless. For a circular pipe running full-bore $r = D/4$ and the Chezy formula (1) becomes $V = \frac{1}{2}C\sqrt{DS}$. But from the definition of f it follows that $V = \sqrt{\frac{2g}{f}} \cdot \sqrt{DS}$ showing that the relation between Chezy's coefficient C and the friction factor f is:

$$C = \sqrt{\frac{8g}{f}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For flow in smooth pipes, Prandtl and Von Karman established by reasoning and experiment the smooth-pipe law:

$$\frac{1}{\sqrt{f}} = 2 \log_{10} \frac{R\sqrt{f}}{2{\cdot}51} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Later Nikuradse, working with artificially roughened pipes, gave:

$$\frac{1}{\sqrt{f}} = 2 \log_{10} \left(\frac{3{\cdot}7D}{K}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

as the law for rough pipes, K being a linear dimension, namely, the equivalent diameter of the grains used by him for roughening the boundary.

Colebrook-White transition formulae

The gap between these two limiting conditions of flow was filled by Colebrook and White in presenting their transition formula:

$$\frac{1}{\sqrt{f}} = -2 \log_{10}\left[\frac{1}{3{\cdot}7} \cdot \frac{K}{D} + \frac{2{\cdot}51}{R\sqrt{f}}\right] \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This formula, which gives a unique relation between the three dimensionless parameters R, f, and D/K, is of universal application. It embraces the two laws (6) and (7) which appear as limiting cases when either of the two terms inside the bracket becomes negligibly small compared with the other.

Lamont's investigation

In a recent Paper* Lamont presented the results of a mass of reliable data collected from many sources which confirm that the behaviour of pipes conforms

* P. A. Lamont, "A Review of Pipe-Friction Data and Formulae, with a Proposed Set of Exponential Formulae based on the Theory of Roughness". Proc. Instn Civ. Engrs, Pt III, vol. 3, p. 248 (Apr. 1954).

losely with formula (8) which he designated the "Theory of Roughness". He has resented the theory graphically in the usual manner by plotting f against R (both garithmically) for various selected constant values of D/K. The grid so formed is ne background to his charts and the basis of a group of rational exponential formulae roposed by him as an alternative to the nomographical chart shown in Fig. 14 of is Paper. Lamont's grid covers the whole range of flow from $R = 3 \times 10^3$ to $\times 10^8$. That all sewers lie within this range is clear because for a 4-in. pipe unning full-bore at a velocity of 2 ft/sec the value of R is about 3×10^4.

Dimensional homogeneity of exponential formulae

The various exponential formulae proposed by Lamont all represent tangents to is grid curves. The equation to such a tangent takes the general form:

$$\log f = \log A - n \log R$$

e.,

$$f = A \,.\, R^{-n} \qquad (9)$$

here A is a constant and n denotes the downward slope of the tangent. Putting $= \frac{2gDS}{V^2}$ and $R = VD/\nu$ in the above equation yields the exponential formula:

$$V = B \,.\, D^{\alpha} S^{\beta} \qquad (10)$$

here $B = \left(\frac{2g}{A\nu^n}\right)$; $\alpha = \frac{1+n}{2-n}$ and $\beta = \frac{1}{2-n}$ showing that Lamont's formulae atisfy the criterion:

$$3\beta - \alpha = 1 \qquad (4)$$

hich, as already stated, is merely the condition required to render the formula (10) imensionally homogeneous.

Transformed Colebrook-White formula

Reverting to formula (8) it may possibly have escaped the attention of Lamont nd others that in order to put the formula into a shape acceptable to the designing ngineer, it is only necessary to give f and R their defined values of $f = \frac{2gSD}{V^2}$ and $= \frac{VD}{\nu}$. This done, the formula may be written:

$$V = -\sqrt{8DSg} \,.\, \log_{10}\left[\frac{1}{3{\cdot}7} \cdot \frac{K}{D} + \frac{5{\cdot}02\nu}{D\sqrt{8DSg}}\right] \qquad (8a)$$

r, to make it more general by putting $D = 4r$,

$$V = -\sqrt{32rSg} \,.\, \log_{10}\left[\frac{K}{14{\cdot}8r} + \frac{1{\cdot}255\nu}{r\sqrt{32rSg}}\right] \qquad (8b)$$

n these two formulae the negative sign is required because the expression inside the quare bracket is always less than unity and its logarithm therefore is always negative.

The Author regards the Colebrook-White formula (8) as the last word in the hysics of flow, and would emphasize the importance of Lamont's valuable contribuon. Except with smooth pipes, the necessity to assign a suitable value to the oughness dimension K (in feet for British units) is inescapable. In this regard the alues recommended by Lamont for different surfaces are of great assistance to

engineers. In applying the Theory of Roughness to the solution of practical problems, whether to use Lamont's nomographic chart, his proposed exponential formula, or the Author's formula (8b), is a matter for each engineer to decide individually.

The Paper was received on 22 July, 1955.

CORRESPONDENCE on the Paper should be forwarded to reach the Institution before 15 December, 1956. Contributions should not exceed 1,200 words.—SEC.

Paper No. 5955

THE EFFECT OF DEFLEXION DUE TO SHEAR ON THE STRESSES AND DEFLEXIONS OF A PLANE GRILLAGE OF BEAMS

by

*** Stanley Kendrick**

(Ordered by the Council to be published in abstract form) †

Explanatory

Precise analyses, including shear deflexion but neglecting torsional rigidity, of two very different types of grillage have been carried out. The relaxation method of analysis was used and the relaxation patterns considered a unit lateral displacement (bending plus shear deflexion) and unit rotations (bending rotation only). Discontinuities in total slopes over beam intersections make their use for relaxation purposes impracticable but they can be calculated if required when the relaxation has been completed.

The first grillage A, shown in Fig. 1, consists of three uniform girders intersected by nine uniform stiffeners and is subjected to a central concentrated load of 10 tons. The second grillage B, shown in Fig. 2, is an idealization of a caisson at the entrance to a lock and is subjected to hydrostatic pressure. The validity of the idealization is very dubious but it provides a valuable illustration of the effect of shear in grillages composed of very deep beams.

The results for deflexions and bending moments both including and neglecting the effects of shear deflexion are given in Tables 1 to 4.

Table 1.—Intersection displacements in grillage A in inches

Point (*see* Fig. 1)	Shear included	Shear neglected	Point (*see* Fig. 1)	Shear included	Shear neglected
0	0·155	0·144	5	0·0110	0·0102
1	0·104	0·0995	6	0·0249	0·0234
2	0·0694	0·0660	7	0·000348	0·000292
3	0·0441	0·0421	8	0·00649	0·00580
4	0·0528	0·0505	9	0·000236	0·000159

* The Author is a Senior Scientific Officer at the Naval Construction Research Establishment, Dunfermline, Fifeshire.

† The full MS. and illustrations may be seen in the Institution Library.—Sec.

TABLE 2.—INTERSECTION DISPLACEMENTS IN GRILLAGE B IN INCHES

Point (*see* Fig. 2)	1	2	3	4	5
Shear included	0·159	0·282	0·397	0·426	0·124
Shear neglected	0·0965	0·187	0·274	0·299	0·0962

TABLE 3.—BENDING MOMENTS IN GRILLAGE A IN TONS-IN.

Point (*see* Fig. 1)	Shear included		Shear neglected	
	Girders	Stiffeners	Girders	Stiffeners
0	153	100	153	101
1	−6·64	56·0	−8·24	58·2
2	45·3	−14·4	46·3	−14·4
3	−34·1	19·5	−34·0	19·8
4	11·6	1·60	11·5	1·73
5	−25·8	4·33	−25·7	4·44
6	−13·7	5·02	−13·7	5·60
7	−11·5	0·0822	−11·5	0·144
8	−14·6	1·77	−14·7	1·70
9	−6·78	0·111	−6·15	0·0314

TABLE 4.—BENDING MOMENTS IN GRILLAGE B IN THOUSANDS OF TONS-INCHES

Point (*see* Fig. 2)	Shear included		Shear neglected	
	Horizontal	Vertical	Horizontal	Vertical
1	25·4	15·1	23·4	15·6
2	67·9	9·28	64·9	9·90
3	93·9	0	96·6	0
4	98·4	0	100	0
5	20·4	0	20·4	0

DISCUSSION OF RESULTS

The values given in the Tables show that the effect of shear deflexion on bending moments and hence on stresses is very much less than the effect on the total displacements. This result which, if generally true, is of considerable importance can be explained by examining the effect of shear deflexions in simpler structures.

For any statically determinate problem in beam structures it is obvious by definition that the bending moments and hence bending stresses by Bernouilli–Euler theory are independent of shear deflexion. This proposition is of course subject to the limitation that deflexions under loading are not so large as to affect, appreciably, the conditions of statical equilibrium. For some simple beam problems which are not statically determinate the result still holds that bending stresses are independent of

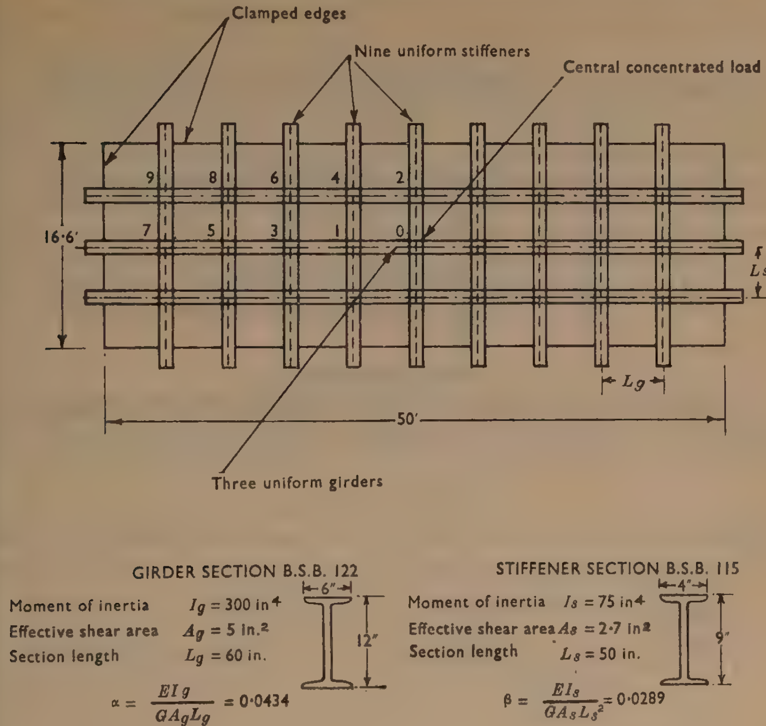


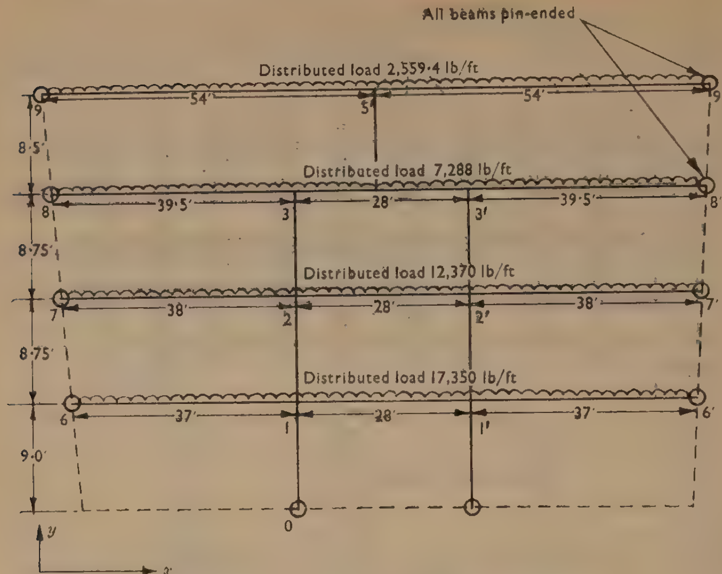
Fig. 1.—Grillage A

:he effect of shear deflexion. An illustration is provided by the lateral loading of a :lamped-ended beam. Provided that the distribution of lateral loading and bending stiffness is symmetrical about the mid-point of the beam, the bending slopes at the ends and at the mid-point are obviously zero. Since this bending slope condition is sufficient to determine the unknown end clamping moment without reference to shear effects, the bending-moment distribution is independent of shear effects.

It seems reasonable to conclude that bending moments are always affected much ess than deflexions by the inclusion of shear deflexion. Since shear deflexions are unlikely to predominate in practical grillages this leads to the further proposition that in the evaluation of bending moments the effect of shear deflexion may always be neglected. From this it follows that the additional shear deflexions may be estimated from the distribution of shear obtained by neglecting shear deflexions.

This result, that bending moments may be evaluated without reference to shear deflexion, is not surprising. However, it has been suggested by Vedeler[1] that this may not be so. In an investigation into the strength of the bottom stiffeners in the cargo tank of a tanker, Vedeler obtained the result that shear deflexion reduced the principal bending moments by nearly 20%. Although this reduction is not important, considering the approximate nature of the idealization, it is rather more

[1] G. Vedeler, "Grillage Beams in Ships and other Structures". Grondahl, Oslo, 1945.



Beam	Section	Moment of inertia: in.²/ft²	Shear area: in.²
9-9'	9"×17/40"; 17/40"; 25"×1"; 13·8'	19,630	140·6
8-8'	21"×17/40"; 17/40"; 50"×1"; 9·8'; 13·8'	29,690	140·6
7-7'	21"×17/40"; 5"×10"; 17/40"; 50"×1"; 13·8'	29,503	151·3
6-6'	10·5"×17/40"; 50"×1"; 6'×17/40"; 5'×17/40"; 10'-8"	25.672	86·7
0-3	21"×17/40"; 17/40"; 25"×1"; 13·7'	19.400	139·5
4-5	15/40"; 25"×1"; 13·7'	17,000	123

FIG. 2.—GRILLAGE B

han would be expected from the results for the grillages analysed in this Paper. This discrepancy may, however, be explained in several ways.

First there appears to be an arithmetical error in Vedeler's evaluation of the bending moments neglecting shear. Thus Vedeler's values of $\alpha = 1/(129{\cdot}2)$, $\beta = 1/(83{\cdot}2)$, appear to be wrong, the correct values being $\alpha = 1/(134)$, $\beta = 1/(46{\cdot}13)$. Using the new values for α and β the bending-moment reduction is found to be more like 5%.

A more fundamental error appears to be in assuming that by adding plating to a grillage, the effective moment of inertias of the girders are increased whereas the effective shear areas are unchanged. This assumption will always lead to the result that the effect of adding plating is to increase the percentage of shear deflexion. One obviously wrong consequence of this assumption is that plating can be weakened by adding girders of very small shear area.

ACKNOWLEDGEMENT

This Paper describes work carried out by the Author in connexion with his duties at the Naval Construction Research Establishment, Dunfermline, and is published by permission of the Admiralty.

The Paper, which was received on 7 July, 1955, is accompanied by four sheets of diagrams, from some of which the Figures in the text have been prepared.

Paper No. 6109

THE EFFECT OF SURGE ON THE DESIGN OF CRANE GANTRY GIRDERS: TESTS ON A 50-FT-SPAN CRANE GANTRY GIRDER

by

* John Stanley Terrington, B.Sc.(Eng.), A.M.I.C.E.

(Ordered by the Council to be published in abstract form) †

Introduction

Existing data for the design of gantry girders on which electric overhead travelling cranes run is limited and the method so far used of calculating the stresses is empirical. Accordingly guidance on the correct method of design and the best form of section to resist lateral surge arising from operation of the crane is needed. This need has been emphasized by frequent local failures in flanges and fastenings and excessive twisting of the gantry girders causing crane-wheel wear and "crabbing" of the crane.

As part of an investigation which is planned to throw light on these problems, it was decided to examine the behaviour of a full-scale crane gantry girder tested under known loads and the Paper describes tests on a full-scale crane gantry girder of a particular type of section. Results of the tests are discussed and conclusions are drawn from them.

Tests

A 50-ft.-span gantry girder was tested; this comprised a welded plate girder, having a riveted lattice girder parallel to it, both girders being about 5 ft deep and at approximately 5-ft centres. The top flanges of the girders were connected by a walkway of $\frac{3}{8}$-in. plate forming the web of the surge girder, and the bottom flanges of both girders were connected at the panel points of the auxiliary girder by single horizontal angles. At the panel points it was possible to insert or remove diagonal struts or ties connecting the bottom of the main girder to the top of the auxiliary girder.

The girder was set up on bearings in the fabrication shop and was surrounded by a test rig consisting of two deep lattice girders built largely of channel sections and cross-braced top and bottom. Two vertical loads were imposed on the crane rail immediately over the main girder at 18-ft centres to simulate crane-wheel loads, by means of jacks with proving rings strutted between the rail and overhead cross-beams which spanned between posts in each girder of the test rig. Some of the tests were carried out with vertical loads only, and some with a simultaneous side load at each of the two vertical loading points, first from one side and then from the other, to simulate side-surge from the crane wheels.

The loadings were applied in five groups. Lateral loads of 10% and 5% of the simultaneous vertical loads were both exerted in each direction, and also vertical

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* The Author is Head of the Civil Engineering Section (Plant Engineering Division), The British Iron and Steel Research Association.

† The full MS. and illustrations may be seen in the Institution Library.—Sec.

oads alone. In all groups a vertical load of up to 60 tons was applied, comprising wo loads of 30 tons from each of the two cross-beams; this load was applied in six ncrements of 10 tons (5 tons at each cross-beam).

Two major series of tests were undertaken, for deflexion and for strain. For the leflexions, micrometer dial gauges were used at mid-span and at the supports. Six gauges were used at each section to indicate the vertical and lateral movement at the orners of the girder. In all about 4,250 readings were taken.

For the strains, electrical resistance strain gauges were fixed at points located on hree cross-sections of the girder, one section at mid-span, one section at 10-ft 8 in. rom mid-span (between the point load and the support), and one section at 21 ft 5½ in. from mid-span (near the support). At each of the three sections, gauges were placed at nineteen points where it was of particular interest to know the actual trains. Since many of the points occurred on a plate, as for example the web, the lirection of the stresses was not known so that at such points three gauges in "delta" formation were used. About fifty gauges per section, totalling about 150 active gauges in all, were accordingly required. These gauges with their compensating gauges were connected to multiple switch-boxes, bridge circuits, amplifiers, and an automatic pen-recording instrument. A method of automatic recording with a nultiple switch-box was specially developed for these tests and made possible the arge number of readings—about 52,000—taken. The amplification required for points lightly stressed was more than for those where higher stresses occurred. Sets of readings, therefore, had to be taken at different "gains" or degrees of elecrical magnification, thus increasing the readings for some points on the sections.

Results obtained

Deflexions.—From the measured deflexions, the net deflexions of the mid-span ection relative to the ends have been computed. Although with very slender ections the deflexions are not necessarily linear when the section twists, it was found that with the range of vertical and lateral loading employed, the deflexions vere linear with the load. For purposes of comparison, it was accordingly possible o reduce the net deflexions to inches per ton of total load applied, i.e., ½ ton at each of the two points 18 ft apart.

From the measured deflexions, and taking into account the distance apart of the gauges, the angle of twist of both the undersides of the girders and the sides of the girder has been evaluated in radians per ton of load.

Deflexion phenomena included the following:—As the lateral load changed from a maximum of 10% in one direction, through the position of vertical load only, to a naximum of 10% in the other direction, the net deflexions and angle of twist showed lefinite trends. The auxiliary girder deflects but by only about one-fifth to onewentieth of the deflexion of the main girder. The effect of the diagonal ties on the nain girder was not marked, but the ties increased the deflexion of the auxiliary girder by about 50%. The diagonal ties did not affect the lateral deflexion at the op of the girder, which was relatively small, but they increased the lateral deflexion at the bottom of the girder and correspondingly the angle of twist of the main and auxiliary girders. The measured deflexions, after an appropriate allowance had been made for deflexion excluding torsion, showed that the centre of rotation of this girder was above the level of the walkway. This was also shown by the progressive ncrease of the lateral deflexion and angle of twist when the vertical load acts and he lateral load was changed progressively from a maximum in the direction "from he auxiliary girder" through zero to a maximum in the direction "towards the

auxiliary girder." The lateral deflexion of the bottom flanges of the main and auxiliary girders was about the same as the vertical deflexion of the main girder.

Stresses.—The deflexions made by the pen recorder had to be read and tabulated for the calibration and the load and their corresponding zeros. The large number of readings necessitated the use of an electronic computer which yielded the values of the stresses in the single gauges and, in the delta gauges, the maximum and minimum principal stresses and the angle in radians which they made with the delta. The results from the electronic computer were averaged per ton of load (i.e., per two ½-ton loads at 18-ft centres) on the girder.

The magnitude and direction of the measured stresses were plotted diagrammatically and graphically.

Correlation with theory.—The stresses and the angle of rotation of the girder from the loading have been calculated by a method which has been propounded by the Author. In this method the fundamental equations for bending and torsion have been developed to apply for the first time to a structural section having asymmetry of this complexity. The stresses calculated by this method have been found to agree closely with the measured values.

Conclusions

The extensive stress measurements have provided a comprehensive picture of the actual stress distribution in this compound full-scale girder section under simulated wheel loads which exert vertical, lateral, and torsional forces. The compound girder comprising the main girder, surge girder, and auxiliary girder act largely as a complete unit, since under the action of vertical or vertical and lateral wheel loads the girder deflects both vertically and laterally and twists. Resistance to torsion is afforded by shear forces developed in the component girders. The effects of these additional shear forces are negligible on the main girder but of considerable importance to the auxiliary and surge girders where reversal of stress and local contraflexure can produce fatigue effects. The measured deflexions and stresses confirmed the position of the centre of rotation of this section which behaved like an inverted channel. The tests demonstrated the conditions of this type of section in which there was no diagonal shear lacing between the bottom flanges, but the merits or otherwise of lacing on all faces, or alternatively providing articulations between the component girders, have to be left to further tests. Except for the compressive stresses in the main girder, the makers' calculations, estimated in accordance with the usual empirical assumptions, are in fair agreement with the measured stresses. The rail seems to act as part of the complete section but the effect of the diagonal ties (at least in this new girder) was not marked. Although this shape of girder is resistant to torsional and side loads, the test results have corroborated the method of calculation developed by the Author and have revealed the principles involved.

The Paper, which was received on the 23rd July, 1955, is accompanied by five photographs and seventeen sheets of diagrams.

Paper No. 6111

DESIGN OF BRIDGE PIERS EMBEDDED IN COHESIONLESS MATERIAL, TAKING INTO ACCOUNT THEIR FLEXIBILITY

by

* Vladimir Karmalsky, B.C.E., and Gerald Korner, B.C.E.

(Ordered by the Council to be published in abstract form) †

Stress distribution in the foundation material

HERE the ground is elastic, homogeneous, and isotropic:

$$\text{Stress } f = Ky = \text{modulus of earth reaction} \times \text{displacement} \quad . \quad (1)‡$$

In general, horizontal and vertical earth-reaction moduli are different.

The horizontal modulus of earth reaction at any depth x is usually given approxi-ately by[1, 2]:

$$K = K_h \cdot \left(\frac{x}{h}\right)^n$$

ere h denotes pier depth below ground surface and n is a positive number greater less than unity.

In uniform cohesionless soils n is near unity,[1, 2] but in other cases n, and similarly can be determined only by experiment. For cohesionless soils, experience is ailable to estimate K from boring data. This Paper deals only with $n = 1$. en:

$$K = K_h \cdot \frac{x}{h} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

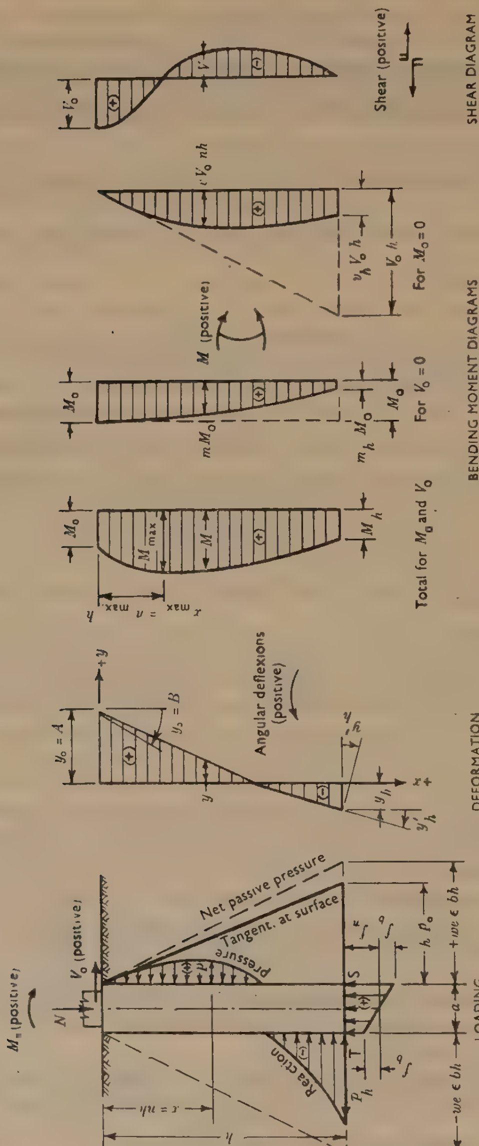
Where no field tests are conducted and only boring logs are available K has to be ermined by soil classification. For example, Groeger[3] has classified soils behav- elastically, according to their vertical earth-reaction moduli and corresponding missible foundation pressures. From this results the empirical relation:

$$K_v = \frac{\text{Permissible foundation pressure in kg/sq. cm}}{1 \text{ cm settlement}} \quad . \quad . \quad (3)$$

gross foundation pressure mentioned includes the weight of overlying material. hen K_h can be assumed equal to the vertical foundation modulus of the soil bedding the pier, at base level.

Equation (3) and the assumption in regard to K_h were not substituted into the l equations so that any better method if known, or field tests, may be used.

Mr Karmalsky is a supervising engineer for design of major steel structures in the Bridge Section of the Department of Main Roads, New South Wales, Australia.
Mr Korner is an engineer in the same section.
The full MS. and illustrations may be seen in the Institution Library.—Sec.
The notation is given on p. 547.
The references are given on p. 548.



SHEAR DIAGRAM
Shear (positive)
BENDING MOMENT DIAGRAMS
M (positive)
For $M_o = 0$
For $V_o = 0$
Total for M_o and V_o
DEFORMATION
Angular deflexions (positive)
LOADING
Net passive pressure
Tangent. at surface
Pressure
Reaction
V_o (positive)
M_o (positive)

FIG. 1

The pressure is assumed proportional to the pier displacement. The limit of proportionality should not be exceeded and is taken as the Rankine passive pressure for the depth considered.[2, 4]

The net passive pressure p_n at depth x then becomes:

$$p_n = wex$$

where

$$e = \frac{1 + \sin \phi}{1 - \sin \phi} - \frac{1 - \sin \phi}{1 + \sin \phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Pier embedded in cohesionless material

In this case let M_0 and V_0 act at ground surface level on the pier embedded in cohesionless material, together with a direct load N (see Fig. 1). The equations of the elasticity theory for shear, moment, and deflexion are:

$$EI_1 \,.\, y'' = + M \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$EI_1 \,.\, y^{(3)} = + V \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and differentiating in equation (6), the differential equation of the elastic line is:

$$EI_1 \,.\, y^{(4)} = - p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Substituting for p (pressure per unit length), as a function of x and y gives:

$$p = \epsilon b f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where ϵ is a form factor depending on the pier cross-section, assumed symmetrical about the y-axis. For a rectangular section $\epsilon = 1$, and for a circular section $\epsilon = \pi/4$. Substituting into equation (8) for f from equation (1) and for K from equation (2) then:

$$p = \frac{K_h}{h} \,.\, \epsilon b x_y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Substituting p from equation (9) into equation (7) the differential equation of elastic line is:

$$y^{(4)} = - cxy \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where

$$c = \frac{K_h \,.\, \epsilon b}{EI_1 \,.\, h}$$

The solution of equation (10) cannot be given in a closed form and is therefore expressed as a series using Maclaurin's theorem.

Then

$$\begin{aligned} y = A1 &\left\{1 - \frac{cx^5}{5P_4} + \frac{(cx^5)^2}{5P_4 \,.\, 10P_4} - \frac{(cx^5)^3}{5P_4 \,.\, 10P_4 \,.\, 15P_4} + \ldots\right\} \\ + Bx &\left\{1 - \frac{cx^5}{6P_4} + \frac{(cx^5)^2}{6P_4 \,.\, 11P_4} - \frac{(cx^5)^3}{6P_4 \,.\, 11P_4 \,.\, 16P_4} + \ldots\right\} \\ + Cx^2 &\left\{1 - \frac{cx^5}{7P_4} + \frac{(cx^5)^2}{7P_4 \,.\, 12P_4} - \frac{(cx^5)^3}{7P_4 \,.\, 12P_4 \,.\, 17P_4} + \ldots\right\} \\ + Dx^3 &\left\{1 - \frac{cx^5}{8P_4} + \frac{(cx^5)^2}{8P_4 \,.\, 13P_4} - \frac{(cx^5)^3}{8P_4 \,.\, 13P_4 \,.\, 18P_4} + \ldots\right\} \quad . \quad (11) \end{aligned}$$

where $11P_4 = 11 \,.\, 10 \,.\, 9 \,.\, 8$, $15P_4 = 15 \,.\, 14 \,.\, 13 \,.\, 12$, etc., and A, B, C, D are integration constants. The series y, y', y'', and $y^{(3)}$, obtained by differentiating equation (11), converge uniformly.

At $x = 0$ the moment is M_0. Substituting into equation (5):

$$EI_1 \cdot y_0'' = + M_0$$

Substituting for y_0'' by differentiating equation (11) twice and using $x = 0$:

$$C = + \frac{M_0}{2EI_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

At $x = 0$, the shear is V_0 and substituting into equation (6):

$$EI_1 \cdot y_0^{(3)} = + V_0$$

Substituting for $y_0^{(3)}$ by differentiating equation (11) three times and using $x = 0$:

$$D = + \frac{V_0}{6EI_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

A and B are obtained from end conditions at the base. From equation (11), and by differentiating equation (11), and using $x = 0$: $y_0 = A$; $y_0' = B$; i.e., A is the ordinate and B the slope of the elastic curve at the ground surface. For the calculation of A and B, values at $x = h$, of y_h, y_h', y_h'', and $y_h^{(3)}$ will be required. They are obtained by successive differentiation of equation (11) and using $x = h$.

Introducing: $$k_h = ch^5 = \frac{K_h \,.\, \epsilon b h^5}{EI_1}; \; u = \frac{K_v \,.\, I_b}{K_h \,.\, \epsilon b h^3}$$

Then:

$$y_h = AA_0 + BhB_0 + \frac{M_0}{2EI_1} h^2 C_0 + \frac{V_0 \,.\, h}{6EI_1} h^2 D_0$$

$$y_h' = - A \frac{K_h \,.\, \epsilon b h^3}{24EI_1} A_1 + BB_1 + \frac{M_0}{EI_1} hC_1 + \frac{V_0 \,.\, h}{2EI_1} hD_1$$

$$y_h'' = - A \frac{K_h \,.\, \epsilon b h^2}{6EI_1} A_2 - B \frac{K_h \,.\, \epsilon b h^3}{12EI_1} B_2 + \frac{M_0}{EI_1} C_2 + \frac{V_0 \,.\, h}{EI_1} D_2$$

$$y_h^{(3)} = - A \frac{K_h \,.\, \epsilon b h}{2EI_1} A_3 - B \frac{K_h \,.\, \epsilon b h^2}{3EI_1} B_3 - \frac{M_0}{EI_1 . h} \cdot \frac{k_h}{8} C_3 + \frac{V_0}{EI_1} D_3$$

where:

$$A_0 = 1 - \frac{k_h}{5P_4} + \frac{k_h^2}{5P_4 \,.\, 10P_4} - \frac{k_h^3}{5P_4 \,.\, 10P_4 \,.\, 15P_4} + \cdots$$
$$\simeq 1 - 0{\cdot}833\,333 \times 10^{-2} k_h + 0{\cdot}165\,344 \times 10^{-5} k_h^2 - 0{\cdot}504\,713 \times 10^{-10} k_h^3$$
$$+ 0{\cdot}434\,050 \times 10^{-15} k_h^4$$

$$B_0 = 1 - \frac{k_h}{6P_4} + \frac{k_h^2}{6P_4 \,.\, 11P_4} - \frac{k_h^3}{6P_4 \,.\, 11P_4 \,.\, 16P_4} + \cdots$$
$$\simeq 1 - 0{\cdot}277\,778 \times 10^{-2} k_h + 0{\cdot}350\,730 \times 10^{-6} k_h^2 - 0{\cdot}802\,952 \times 10^{-11} k_h^3$$
$$+ 0{\cdot}559\,003 \times 10^{-16} k_h^4$$

$$C_0 = 1 - \frac{k_h}{7P_4} + \frac{k_h^2}{7P_4 \,.\, 12P_4} - \frac{k_h^3}{7P_4 \,.\, 12P_4 \,.\, 17P_4} + \cdots$$
$$\simeq 1 - 0{\cdot}119\,048 \times 10^{-2} k_h + 0{\cdot}100\,208 \times 10^{-6} k_h^2 - 0{\cdot}175\,435 \times 10^{-11} k_h^3$$
$$+ 0{\cdot}999\,288 \times 10^{-17} k_h^4$$

$$D_0 = 1 - \frac{k_h}{8P_4} + \frac{k_h^2}{8P_4 \,.\, 13P_4} - \frac{k_h^3}{8P_4 \,.\, 13P_4 \,.\, 18P_4} + \cdots$$
$$\simeq 1 - 0{\cdot}595\,238 \times 10^{-3} k_h + 0{\cdot}346\,875 \times 10^{-7} k_h^2 - 0{\cdot}472\,325 \times 10^{-12} k_h^3$$

$$A_1 = 1 - \frac{k_h}{5P_1 \,.\, 9P_3} + \frac{k_h^2}{5P_1 \,.\, 10P_4 \,.\, 14P_3} - \frac{k_h^3}{5P_1 \,.\, 10P_4 \,.\, 15P_4 \,.\, 19P_3} + \cdots$$
$$\simeq 1 - 0{\cdot}396\,825 \times 10^{-3}k_h + 0{\cdot}181\,697 \times 10^{-7}k_h^2 - 0{\cdot}208\,344 \times 10^{-12}k_h^3$$

$$B_1 = 1 - \frac{k_h}{1P_1 \,.\, 5P_3} \times \frac{k_h^2}{1P_1 \,.\, 6P_4 \,.\, 10P_3} - \frac{k_h^3}{1P_1 \,.\, 6P_4 \,.\, 11P_4 \,.\, 15P_3} + \cdots$$
$$\simeq 1 - 0{\cdot}166\,667 \times 10^{-1}k_h + 0{\cdot}385\,802 \times 10^{-5}k_h^2 - 0{\cdot}128\,472 \times 10^{-9}k_h^3 + 0{\cdot}117\,391 \times 10^{-14}k_h^4$$

$$C_1 = 1 - \frac{k_h}{2P_1 \,.\, 6P_3} + \frac{k_h^2}{2P_1 \,.\, 7P_4 \,.\, 11P_3} - \frac{k_h^3}{2P_1 \,.\, 7P_4 \,.\, 12P_4 \,.\, 16P_3} + \cdots$$
$$\simeq 1 - 0{\cdot}416\,667 \times 10^{-2}k_h + 0{\cdot}601\,251 \times 10^{-6}k_h^2 - 0{\cdot}149\,120 \times 10^{-10}k_h^3 + 0{\cdot}109\,922 \times 10^{-15}k_h^4$$

$$D_1 = 1 - \frac{k_h}{3P_1 \,.\, 7P_3} + \frac{k_h^2}{3P_1 \,.\, 8P_4 \,.\, 12P_3} - \frac{k_h^3}{3P_1 \,.\, 8P_4 \,.\, 13P_4 \,.\, 17P_3} + \cdots$$
$$\simeq 1 - 0{\cdot}158\,730 \times 10^{-2}k_h + 0{\cdot}150\,313 \times 10^{-6}k_h^2 - 0{\cdot}283\,395 \times 10^{-11}k_h^3 + 0{\cdot}170\,391 \times 10^{-16}k_h^4$$

$$A_2 = 1 - \frac{k_h}{5P_2 \,.\, 8P_2} + \frac{k_h^2}{5P_2 \,.\, 10P_4 \,.\, 13P_2} - \frac{k_h^3}{5P_2 \,.\, 10P_4 \,.\, 15P_4 \,.\, 18P_2} + \cdots$$
$$\simeq 1 - 0{\cdot}892\,857 \times 10^{-3}k_h + 0{\cdot}635\,938 \times 10^{-7}k_h^2 - 0{\cdot}989\,633 \times 10^{-12}k_h^3$$

$$B_2 = 1 - \frac{k_h}{6P_2 \,.\, 9P_2} + \frac{k_h^2}{6P_2 \,.\, 11P_4 \,.\, 14P_2} - \frac{k_h^3}{6P_2 \,.\, 11P_4 \,.\, 16P_4 \,.\, 19P_2} + \cdots$$
$$\simeq 1 - 0{\cdot}462\,963 \times 10^{-3}k_h + 0{\cdot}231\,250 \times 10^{-7}k_h^2 - 0{\cdot}281\,738 \times 10^{-12}k_h^3$$

$$C_2 = 1 - \frac{k_h}{2P_1 \,.\, 5P_2} + \frac{k_h^2}{2P_2 \,.\, 7P_4 \,.\, 10P_2} - \frac{k_h^3}{2P_2 \,.\, 7P_4 \,.\, 12P_4 \,.\, 15P_2} + \cdots$$
$$\simeq 1 - 0{\cdot}250\,000 \times 10^{-1}k_h + 0{\cdot}661\,376 \times 10^{-5}k_h^2 - 0{\cdot}238\,592 \times 10^{-9}k_h^3 + 0{\cdot}230\,835 \times 10^{-14}k_h^4$$

$$D_2 = 1 - \frac{k_h}{6P_1 \,.\, 6P_2} + \frac{k_h^2}{6P_2 \,.\, 8P_4 \,.\, 11P_2} - \frac{k_h^3}{6P_1 \,.\, 8P_4 \,.\, 13P_4 \,.\, 16P_2} + \cdots$$
$$\simeq 1 - 0{\cdot}555\,556 \times 10^{-2}k_h + 0{\cdot}901\,876 \times 10^{-6}k_h^2 - 0{\cdot}240\,886 \times 10^{-10}k_h^3 + 0{\cdot}187\,430 \times 10^{-15}k_h^4$$

$$A_3 = 1 - \frac{k_h}{5P_3 \,.\, 7P_1} + \frac{k_h^2}{5P_3 \,.\, 10P_4 \,.\, 12P_1} - \frac{k_h^3}{5P_3 \,.\, 10P_4 \,.\, 15P_4 \,.\, 17P_1} + \cdots$$
$$\simeq 1 - 0{\cdot}238\,095 \times 10^{-2}k_h + 0{\cdot}275\,573 \times 10^{-6}k_h^2 - 0{\cdot}593\,780 \times 10^{-11}k_h^3 + 0{\cdot}394\,590 \times 10^{-16}k_h^4$$

$$B_3 = 1 - \frac{k_h}{6P_3 \,.\, 8P_1} + \frac{k_h^2}{6P_3 \,.\, 11P_4 \,.\, 13P_1} - \frac{k_h^3}{6P_3 \,.\, 11P_4 \,.\, 16P_4 \,.\, 18P_1} + \cdots$$
$$\simeq 1 - 0{\cdot}104\,167 \times 10^{-2}k_h + 0{\cdot}809\,376 \times 10^{-7}k_h^2 - 0{\cdot}133\,825 \times 10^{-11}k_h^3 + 0{\cdot}729\,135 \times 10^{-17}k_h^4$$

$$C_3 = 1 - \frac{k_h}{7P_3 \,.\, 9P_1} + \frac{k_h^2}{7P_3 \,.\, 12P_4 \,.\, 14P_1} - \frac{k_h^3}{7P_3 \,.\, 12P_4 \,.\, 17P_4 \,.\, 19P_1} + \cdots$$
$$\simeq 1 - 0{\cdot}529\,101 \times 10^{-3}k_h + 0{\cdot}286\,310 \times 10^{-7}k_h^2 - 0{\cdot}369\,337 \times 10^{-12}k_h^3$$

$$D_3 = 1 - \frac{k_h}{6P_1 . 5P_1} + \frac{k_h^2}{6P_1 . 8P_4 . 10P_1} - \frac{k_h^3}{6P_1 . 8P_4 . 13P_4 . 15P_1} + \ldots$$
$$\simeq 1 - 0{\cdot}333\,333 \times 10^{-1}k_h + 0{\cdot}992\,063 \times 10^{-5}k_h^2 - 0{\cdot}385\,417 \times 10^{-9}k_h^3 + 0{\cdot}393\,604 \times 10^{-14}k_h^4$$

The number of terms, when given in decimal form, was based on $k_h < 1{,}000$, with an accuracy of about 0·001%, the high accuracy being required for formulae for A and B, derived later. In some exceptional cases higher accuracy may be required.

For the calculation of the bending moment and shear, y'' and $y^{(3)}$ are required, then:

$$y'' = -A\frac{K_h . \epsilon b}{6EI_1 . h} . A_2' . x^3 - B . \frac{K_h . \epsilon b}{12EI_1 . h} . B_2' . x^4 + \frac{M_0}{EI_1} . C_2' + \frac{V_0}{EI_1} . D_2' . x$$

$$y^{(3)} = -A . \frac{K_h . \epsilon b}{2EI_1 . h} . A_3' . x^2 - B\frac{K_h . \epsilon b}{3EI_1 . h} . B_3' . x^3 - \frac{M_0}{EI_1}\frac{K_h . \epsilon b}{8EI_1 . h} . C_3' . x^4 + \frac{V_0}{EI_1} . D_3'$$

where A_2', B_2', C_2', D_2', A_3', B_3', C_3', and D_3' are equal to the corresponding constants A_2, B_2, C_2, D_2, A_3, B_3, C_3, and D_3 with cx^5 substituted for k_h in the latter.

Hence: $$A_2' = 1 - \frac{cx^5}{5P_2 . 8P_2} + \frac{(cx^5)^2}{5P_2 . 10P_4 . 13P_2} - \frac{(cx^5)^3}{5P_2 . 10P_4 . 15P_4 . 18P_2} + \ldots, \text{ etc.}$$

For a pier embedded in cohesionless material, end conditions at the base for $x = h$ are:—

(i) The bending moment equals the base pressure moment M_h.
(ii) The shear is zero.

The base pressure f_b arising from the rotation y_h' is obtained from equation (1) by substituting K_v for K and $\mp\frac{1}{2}ay_h'$ for the vertical displacement of T and S:

$$f_b = \pm K_v . \frac{a}{2} . y_h'$$

or

$$f_b = \mp \frac{M_h}{I_b} . \frac{a}{2}$$

$$\therefore M_h = -K_v . I_b . y_h' \quad \ldots \quad (14)$$

Substituting for $M = M_h$ into equation (5), for $V = 0$ into equation (6), and $h = x$:

$$+EI_1 . y_h'' = -K_v . I_b . y_h' \quad \ldots \quad (15)$$

and

$$+EI_1 . y_h^{(3)} = 0 \quad \ldots \quad (16)$$

Solving equations (15) and (16) by substituting for y_h', y_h'', and $y_h^{(3)}$, separating the effects of M_0 and V_0, and superimposing:

$$A = y_0 = \frac{A_m}{K_h . \epsilon bh^2} . M_0 + \frac{A_v}{K_h . \epsilon bh^2} . V_0 . h \quad \ldots \quad (17)$$

$$B = y_0' = \frac{B_m}{K_h . \epsilon bh^3} . M_0 + \frac{B_v}{K_h . \epsilon bh^3} . V_0 . h \quad \ldots \quad (18)$$

where $$B_m = \frac{-12\{3A_3(C_2 + uk_h . C_1) + \frac{1}{8}k_h . C_3(A_2 + \frac{1}{4}uk_h . A_1)\}}{4B_3(A_2 + \frac{1}{4}uk_h . A_1) - 3A_3(B_2 - 12uB_1)} \quad \ldots \quad (19)$$

$$B_v = \frac{-12\{3A_3(D_2 + \frac{1}{2}uk_h . D_1) - D_3(A_2 + \frac{1}{4}uk_h . A_1)\}}{4B_3(A_2 + \frac{1}{4}uk_h . A_1) - 3A_3(B_2 - 12uB_1)} \quad \ldots \quad (20)$$

$$A_m = -\frac{2B_3}{3A_3} \cdot B_m - \frac{C_3}{4A_3} \cdot k_h \quad . . . . . . (21)$$

$$A_v = -\frac{2\,B_3}{3A_3} \cdot B_v + \frac{2D_3}{A_3} \quad . . . . (22)$$

Equilibrium criteria are.—(i) The slope of the tangent to the horizontal reaction pressure curve at the ground surface must be less than that of the net passive pressure curve.

(ii) The horizontal reaction pressure at $x = h$ must be less than the net passive pressure at this depth.

From (i):

$$\left(\frac{dp}{dx}\right)_{x=0} = p_0' < + we\epsilon b$$

and differentiating equation (9):

$$\frac{dp}{dx} = p' = \frac{K_h \,.\, \epsilon b}{h} xy' + \frac{K_h \,.\, \epsilon b}{h} y \quad . . . . . . (23)$$

for $x = 0$, $y = y_0 = A$, therefore:

$$\frac{K_h \,.\, A}{h} < + we \quad . . . . (24)$$

From (ii):

$$p_h < - we\epsilon bh$$

and by substituting $p_h = K_h \,.\, \epsilon b y_h$ in this equation:

$$K_h \cdot y_h < - weh \quad . . . . (25)$$

(iii) The foundation pressure below the pier base must not exceed the permissible values.

(iv) The soil stress below the pier base must always be compressive.

The stress resulting from the direct force N is:

$$f_n = + \frac{N}{A_b} \quad . . . . (26)$$

The base pressures at T and S from the bending moment M_h are:

$$f_b = \mp \frac{M_h}{I_b} \cdot \frac{a}{2} \quad . . . . (27)$$

where M_h is obtained from equation (14) by substituting for y_h'. Hence:

$$M_h = - K_v \,.\, I_b \left(- A \frac{K_h \,.\, \epsilon b h^3}{24EI_1} \,.\, A_1 + BB_1 + \frac{M_0}{EI_1} \,.\, hC_1 + \frac{V_0 \,.\, h}{2EI_1} \,.\, hD_1 \right)$$

and substituting for A and B from equations (17) and (18):

$$M_h = m_h \,.\, M_0 + v_h \,.\, V_0 \,.\, h \quad . . . . . . (28)$$

where $m_h = - u\{B_1 \,.\, B_m + k_h(C_1 - \tfrac{1}{24} A_1 \,.\, A_m)\}$ . . . . . (29)

and $v_h = - u\{B_1 \,.\, B_v + \tfrac{1}{2}k_h(D_1 - \tfrac{1}{12} A_1 \,.\, A_v)\}$ . . . . . (30)

$$\text{Total stress at T and S} = f_n + f_b \quad . . . . . (31)$$

Bending moment at $x = n \,.\, h$ follows from equation (5), by substituting for y'':

$$M = - A \frac{K_h \,.\, \epsilon b}{6h} A_2' \,.\, x^3 - B \,.\, \frac{K_h \,.\, \epsilon b}{12h} \,.\, B_2' \,.\, x^4 + M_0 \,.\, C_2' + V_0 \,.\, D_2' \,.\, x$$

Substituting for A and B from equations (17) and (18):

$$M = m \, . \, M_0 + v \, . \, V_0 \, . \, n \, . \, h \qquad (32)$$

where
$$m = C_2' - \frac{n^3}{12}(2A_2' \, . \, A_m + B_2' \, . \, B_m \, . \, n) \qquad (33)$$

and
$$v = D_2' - \frac{n^2}{12}(2A_2' \, . \, A_v + B_2' \, . \, B_v \, . \, n) \qquad (34)$$

Bending moment at the base.—Alternatively, m_h and v_h can also be obtained from equations (33) and (34) by substituting 1 for n, A_2 for A_2', B_2 for B_2', C_2 for C_2', and D_2 for D_2':

$$m_h = C_2 - \tfrac{1}{12}(2A_2 \, . \, A_m + B_2 \, . \, B_m) \qquad (35)$$
$$v_h = D_2 - \tfrac{1}{12}(2A_2 \, . \, A_v + B_2 \, . \, B_v) \qquad (36)$$

Maximum bending moment is assumed to occur at a section for which $n < \frac{1}{2}$. Furthermore, assuming $k_h < 1{,}000$, i.e., $k_h \, . \, n^5 < 30$:

$$A_2' \simeq B_2' \simeq 1$$
$$C_2' \simeq 1 - 0{\cdot}0250 k_h \, . \, n^5$$
and
$$D_2' \simeq 1 - 0{\cdot}0056 k_h \, . \, n^5$$

giving results to within about 3%. Substituting A_2', C_2', and D_2' into equations (33) and (34):

$$m \simeq 1 - 0{\cdot}0250 k_h \, . \, n^5 - \frac{n^3}{12}(2A_m + B_m \, . \, n) \qquad (37)$$

and
$$v \simeq 1 - 0{\cdot}0056 k_h \, . \, n^5 - \frac{n^2}{12}(2A_v + B_v \, . \, n) \qquad (38)$$

The section $x_{max.} = n_{max.} \, h$, at which the maximum bending moment occurs, is

where
$$EI_1 \, . \, y^{(3)} = V = 0$$

Substituting for $y^{(3)}$:

$$- A \, . \, \frac{K_h \, . \, \epsilon b}{2h} \, . \, A_3' \, . \, x^2_{max.} - B \, . \, \frac{K_h \, . \, \epsilon b}{3h} \, . \, B_3' \, . \, x^3_{max.} - M_0 \frac{K_h \, . \, \epsilon b}{8EI_1 \, . \, h} \, . \, C_3' \, . \, x^4_{max.} + V_0 \, . \, D_3' = 0$$

where
$$A_3' \simeq 1 - 0{\cdot}238 \times 10^{-2} k_h \, . \, n^5_{max.}$$
$$B_3' \simeq C_3' \simeq 1$$
and
$$D_3' \simeq 1 - 0{\cdot}333 \times 10^{-1} k_h \, . \, n^5_{max.}$$

This gives results to within about 3%.

Substituting into the above equation, then:

$$0{\cdot}00357 k_h \, . \, \frac{A_m + A_v \, . \, r}{B_m + B_v \, . \, r} \, . \, n^7_{max.} - \frac{0{\cdot}1 k_h \, . \, r}{B_m + B_v \, . \, r} \, . \, n^5_{max.} - \frac{3k_h}{8(B_m + B_v \, . \, r)} \, . \, n^4_{max.}$$
$$- n^3_{max.} - \frac{3(A_m + A_v \, . \, r)}{2(B_m + B_v \, . \, r)} \, . \, n^2_{max.} + \frac{3r}{B_m + B_v \, . \, r} = 0 \qquad (39)$$

where
$$r = \frac{V_0 \, . \, h}{M}$$

From this equation, $n_{max.}$ is found by trial and error.

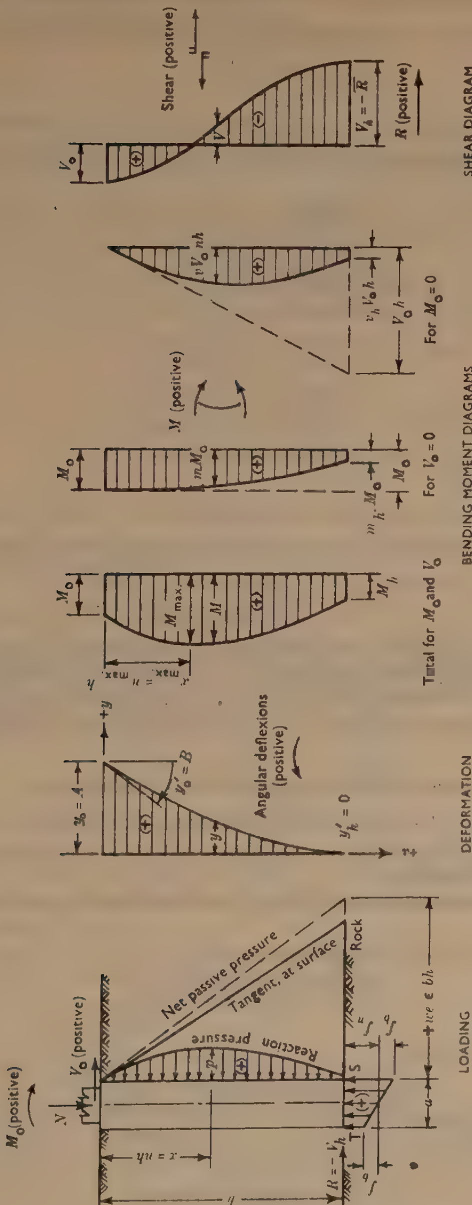

LOADING
Net passive pressure
Tangent, at surface
Reaction pressure
Rock
M₀ (positive)
V₀ (positive)
DEFORMATION
Angular deflexions (positive)
BENDING MOMENT DIAGRAMS
Total for M₀ and V₀
For V₀ = 0
For M₀ = 0
M (positive)
SHEAR DIAGRAM
Shear (positive)
R (positive)

FIG. 2

Pier embedded in cohesionless material with base on rock

In this case (see Fig. 2) the end conditions of the base for $x = h$ are:

$$y_h = 0 \quad \ldots \quad (40)$$

and

$$y_h' = 0 \quad \ldots \quad (41)$$

Solving equations (40) and (41) by substituting for y_h and y_h', separating effects of M_0 and V_0, and superimposing; then A and B are given by equations (17) and (18) where for this case:

$$A_m = \frac{+\,12k_h(2B_0 \,.\, C_1 - C_0 \,.\, B_1)}{24A_0 \,.\, B_1 + k_h \,.\, B_0 \,.\, A_1} \quad \ldots \quad (42)$$

$$A_v = \frac{+\,4k_h(3B_0 \,.\, D_1 - D_0 \,.\, B_1)}{24A_0 \,.\, B_1 + k_h \,.\, B_0 \,.\, A_1} \quad \ldots \quad (43)$$

$$B_m = \frac{-\,k_h(24A_0 \,.\, C_1 + \tfrac{1}{2}k_h \,.\, C_0 \,.\, A_1)}{24A_0 \,.\, B_1 + k_h \,.\, B_0 \,.\, A_1} \quad \ldots \quad (44)$$

$$B_v = \frac{-\,k_h(12A_0 \,.\, D_1 + \tfrac{1}{6}k_h \,.\, D_0 \,.\, A_1)}{24A_0 \,.\, B_1 + k_h \,.\, B_0 \,.\, A_1} \quad \ldots \quad (45)$$

The horizontal reaction at the base for $x = h$ is $R = -\,V_h$ and from equation (6):

$$R = -\,EI_1 \,.\, y_h^{(3)}$$

Substituting for $y_h^{(3)}$, and for A and B from equations (17) and (18):

$$R = \tfrac{1}{6}h\{3A_3(A_m \,.\, M_0 + A_v \,.\, V_0 \,.\, h) + 2B_3(B_m \,.\, M_0 + B_v \,.\, V_0 \,.\, h) + \tfrac{3}{4}k_h \,.\, C_3 \,.\, M_0\} - D_3 \,.\, V_0 \quad \ldots \quad (46)$$

Equations (24) to (39), with the exception of equations (29) and (30), are independent of end conditions and apply as before, provided that the above values for A_m, A_v, B_m, and B_v are substituted. Equation (28) applies only if m_h and v_h are substituted from equations (35) and (36). Equation (25) is not required since $P_h = 0$.

Applied forces acting in two directions

Generally, M_0 and V_0 act in two directions. For simplicity rectangular or circular sections will be considered. Suffixes t and l refer to transverse and longitudinal directions respectively.

For rectangular sections criteria of equations (24) and (25) are calculated for each direction separately.

For circular sections the first criterion is:

$$\frac{\sqrt{(K_h \,.\, A_t)^2 + (K_h \,.\, A_l)^2}}{h} < we \quad \ldots \quad (47)$$

and the second criterion is:

$$K_h \,.\, \sqrt{(y_h)_t^2 + (y_h)_l^2} < weh \quad \ldots \quad (48)$$

The resultant base pressures are:

Rectangular base: $$f_b = (f_b)_t + (f_b)_l \quad \ldots \quad (49)$$

Circular base: $$f_b = \sqrt{(f_b)_t^2 + (f_b)_l^2} \quad \ldots \quad (50)$$

$$\text{Total stress} = f_n \mp f_b \quad \ldots \quad (51)$$

The maximum bending moment:

$$M_{\max.} = \sqrt{(M_t^2 + M_l^2)_{\max.}} \quad \ldots \quad (52)$$

where M_t and M_l are given by equation (32). $M_{\max.}$ is obtained from equation (52) by trial and error for different values of n.

The resultant horizontal reaction (base on rock) is:

$$R = \sqrt{(R_t^2 + R_l^2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

The two-legged framed pier embedded in cohesionless material

In this case (Fig. 3) cutting the frame at the ground surface, the unknowns M_0 and V_0 act on two single columns AS and DT below it, and on the frame with cantilevers SB and TE above it. Then $y_0' = B$ and $y_0 = A$ of the portion below the ground surface, follow from previous formulae. M_0 and V_0 follow by equating deformations at the ground surface of portions above and below it:

$$y_0' = \phi_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

$$y_0 = d_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

where

$$\phi_0 = \phi_b + \beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

$$d_0 = d_t + s\phi_b + d_b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

and d_t denotes the displacement of joint B due to temperature change.

For antisymmetrical loading only equation (54) is required since V_0 is known. With no external load on SB:

$$M_1 = M_0 - \frac{s}{h} \cdot V_0 \cdot h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

The joint moments of the frame, including M_2 at B of brace BE, are obtained in terms of M_0 and V_0 by any frame calculation method. β and d_b (by moment area method) are substituted into equations (56) and (57):

$$\phi_0 = \phi_b + \frac{s}{EI_1} \cdot M_0 - \frac{s^2}{2EI_1} \cdot V_0 \quad . \quad . \quad . \quad . \quad . \quad (59)$$

$$d_0 = d_t + s\phi_b + \frac{s^2}{2EI_1} \cdot M_0 - \frac{s^3}{3EI_1} \cdot V_0 \quad . \quad . \quad . \quad . \quad (60)$$

Substituting into equations (54) and (55) for y_0' from equation (18), for ϕ_0 from equation (59), for y_0 from equation (17), and for d_0 from equation (60), the two deformation equations are:

$$\left(\frac{B_m}{k_h} \cdot h - s\right) M_0 + \left(\frac{B_v}{k_h} \cdot h + \frac{s^2}{2h}\right) V_0 \cdot h - EI_1 \cdot \phi_b = 0 \quad . \quad (61)$$

and

$$\left(\frac{A_m}{k_h} \cdot h^2 - \frac{1}{2}s^2\right) M_0 + \left(\frac{A_v}{k_h} \cdot h^2 + \frac{s^3}{3h}\right) V_0 \cdot h - sEI_1 \cdot \phi_b = EI_1 \cdot d_t \quad (62)$$

A_m, A_v, B_m, B_v are substituted from previously derived formulae. Values of ϕ_b obtained by moment area method are:

a) Symmetrical loading

No load on BE: $$EI_1 \cdot \phi_b = -\frac{I_1}{I_2} \cdot \frac{l}{2} M_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

Uniform load q on BE: $$EI_1 \cdot \phi_b = -\frac{I_1}{I_2} \cdot \frac{ql^3}{24} - \frac{I_1}{I_2} \cdot \frac{l}{2} M_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

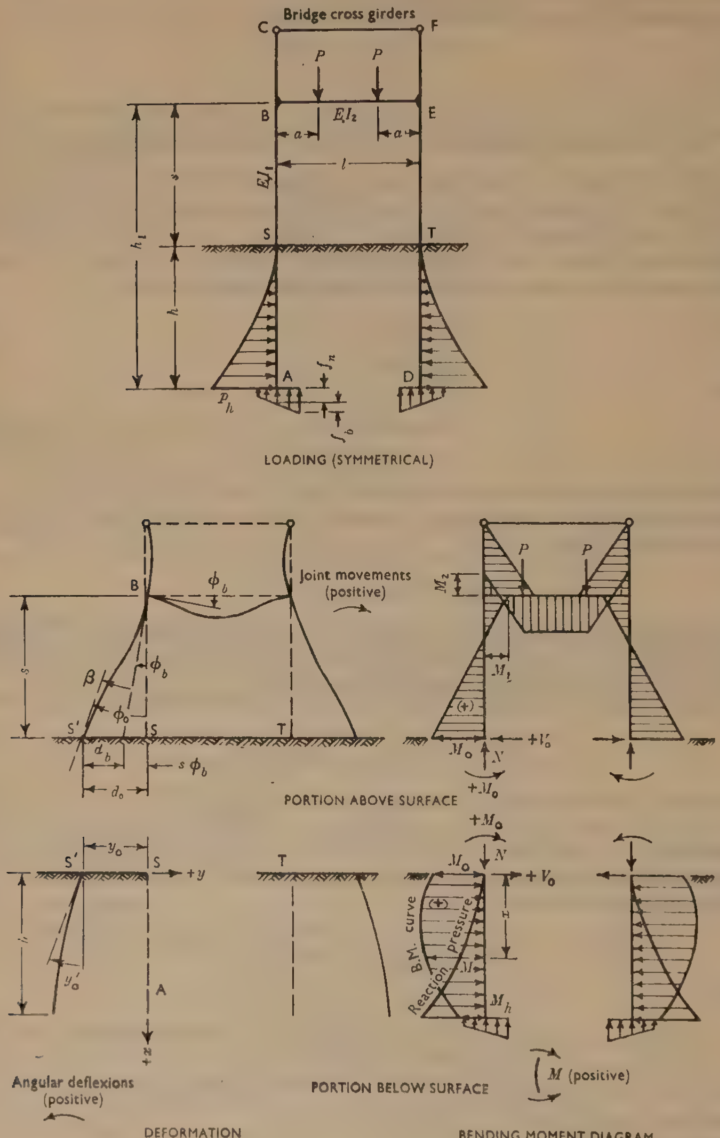

Bridge cross girders
C
F
P
P
B
E
EI_2
a
a
l
EI_1
s
h_1
h
S
T
A
D
f_n
f_b
P_h
LOADING (SYMMETRICAL)
B
ϕ_b
Joint movements
(positive)
ϕ_b
β
ϕ_0
S'
S
T
d_b
$s\,\phi_b$
d_0
M_2
M_1
(+)
M_0
N
$+V_0$
$+M_0$
PORTION ABOVE SURFACE
y_0
S'
S
$+y$
T
y_0'
A
$+z$
$+M_0$
M_0
N
$+V_0$
(+)
B.M. curve
Reaction
pressure
M
M_h
Angular deflexions
(positive)
PORTION BELOW SURFACE
M (positive)
DEFORMATION
BENDING MOMENT DIAGRAM

FIG. 3

wo-point loads (distance a from B and E):

$$EI_1 \, . \, \phi_b = -\frac{I_1}{I_2} \cdot \frac{Pa(l-a)}{2} - \frac{I_1}{I_2} \cdot \frac{l}{2} M_2 \quad . \quad . \quad . \quad . \quad . \quad (65)$$

) *Antisymmetrical loading: no load on* BE

$$EI_1 \, . \, \phi_b = -\frac{I_1}{I_2} \cdot \frac{l}{6} M_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

wo antisymmetrical point loads (distance a from B and E, load nearer to B down-
ards):

$$EI_1 \, . \, \phi_b = -\frac{I_1}{I_2} \cdot \frac{Pa(l-a)(l-2a)}{6l} - \frac{I_1}{I_2} \cdot \frac{l}{6} M_2 \quad . \quad . \quad (67)$$

) *Rise or fall of temperature t of* BE
= coefficient of expansion)

$$EI_1 \, . \, d_t = \mp \tfrac{1}{2} EI_1 \, . \, atl \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

Acknowledgements

The Authors are indebted to Mr H. M. Sherrard, Commissioner, Department of
ain Roads, New South Wales, for permission to publish the Paper and to Messrs
. T. Britton, G. D. Black, and K. T. Jones for assistance in its preparation.
The views expressed in the Paper are not necessarily those of the Department,
nce many are personal views of the Authors.

Notation

$_b$ denotes pier base area
,, pier width parallel to Y-axis
,, pier width parallel to Z-axis
, d_b denote horizontal deflexions of pier leg at ground surface (Fig. 3)
denotes displacement of joint B due to temperature change (Fig. 3)
,, Young's modulus
,, any ordinate of a stress distribution curve
,, maximum vertical base pressure due to bending moment at the base
,, vertical base pressure due to direct force N at the base
,, pier depth below ground surface
,, distance of bottom brace of pier frame from base (Fig. 3)
,, moment of inertia about Z-axis of embedded pier leg
,, moment of inertia about Z-axis brace
,, moment of inertia about Z-axis of base
,, horizontal earth-reaction modulus at depth x
,, horizontal earth-reaction modulus at depth h
,, vertical earth-reaction modulus at pier base
,, distance between pier frame legs (Fig. 3)
,, bending moment in pier leg at depth x below ground surface
$_1$, M_2 denote joint moments of pier frame at bottom brace (Fig. 3)
denotes direct load at base
,, horizontal soil-reaction pressure per unit length
,, net passive pressure at depth x
,, horizontal reaction at the base
,, distance of bottom brace of pier frame from ground surface (Fig. 3)

V ,, shear at depth x below ground surface
w ,, unit weight of water-borne foundation material
x ,, depth below ground surface
y ,, horizontal deflexion at point x
ϕ ,, angle of repose of water-borne foundation material
β, ϕ_o, ϕ_b denote deflexion angles of pier leg at ground surface (Fig. 3)

Suffixes o, h refer to any quantities at ground surface and at depth h below ground surface, respectively.

Suffixes t and l refer to transverse and longitudinal directions, respectively.

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2. K. Terzaghi, "Theoretical Soil Mechanics." Wiley, New York (1943).
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APPENDIX

The theory will be applied by checking one of the piers of the Iron Cove Bridge, Sydney, built recently (see Fig. 4). The original design was prepared by Karmalsky using another method. This example is given merely to demonstrate the application of the theory, since the pier was actually not embedded in an ideal cohesionless material. Corrections could not be made on account of the absence of material tests.

Constants (see also Fig. 5)

The pier is embedded in different layers of material. However, the materials in the critical positions are rock at the base and 25 ft of cohesionless material at the top. K_h was based on an estimated average permissible bearing pressure of the soil on the sides of the embedded pier footing at base level of 2·0 tons/sq. ft in addition to the weight of the overlying material.

Substituting into equation (3):

$$K_h = 12 \times \frac{0{\cdot}91}{0{\cdot}39}(2{\cdot}24 \times 2{\cdot}0 + 0{\cdot}064 \times 16{\cdot}0 + 0{\cdot}110 \times 64{\cdot}4)$$

$$= 353{,}000 \text{ lb/cu. ft}$$

Hence

$$k_h = \frac{353 \times \frac{\pi}{4} \times 13{\cdot}0 \times 64{\cdot}4^4}{576 \times 10^3 \times 1{,}082} \simeq 100$$

Substituting for k_h, A_0 to D_3 are obtained.

$$A_0 = +\,0{\cdot}183151 \qquad A_1 = +\,0{\cdot}960499$$
$$B_0 = +\,0{\cdot}725721 \qquad B_1 = -\,0{\cdot}628215$$
$$C_0 = +\,0{\cdot}881952 \qquad C_1 = +\,0{\cdot}589331$$
$$D_0 = +\,0{\cdot}940823 \qquad D_1 = +\,0{\cdot}842773$$

$$A_2 = +0{\cdot}911350 \qquad A_3 = +0{\cdot}764655$$
$$B_2 = +0{\cdot}953935 \qquad B_3 = +0{\cdot}896641$$
$$C_2 = -1{\cdot}434101 \qquad C_3 = +0{\cdot}947376$$
$$D_2 = +0{\cdot}453439 \qquad D_3 = -2{\cdot}234512$$

ıbstituting into equations (42) to (45):

$$A_m = +25{\cdot}2647 \qquad B_m = -67{\cdot}1399$$
$$A_v = +14{\cdot}4950 \qquad B_v = -25{\cdot}2647$$

ıbstituting then into equations (35) and (36):

$$m_h = +0{\cdot}066 \qquad v_h = +0{\cdot}260$$

equations (61) and (62) the following values are required:

$$\frac{B_m \,.\, h}{k_h} - s = -62{\cdot}7381 \text{ ft} \qquad \frac{A_m \,.\, h^2}{k_h} - \frac{s^2}{2} = +857{\cdot}693 \text{ sq. ft}$$

$$\frac{B_v \,.\, h}{k_h} + \frac{s^2}{2h} = -13{\cdot}3182 \text{ ft} \qquad \frac{A_v \,.\, h^2}{k_h} + \frac{s^3}{3h} = +603{\cdot}128 \text{ sq. ft}$$

PIER UNDER TRANSVERSE LOADS

) *Joint moments at* B *of frame portion above ground surface*

These were obtained by moment distribution for dead load of brace and temperature ange, and by statics for transverse wind, see Table 1.

For transverse wind from right $V_0 = -\frac{1}{2}(133{\cdot}4 + 9{\cdot}2) = -71{,}300$ lb.

TABLE 1

Member	Joint moments at B of frame portion above the surface in thousands of lb.-ft		
	Dead load of brace	$+25°$F temperature change	Transverse wind ←
$\text{A} = M_1$	$+M_0 - 0{\cdot}30280V_0 \,.\, h$	$+M_0 - 0{\cdot}30280V_0 \,.\, h$	$+M_0 + 1{,}390$
$\text{E} = M_2$	$-0{\cdot}126M_0 + 0{\cdot}03815V_0 \,.\, h - 844{\cdot}28$	$-0{\cdot}126M_0 + 0{\cdot}03815V_0 \,.\, h$	$-M_0 - 3{,}951$
$\text{C} = M_3$	$-0{\cdot}874M_0 + 0{\cdot}26465V_0 \,.\, h + 844{\cdot}28$	$-0{\cdot}874M_0 + 0{\cdot}26465V_0 \,.\, h$	$+2{,}561$

) $EI_1 \,.\, \phi_b$ and $EI_1 \,.\, d_t$

Using the appropriate equation for $EI_1 \,.\, \phi_b$ and thereafter substituting for M_2 from ıble 1, then:—

ead load of brace, from equation (64):

$$EI_1 \,.\, \phi_b = -\frac{1{,}080}{143} \times \frac{5{\cdot}25 \times 47^3}{24} - \frac{1{,}080}{143} \times \frac{47}{2}M_2$$
$$= (+22{\cdot}3628M_0 - 6{\cdot}7710V_0 \,.\, h - 21{,}603) \times 1{,}000 \text{ lb/sq. ft}$$

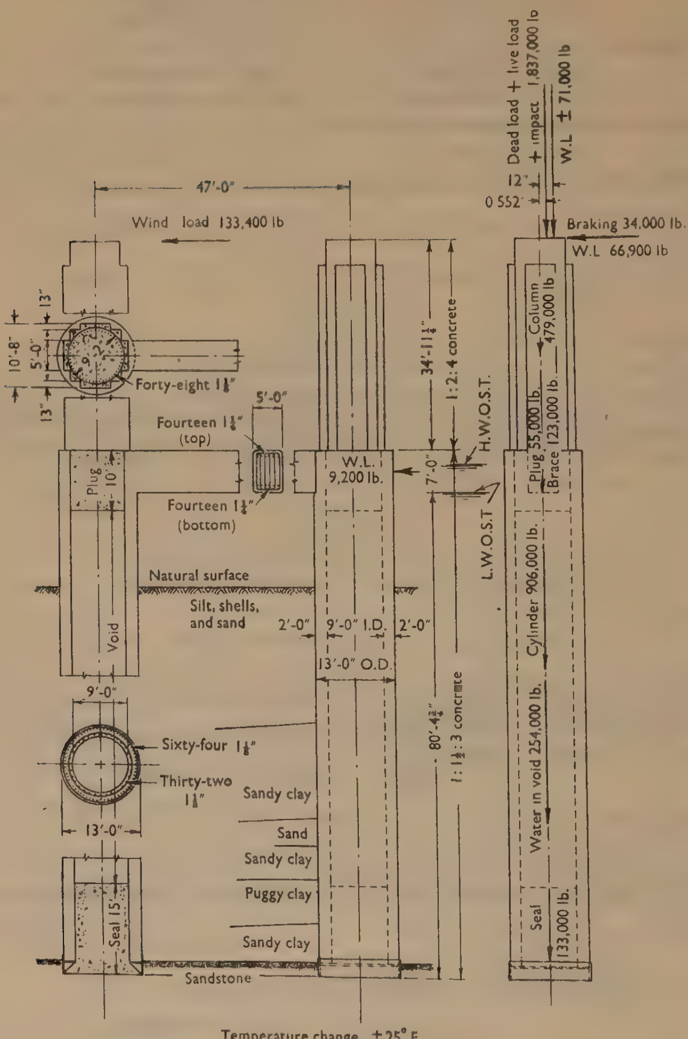
25°F temperature change, from equations (63) and 68):

$$EI_1 \,.\, \phi_b = -\frac{1{,}080}{143} \times \frac{47}{2}M_2 = (+22{\cdot}3628M_0 - 6{\cdot}7710V_0 \,.\, h) \times 1{,}000 \text{ lb/sq. ft}$$

$$EI_1 \,.\, d_t = -\tfrac{1}{2} \times 576 \times 10^3 \times 1{,}080 \times 0{\cdot}000005 \times 25 \times 47$$
$$= -1{,}827{,}360 \times 1{,}000 \text{ lb/cu. ft}$$

ansverse wind, from equation (66):

$$EI_1 \,.\, \phi_b = -\frac{1{,}080}{143} \times \frac{47}{6}M_2 = (+59{\cdot}1608M_0 + 233{,}744) \times 1{,}000 \text{ lb/sq. ft}$$



Critical load combinations

Case A: Dead load + live load + impact + 25° F temperature change + longitudinal wind

Case B: Dead load + live load + impact + 25° F temperature change + transverse wind + braking

FIG. 4.—IRON COVE BRIDGE PIER

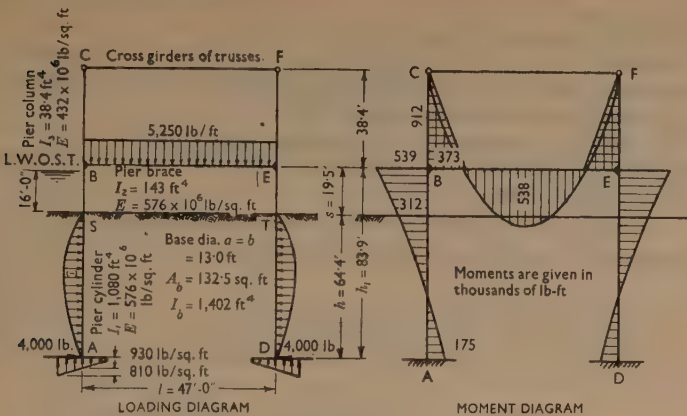


FIG. 5.—DEAD LOAD OF PIER BRACE

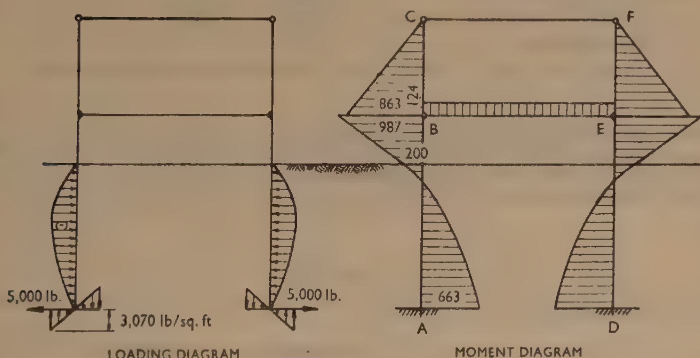


FIG. 6.—DEAD LOAD + 25°F TEMPERATURE CHANGE

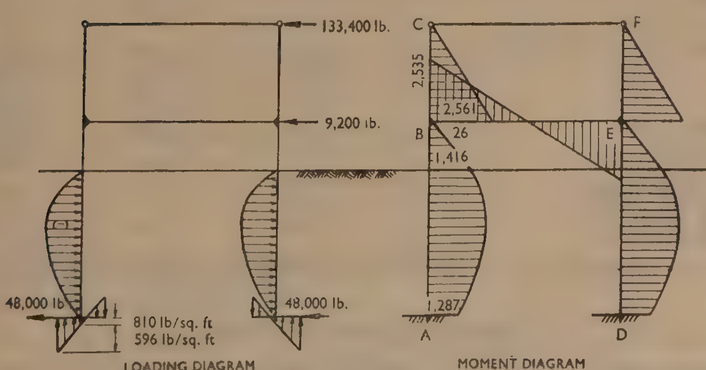


FIG. 7.—TRANSVERSE WIND LOAD

(3) *Deformation equations*

Dead load of brace equations (61) and (62):

$$-62{\cdot}7381M_0 - 13{\cdot}3182V_0 \,.\, h - 22{\cdot}3628M_0 + 6{\cdot}7710V_0 \,.\, h + 21{,}603 = 0$$
$$+857{\cdot}693M_0 + 603{\cdot}128V_0 \,.\, h - 436{\cdot}075M_0 + 132{\cdot}034V_0 \,.\, h + 421{,}262 = 0$$

+ 25°F temperature change equations (61) and (62):

$$-62{\cdot}7381M_0 - 13{\cdot}3182V_0 \,.\, h - 22{\cdot}3628M_0 + 6{\cdot}7110V_0 \,.\, h = 0$$
$$+857{\cdot}693M_0 + 603{\cdot}128V_0 \,.\, h - 436{\cdot}075M_0 + 132{\cdot}034V_0 \,.\, h = -1{,}827{,}360$$

Transverse wind equation (61):

$$-62{\cdot}7381M_0 + 13{\cdot}3182 \times 4{,}592 - 59{\cdot}1608M_0 - 233{,}744 = 0$$

Solving equations, M_0 and V_0 were obtained, see Table 2.

TABLE 2

Loading	M_0	$V_0 \,.\, h$	V_0
	1,000 lb.-ft	1,000 lb.-ft	1,000 lb.
Dead load of brace . . .	+ 312	− 752	− 11·7
+ 25°F temperature change	+ 200	− 2,600	− 40·4
Transverse wind ← . . .	− 1,416	− 4,592	− 71·3

(4) *Joint moments at* B; M_h, K_h, A, R, N, f_n, *and* f_b:

Joint moments at B were obtained by substituting for M_0 and $V_0 \,.\, h$ into Table 1; M_h, $K_h \,.\, A$, and R by substituting into equations (28), (17), and (46); N by statics; and f_n and f_b from equations (26) and (27). The results are shown in Table 3 and Figs 5, 6, and 7.

TABLE 3

	Dead load of brace	+ 25°F temp. change	Transverse wind ←
V_0 (1,000 lb.)	− 11·7	− 40·4	− 71·3
M_0 (1,000 lb.-ft)	+ 312	+ 200	− 1,416
Joint moment at B (1,000 lb-ft) . .			
,, of BA	+ 539	+ 987	− 26
,, of BE	− 912	− 124	− 2,535
,, of BC	+ 373	− 863	+ 2,561
M_h (1,000 lb.-ft)	− 175	− 663	− 1,287
R (1,000 lb.)	+ 4	− 5	− 48
N (1,000 lb.)	123	0	108
$K_h \,.\, A$ (1,000 lb/sq. ft)	− 0·071	− 0·771	− 2·417
f_n (1,000 lb/sq. ft)	+ 0·93	0	+ 0·81
f_b (1,000 lb/sq. ft)	± 0·81	± 3·07	± 5·96

PIER UNDER LONGITUDINAL LOADS

By substituting for M_0 and V_0, values of M_h, $K_h \,.\, A$, and R were obtained from equations (28), (17), and (46), and f_n and f_b from equations (26) and (27) for the given N and are shown in Table 4.

TABLE 4

Loading	M_0	V_0	M_h	R	N	$K_h . A$	f_n	f_b
	1,000 lb.-ft	1,000 lb.	1,000 lb.-ft	1,000 lb.	1,000 lb.	1,000 lb/ sq. ft	1,000 lb/ sq. ft	1,000 lb/ sq. ft
Dead load of superstructure and pier leg + live load . . .	+1,014	0	+ 67	+ 23	+3,627	+0·605	+27·37	∓0·31
Transverse wind vertical load at bearing →	+ 71	0	+ 5	+ 2	+ 71	+0·042	+ 0·54	∓0·02
Braking load → . .	+1,969	+34·0	+ 699	+ 52	0	+1·924	0	∓3·24
Longitudinal wind → .	+3,874	+66·9	+1,375	+102	0	+3·786	0	∓6·38

PIER UNDER COMBINED LOADING

(1) *Base pressure*

The critical load combination occurred with case B. The value of f_b and the total pressure were obtained from equations (50) and (51).

$$f_n = (+0·93 + 0 + 0·81) + (27·37 + 0·54 + 0) \qquad = +29{,}650 \text{ lb/sq. ft}$$

$$f_b = \pm\sqrt{(+0·81 + 3·07 + 5·96)^2 + (+0·31 + 0·02 + 3·24)^2} = \pm 10,450 \quad ,,$$

Total maximum base pressure = (29·65 + 10·45) ÷ 2·24 = + 17·9 tons/sq. ft

Permissible pressure was 11 tons/sq. ft plus pressure due to weight of overlying material plus 25% = 18·4 tons/sq. ft.

(2) *Equilibrium criterion* (i)

The critical load combination occurred with case A.

From equation (47):

$$\frac{\sqrt{(K_h . A_t)^2 + (K_h . A_l)^2}}{h} = \frac{\sqrt{(-0·071 - 0·771)^2 + (+0·605 + 3·786)^2}}{64·4} = 69·4 \text{ lb/cu. ft}$$

For $\phi = 30°$ and $w = 60$ b/cu. ft, $e = 2·67$ from equation (4). Adopting a safety factor of 1·5, the permissible value of $\frac{\sqrt{(K_h . A_t)^2 + (K_h . A_l)^2}}{h}$ is $\frac{we}{1·5} = \frac{0·06 \times 2·67}{1·5} = 107$ lb/cu. ft.

The large margin allows for the different materials which are partly cohesive downward from a level 25 ft below the ground surface.

(3) *R at base*

The critical load combination occurred with case A.

From equation (53):

$$R = \sqrt{(4+5)^2 + (23+102)^2} = 124{,}000 \text{ lb.}$$

(4) *Pier cylinder*

The critical load combination occurred with case A.

By trial and error to satisfy equation (52) $n_{max} = 0·30$.

Substituting into equations (37) and (38):

$$m = 1 - 0·0250 \times 100 \times 0·30^5 - \frac{0·30^3}{12}(2 \times 25·26 - 67·14 \times 0·30) = 0·926$$

$$v = 1 - 0·0056 \times 100 \times 0·30^5 - \frac{0·30^2}{12}(2 \times 14·50 - 25·26 \times 0·30) = 0·838$$

Substituting into equation (32) for the transverse and longitudinal directions:

$$M_t = (+\,312 + 200)0{\cdot}926 + (-\,11{\cdot}7 - 40{\cdot}4)0{\cdot}30 \times 64{\cdot}4 \times 0{\cdot}838 = -\,370{,}000 \text{ lb.-ft}$$

$$M_l = (+\,1{,}014 + 3{,}874)0{\cdot}926 + (0 + 66{\cdot}9)0{\cdot}30 \times 64{\cdot}4 \times 0{\cdot}838 = +\,5{,}607{,}000 \text{ lb.-ft}$$

From equation (52):

$$\max.\ M = \sqrt{(-\,370)^2 + 5{,}607^2} = 5{,}620{,}000 \text{ lb.-ft}$$

The corresponding direct loading = 2,932,000 lb.
Stress in concrete = 475 lb/sq. in.
Stress in reinforcement = nil.

The cylinder wall thickness was governed by the erection stresses due to air pressure.

(5) *Pier column*

The critical load combination at critical section (top of brace) occurred with case B: maximum bending moment

$$= \sqrt{(373 + 863 + 2{,}561)^2 + (1{,}013 + 71 + 34{\cdot}0 \times 38{\cdot}4)^2}$$

$$= 4{,}490{,}000 \text{ lb.-ft}$$

The corresponding direct loading = 2,387,000 lb.
Stress in concrete = 416 lb/sq. in.
Stress in reinforcement = 736 „ „

The column size was determined by the space required to accommodate the truss bearings.

(6) *Pier brace*

The critical load combination occurred with case B at the inside cylinder face.

Maximum bending moment = – 2,180,000 lb.-ft
Maximum shear = 197,000 lb.
Stress in concrete = 624 lb/sq. in.
Stress in reinforcement = 22,350 „ „
Permissible stress in reinforcement, case B = 22,500 „ „

The Paper, which was received on 28 July, 1955, is accompanied by seven sheet of diagrams from which the Figures in the text have been prepared, and by an Appendix.

CORRESPONDENCE

on Papers published in Proceedings, Part III, December 1955

Paper No. 6024

Streamflow: poly-dimensional treatment of variable factors affecting the velocity in alluvial streams and rivers †

by

Cornelis Toebes

Correspondence

Professor Thomas Blench (Professor of Civil Engineering, University of Alberta, ;anada) complimented the Author on his choice of a popular and useful hydraulic subject o illustrate a valuable method of statistical analysis. Information on impartial methods f viewing hydraulic information was sorely needed, for hydraulic science was not free rom analyses resting on mathematically obscured dynamical anomalies, assumptions, and xtrapolations of which the ordinary reader was unaware. Such analyses played an mportant part in the production of new ideas, but they needed a counterbalance such as urely statistical methods provided.

Some previous comments by Professor Blench had been paraphrased by the Author, esulting in an accidental change of sense. At the bottom of p. 912, the last two sentences hould read: "Now the rate of change of discharge (or of gauge or any other suitable easure of it) may have been omitted in recording; but that does not prevent the verifica- ion of its relevance if some other quantity that is a function of the independent variables as been measured".

The Author, in reply, expressed pleasure that Professor Blench believed the nalysis to be of real value in hydraulic science.

He apologized for the change of sense which had occurred in his paraphrasing of Pro- ssor Blench's earlier comments and explained that the error had arisen as a result of is (the Author's) unfamiliarity with a newly acquired language.

† Proc. Instn Civ. Engrs, Pt III, vol. 4, p. 900 (Dec. 1955).

Paper No. 6057

Soil permeability determination carried out for the River Mourne hydro-electric project, with particular reference to a discharging-well test †

by

Edward Maurice Gosschalk, M.A., A.M.I.C.E.

Correspondence

Mr E. M. Wilson observed that the Mourne valley had not been surveyed in detail by the Geological Survey and there are no published memoirs of the Survey about it. Charlesworth[8] showed no drumlins in the Mourne valley and marked several "stages of ice retreat". It was probable that the whole area was a vast heterogeneous dump of moraine, sometimes graded into specific particle sizes by water, and more often not. It was unlikely, therefore, that drumlins, which were mounds of boulder clay sculptured by ice, were present in the terrain to the extent inferred in the Paper.

Three other minor points might be mentioned. First, in Fig. 1 (section AA) the ground-water table was shown falling away from the ditch towards rising ground, which was highly improbable. Secondly, in Fig. 4 the graph of water-rise in the pumped well showed a sudden unexpected rise about 12 hours after pumping stopped, at a steeper gradient than immediately after the pump was stopped the first time. Since the final pump stoppage occurred at the same time, it looked as though this might have been due to the non-return valve failing to operate a second time. Thirdly, the reason for the negative results from the dye tests was simply one of time. Even at ten times the average permeability recorded, it would have taken more than 100 hours for dye to reach the pump well from T1, the nearest hole, and pumping continued for only about 5 hours.

Mr Wilson considered that the Author's opinion about the relative permeabilities of different areas of the buried valley was unjustified, in view of the character of the ground and the scarcity of data. The injection tests referred to on pp. 974–5 could not be considered as indicating much more than that permeable conditions existed, and did not justify quantitative comparisons.

As a means of finding an average permeability in such ground the tests were extremely interesting, although since they had only effectively tested the top 30 ft of a valley more than 100 ft deep they could hardly be called conclusive. In the light of the event it would clearly have been better to sink the pump very much deeper, aiming at a draw-down of, say, 90 ft, rather than 30 ft, although with the large percussion casing necessary that would have been a good deal more expensive. By so doing, a better estimate might have been made of likely percolation around the abutment of a dam on the Mourne, since there was a possibility of substantially increased permeability beyond a limiting hydraulic gradient. Although it would appear from the tests that serious percolation was unlikely, a single test, conducted on the top third of the overlying material and about 600 ft from the river was scarcely enough.

The Author, in reply, observed that Mr Wilson's comments on the geological history of the Mourne Valley amplified his own. The prevalence of drumlins was not directly relevant to the tests carried out, but if the moulded hills in the district were not

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† Proc. Instn Civ. Engrs, Part III, vol. 4, p. 972 (Dec. 1955).

[8] J. K. Charlesworth, "The glacial geology of the north-west of Ireland". Proc. R. Irish Acad., vol. 36, 1924, p. 174.

drumlins, nevertheless it was agreed that the valley floor was covered by glacial drift, of composition indicated by the samples (Table 1 of the Paper).

The ground-water levels under the high ground shown in Fig. 1 (section AA) had been assumed by linking those observed in the wells T5 and S1. No intermediate wells had been drilled, but evidently the ground-water level dropped towards the river. The vertical scale of the sketch was distorted to 5 times its horizontal scale and that exaggerated the gradient, which was no doubt less regular than shown.

The unexpected rise in water level experienced long after pumping stopped was discussed on p. 985. It had happened just before the pump had been stopped, and could not therefore have been caused by a failure of the non-return valve as suggested.

It was agreed that the dye tests confirmed a low permeability. They might have revealed an appreciable local flow between two wells if such had existed, whereas the flow into the pumped well had come from all directions. As it had turned out, the minute quantities of dye reaching the hole would not have been detectable. The Author believed that more information could be derived from the injection tests than Mr Wilson's dismissal of them implied. It should be explained that a considerable number of borings, not detailed here, had been made in the valley since 1948, and results were all consistent with the soil patterns as described in the Paper. There was in fact a substantial block of data showing comparative conditions in different parts of the valley.

It was a misconception to suppose that the discharging-well method tested only the top 30 feet of the deep valley. As described in the Paper, the pumped well had been cased with perforated tube to a depth of 86 ft, and ground-water could thus enter the well with horizontal flow down to that depth, the hydraulic gradient towards the well affecting the ground-water at all levels. The permeability calculated was the average for the full depth of overburden through which water percolated. Even if the well were not extended the full depth, ground-water would percolate towards the well at all levels, but in the vicinity of the well flow-lines would be forced upwards to enter the well and calculations based on horizontal flow close to the well would be invalid. Despite that defect, resort to that method would sometimes be essential, for economic reasons.

The Author knew of no evidence to suggest the existence of a limiting hydraulic gradient beyond which percolation might increase at a greater rate, and considered such an event improbable.

CORRESPONDENCE

on a Paper published in Proceedings, Part III, April 1956

Structural Paper No. 46

"The stanchion problem in frame structures designed according to ultimate carrying capacity"†

by

Michael Rex Horne, M.A., Ph.D., A.M.I.C.E.

Correspondence

Professor J. F. Baker (Professor of Mechanical Sciences, University of Cambridge) observed that Part 1 of the Paper provided the steelwork designer with a valuable statement of the stanchion problem. It would be clear from section (2) of Part 1 why it had not yet been possible at Cambridge to set out a plastic design method for multi-storey frames of comparable simplicity to that for single-storey structures. However, though it was so complex, the behaviour of compression members when stressed beyond the elastic limit was now understood and it was only a matter of time, and much further work on actual designs, before a method taking full advantage of the plastic strengths of beams and columns was evolved. In the meantime a practicable, rational, and more economical method of design could be used by proportioning the beams so that they failed by the development of plastic hinges while the stanchions still remained elastic. The stanchions must, of course, be proportioned to take the real reactions from the beams and so the orthodox or "simple" method of B.S. 449 was not satisfactory. The Steel Structures Research Committee's first simplified method of stanchion design could be used for most orthodox buildings, but even there the simplifications introduced to make that method more attractive to the steelwork designer 20 years ago meant loss of economy in steel. The Author had given an attractive method of determining the adequacy of a stanchion section once the axial load and end moments were known which, whilst quite simple to apply, avoided that loss of economy. It was oddly satisfying, incidentally, to the original members of the team to see the S.S.R.C. curves verified after the passage of so many years, though they would have been severely shocked if that had not been so. The worst end moments on the stanchion length must, of course, still be determined and there the critical loading conditions considered in the S.S.R.C. Final Report and more recently in vol. 1 of "The Steel Skeleton" (see reference 10, p. 43) pointed the way.

Professor F. Campus and **Professor C. Massonnet** (Université de Liège) said they had recently completed[39, 40] an extensive series of buckling tests on eccentrically and obliquely loaded I-section stanchions of mild steel, simply supported at their ends. Since those tests gave a means to control the accuracy of the Author's design method, it seemed interesting:

(1) to indicate briefly the main features of the test programme and equipment;
(2) to compare the experimental results obtained with those calculated by the Author's method;

† Proc. Instn Civ. Engrs, Part III, vol. 5, p. 105 (Apr. 1956).
[39] References 39–43 are given on p. 563 at the end of this contribution.

(3) to indicate the new interaction formulae which had been deduced from the tests and which were now under consideration by an official committee prior to being introduced in the Belgian specifications on the design of steel structures.

Brief description of the buckling tests

Two types of stanchions were used; one with wide flanges, grey profile DIE; the other with narrow flanges, continental normal profile PN 22 standard.

The slenderness ratios adopted in the tests were: $\lambda = 40, 60, 80, 100, 130$, and 175.

The maximum test length in the machine being about 5 m, the longest DIE stanchions ($\lambda = 130, 175$) were made from profile DIE 10; the shortest ($\lambda = 40, 60, 80, 100$) were made from profile DIE 20.

All stanchions were loaded in the plane of the web (xx).

The ratios of terminal eccentricities used in the tests were $e_2/e_1 = +1, 0, -1$ (see Fig. 34).

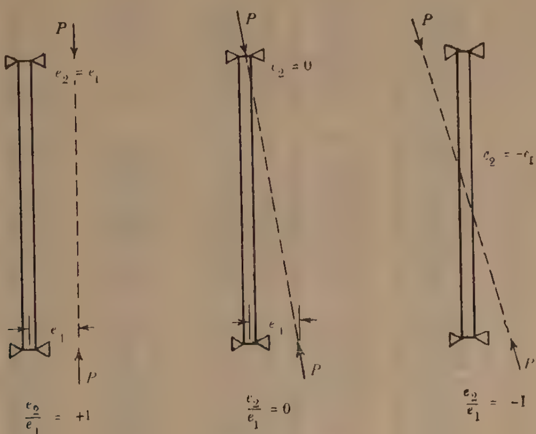

FIG. 34

Finally, the largest end eccentricity e_1 was given three values, equal respectively to 0·5, 1, and 3 times the core radius ρ_x.

Putting

$$\frac{e_1}{\rho_x} = m$$

the extreme stresses were:

$$\sigma_{\min}^{\max} = \frac{P}{A}(1 \pm m)$$

The bearings used to support the stanchions at their ends were hydraulically supported spherical-seated bearings specially made for the tests. The working principle of those bearings was due to Dr Templin.[41]

Owing to the very small friction developed and observed in those bearings, it could be assumed that the ends of the stanchions were simply supported. The stanchions were welded at their ends to thick steel plates bolted to the very rigid bearings so that it could be assumed that there was no warping of the end cross-sections.

Taking into account the tests which were not possible owing to geometrical limitations and three tests with special values of m (see tests Nos 48, 57, and 64 in Table 3), the total

TABLE 3

Profile	(1) λ	(2) $\frac{e_2}{e_1}$	(3) m	(4) $P_{exp.}$	(5) P_{Horne}	(6) $P_{plast.}$	(7) $\frac{P_{exp.}-P_{Horne}}{P_{exp.}}\times 100$	(8) Type of failure	(9) $\frac{P_{exp.}-P_{CM}}{P_{exp.}}\times 100$	(10) Type failure
DIE 20	40	+1	0·5	88,800	82,000	91,500	7·65	B	−3·1	B
			1	66,800	64,600	68,600	3·29	B	−3·0	B
			3	35,800	32,750	34,300	8·52	B	3·2	B
		0	0·5	95,000	100,000	91,500	4·74	P	0·0	P
			1	78,800	82,100	68,600	12·95	P	0·5	P
			3	—	—	—	—	—	—	—
		−1	0·5	105,000	106,800	91,500	12·85	P	9·5	P
			1	88,800	92,100	68,600	22·75	P	19·8	P
			3	—	—	—	—	—	—	—
	60	+1	0·5	84,800	80,780	91,500	4·74	B	−3·7	B
			1	64,800	61,500	68,800	5·09	B	−2·0	B
			3	32,800	31,600	34,300	3·66	B	−2·7	B
		0	0·5	93,800	94,000	91,500	2·45	P	−1·3	P
			1	74,800	77,750	68,600	8·29	P	4·7	P
			3	40,300	46,220	34,300	14·9	P	11·5	P
		−1	0·5	108,000	100,700	91,500	8·61	P	5·7	P
			1	80,800	87,000	68,600	15·10	P	11·8	P
			3	—	—	—	—	—	—	—
	80	+1	0·5	71,000	74,500	91,500	−4·93	B	−18·00	B
			1	59,000	57,215	68,600	3·02	B	−6·0	B
			3	32,500	30,210	34,500	7·05	B	0·9	B
		0	0·5	90,800	85,950	91,500	5·34	B	−8·8	B
			1	70,000	71,750	68,600	2·00	P	−1·8	P
			3	39,000	43,700	34,300	12·05	P	8·6	P
		−1	0·5	88,000	91,320	91,500	−3·77	B	−7·0	P
			1	78,000	80,000	68,600	12·05	P	8·7	P
			3	—	—	—	—	—	—	—
	100	+1	0·5	62,000	66,470	91,500	−7·58	B	−23·6	B
			1	53,500	52,500	68,600	1·87	B	−7·6	B
			3	29,000	28,500	34,300	1·72	B	−4·0	B
		0	0·5	82,000	75,520	91,500	7·90	B	−9·4	B
			1	67,000	64,350	68,600	3·80	B	1·3	B
			3	38,100	40,500	34,300	9·97	P	6·6	P
		−1	0·5	78,000	78,000	91,500	0	B	−21·2	B
			1	74,000	70,400	68,600	4·87	P	3·7	P
			3	42,400	49,060	34,300	19·10	P	15·5	P
DIE 10	130	+1	0·5	22,800	19,000	33,250	16·67	B	20·0	B
			1	19,300	16,170	24,850	16·21	B	23·5	B
			3	11,500	9,500	12,490	17·40	B	28·0	B
		0	0·5	25,000	20,600	33,250	17·60	B	16·7	B
			1	24,400	18,065	24,850	25·95	B	28·0	B
			3	15,050	12,630	12,490	16·09	P	19·3	B
		−1	0·5	30,800	20,460	33,250	33·60	B	31·6	B
			1	24,100	19,340	24,850	23·90	B	−22·2	B
			3	16,750	15,340	12,490	25·40	P	21·0	B
	175	+1	0·5	13,800	12,120	33,250	12·18	B	12·7	B
			1	12,400	11,250	24,850	9·28	B	17·8	B
			3	9,050	7,760	12,490	14·25	B	26·4	B
		0	0·5	11,800	12,390	33,250	−5·00	B	−13·3	B
			1	—	—	—	—	—	—	—
			3	10,800	9,900	12,490	8·34	B	19·6	B
		−1	0·5	10,000	7,320	7,421	26·80	B	30·3	B
			1	14,050	12,210	24,850	13·10	B	12·6	B
			3	13,800	10,970	12,490	20·50	B	34·0	B

(3) m	(4) $P_{exp.}$	(5) P_{Horne}	(6) $P_{plast.}$	(7) $\frac{P_{exp.}-P_{Horne}}{P_{exp.}}\times 100$	(8) Type of failure	(9) $\frac{P_{exp.}-P_{CM}}{P_{exp.}}\times 100$	(10) Type of failure
0·5	70,800	60,350	63,400	14·75	B	9·1	B
1	55,800	45,600	47,500	18·29	B	12·8	B
3	28,900	23,000	23,750	20·40	B	15·2	B
0·5	81,300	70,040	63,400	22·00	P	18·5	P
1	68,800	57,870	47,500	31·00	P	27·9	P
3	—	—	—	—	—	—	—
0·5	81,800	74,900	63,400	22·50	P	19·3	P
0	95,300	88,620	95,100	7·01	B	− 3·0	B
3	—	—	—	—	—	—	—
0·5	65,800	58,000	63,400	11·85	B	4·0	B
1	53,800	44,200	47,500	17·85	B	11·3	B
3	24,800	22,576	23,750	8·97	B	2·8	B
0·5	71,800	66,900	63,400	11·70	P	8·3	P
1	59,050	55,830	47,500	19·60	P	16·0	P
3	—	—	—	—	—	—	—
0·5	72,800	71,270	63,400	12·90	P	9·1	P
0·8	71,230	65,530	52,760	25·90	P	22·6	P
3	—	—	—	—	—	—	—
0·5	67,050	54,440	63,400	18·81	B	9·1	B
1	53,800	42,290	47,500	21·40	B	14·6	B
3	28,800	21,970	23,750	23·70	B	18·5	B
0·5	75,500	61,800	63,400	18·15	B	12·6	P
1	58,800	52,540	47,500	19·20	P	20·5	P
3	—	—	—	—	—	—	—
0·5	75,800	65,250	63,400	16·40	P	12·9	P
1	62,300	57,920	47,500	23·80	P	20·6	P
3	—	—	—	—	—	—	—
0·5	59,800	48,750	63,400	18·50	B	5·3	B
1	53,000	39,320	47,500	25·80	B	18·4	B
3	26,900	21,140	23,750	21·42	B	16·7	B
0·5	64,550	53,440	63,400	17·20	B	− 2·1	B
1	53,800	48,350	47,500	11·70	P	8·0	P
3	35,300	30,350	23,750	32·70	P	29·6	P
0·5	65,300	56,000	63,400	14·22	B	− 4·8	B
1	59,800	51,100	47,500	20·60	P	17·2	P
3	—	—	—	—	—	—	—
0·5	45,550	37,020	63,400	18·71	B	17·1	B
1	43,600	32,340	47,500	25·80	B	29·0	B
3	25,550	19,010	23,750	25·60	B	28·40	B
0·5	—	—	—	—	—	—	—
1	44,000	36,250	47,500	17·61	B	15·7	B
3	30,050	26,380	23,750	21·00	P	17·6	P
0·5	44,300	39,400	63,400	11·05	B	3·1	B
1	48,500	38,050	47,500	21·55	B	20·3	B
3	—	—	—	—	—	—	—
0·5	27,050	22,970	63,400	15·10	B	14·6	B
1	25,800	21,500	47,500	16·68	B	22·8	B
3	20,300	14,840	23,750	26·90	B	34·3	B
0·5	25,650	23,470	63,400	8·50	B	4·7	B
1	25,300	22,840	47,500	9·72	B	10·9	B
3	26,650	18,920	23,750	29·00	B	35·6	B
0·5	—	—	—	—	—	—	—
1	25,450	23,280	47,500	8·52	B	8·8	B
3	—	—	—	—	—	—	—

programme of tests comprised ninety-two specimens. Table 3 gave for each test the profile tested, the λ, e_2/e_1, and m values, as well as the experimental collapse load $P_{\text{exp.}}$ in metric tons.

The full test report[39, 40] described in detail the material properties of the steel used, including the distributions of residual stresses in the three profiles, and gave an accurate, but very complicated, solution of the problem of plastic buckling of eccentrically loaded I-struts by bending and torsion.

Comparison between the tests and the Author's design method

For each of the ninety-two stanchions tested, the collapse load had been calculated according to the Author's equations (25). Table 3, column (5), gave the buckling load P_1 which could be obtained by successive approximations from the Author's third equation (25), i.e.:

$$p + N_x f_x = f$$

Column (6) gave the plastic collapse load P_2 deduced from the Author's first equation (25):

$$p + f_x' = 15{\cdot}25$$

The smallest of those two loads, P_1 or P_2, must be taken as the true ultimate load. In column (7) of Table 3, the percentage deviation of that theoretical ultimate load from experiment was given. Finally, the column (8) specified by B (buckling) or P (plasticity) which phenomenon was responsible for the destruction of the stanchion considered.

It was seen that the agreement between theory and experiment was satisfactory since:

(*a*) The Author's method gave safety in all cases except four;

and (*b*) the mean relative error of the buckling formula was 11·2% for the DIE profile and 17·7% for the PN 22 profile.

Some indications about the interaction formula developed by Professors Campus and Massonnet

It had been recognized that the ultimate load of the stanchions could be reached in two different ways:

(*a*) formation of a "plastic hinge" at one end of the stanchion;

(*b*) general elasto-plastic instability;

and that consequently two different formulae had to be considered simultaneously.

The first one was identical with the Author's first inequality formula (25), except that the conventional yield stress was taken equal to 15·88 tons/sq. in. (25 kg/sq. mm) according to the Belgian specification in force.

The buckling formula proposed by Professors Campus and Massonnet was simpler than the Author's corresponding formula. It could be written:

$$\frac{P}{Ap_E} + \alpha \frac{\sqrt{0{\cdot}3(M_x'^2 + M_x''^2) + 0{\cdot}4\, M_x'\, M_x''}}{f_L\, Z_x\left(1 - \dfrac{P}{P_{E_x}}\right)} \leqslant 1$$

In that formula P, A, M_x', M_x'', Z_x, and f_L had the same signification as in the Paper; $p_E = \dfrac{\pi^2 E_t}{\lambda^2}$ was the Engesser-Shanley critical stress for centric plastic buckling in the transverse direction (which could be replaced approximately by the critical stress given in the official specification in use); $P_{E_x} = \dfrac{\pi^2 E I_x}{l^2}$ was the Euler buckling load for elastic buckling of a centrically loaded strut in the plane of maximum rigidity (plane of the web); and α was a reduction factor taking into account the danger of lateral buckling.

For wide-flange profiles, take:

$$\alpha = \begin{cases} 1 - 3{\cdot}72 \times 10^{-7} \left(\dfrac{lh}{bt_1}\right)^2 & \text{for } \dfrac{lh}{bt_1} \leqslant 1{,}000 \\[2ex] \dfrac{528}{lh/bt_1} & \text{for } \dfrac{lh}{bt_1} \geqslant 1{,}000 \end{cases}$$

In those expressions, h, b, and t_1 were dimensions of the cross-section of the stanchion. or profiles other than wide flanged, the value of α from the new proposals made by ‹erensky, Flint, and Brown (see p. 396) could easily be deduced. The expression of the equivalent" uniform moment adopted in the formula, i.e.:

$$M_x = \sqrt{0{\cdot}3(M_x'^2 + M_x''^2) + 0{\cdot}4\, M_x' M_x''}$$

ad been established several years ago by Professor Massonnet[42] by means of the Ritz ıethod. Finally, the term $\left(1 - \frac{P}{P_{Ex}}\right)$ in the denominator of the second fraction took ıto account, approximately, the magnification of lateral deflexion due to beam/column ction.

Table 3 gave in column (9) the percentage relative error of the Belgian method and ı column (10) the mode of failure predicted by that method, which agreed nearly lways with that predicted by the Author's method.

onclusions

Table 4 summarized the results of the comparison of the two design methods by the xperimental results. It was seen that the Author's buckling formula was safer than rofessors Campus and Massonnet's corresponding formula. On the other hand, the ean error of both formulae was approximately the same. The advantage of the greater ıfety of the Author's formula must thus be weighed against its complexity, which was ertainly greater than that of the Belgian formula.

TABLE 4

Appreciation of buckling formulae

Profile	Horne		Campus and Massonnet	
	Safety for	Mean error: %	Safety for	Mean error: %
DIE	30/34	11·2	20/33	15·0
PN	29/29	17·7	25/28	14·0

FURTHER REFERENCES

. F. Campus and C. Massonnet, "*Recherches sur le flambement de colonnes en acier A 37, à profil en double té, sollicitées obliquement*" ("Investigations on the buckling of A 37 steel I-section stanchions, transversely stressed"). Bull. C.E.R.E.S., Liège, vol. 7, 1955, pp. 119 to 338.

. F. Campus and C. Massonnet, "*Recherches sur le flambement de colonnes en acier A 37, à profil en double té, sollicitées obliquement.*" Spec. pub., I.R.S.I.A.

. R. L. Templin, "Hydraulically-supported spherically-seated compression testing machine platens." Proc. A.S.T.M., vol. 42, 1942.

. C. Massonnet, "*Le flambage des barres à section ouverte et à parois minces*" ("The buckling of open-section thin-webbed struts"). Article in "*Hommage de la Faculté des Sciences appliquées à l'Association des Ingénieurs sortis de l'Ecole de Liège.*" Liège, 1947, pp. 126 to 141.

. E. Chwalla, "*Die Stabilitaet lotrecht belasteter Rechteckrahmen*" ("The stability of vertically loaded right-angled frames"). *Bauingenieur*, vol. 19, p. 69 (4 Feb. 1938).

Mr K. G. Eickhoff (Senior Assistant in Research, Department of Engineering, Cambridge) commented upon the Author's design method as applied to a stanchion which was simply supported about the major axis, with elastic loading about the minor axis (Author's case $O_x . E_y$, p. 114).

In Fig. 35 the stanchion AA was a portion of a continuous internal stanchion which was bent about the minor axis by beams framing into it, the far ends of those beams being encastered. For simplicity, Mr Eickhoff considered the case where the stanchion length

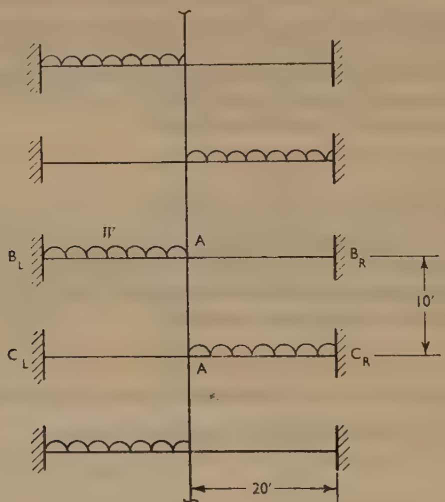

Fig. 35

AA was bent in symmetrical single curvature, alternate beams being loaded as shown by a uniformly distributed load W.

If all the beams had the same section, and that was chosen so that the loaded beams were fully plastic under the loads W, then the moment transmitted to the stanchion was greatest when the remaining beams were unloaded, and that was therefore the design loading (Fig. 35).

If the factored load W was 0·75 ton/ft run, and the frame had a storey height of 10 ft and a beam span of 20 ft, then the full plastic moment of the beam must be 225 tons-in. Using the Author's method of design for stanchions subjected to bending moments about the minor principal axis only (p. 135), and assuming that a continuous range of R.S.J's was available, the minimum stanchion sections which would support various factored stanchion loads P could be calculated. Those were shown in Table 5 (see Fig. 36 for notation).

The yield stress would just be reached at the most highly stressed fibres of those stanchions under the loading shown, but it was known (p. 115) that a stanchion could continue to carry load long after the yield stress had been reached in its extreme fibres. If the stanchions were designed to be elasto-plastic at failure, then the stanchion moment M was reversed in sign and, instead of the stanchion helping to support the beam, the beam helped to support the stanchion.

It could be shown that the most severe loading conditions for an elasto-plastic stanchion were the same as those for a completely elastic stanchion; i.e., alternate beams loaded, the others unloaded. Using that design loading, and the results of an elasto-plastic

nalysis of R.S.J. stanchions recently carried out on an electronic digital computor, Ir Eickhoff had calculated some stanchion sections which would be adequate to carry the oads in Table 5. Those stanchions were shown in Table 6.

The beam (1) was designed to be fully plastic under the load W and the end moment M_1, ut the beam (2) remained elastic under the end moment M_2 and thus provided the

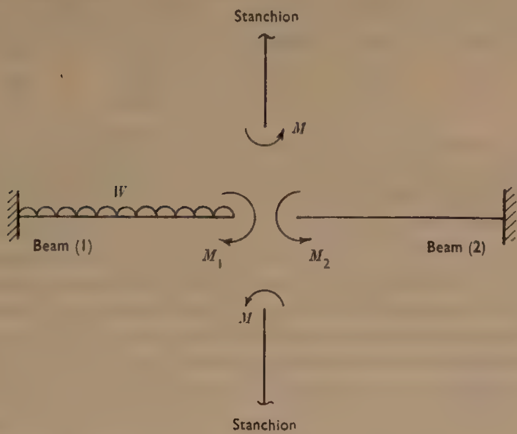

FIG. 36

anchion with the elastic restraint that it needed, the restraining moment which the stanion required being small compared with the end moment of the loaded beam. Comaring Tables 5 and 6, it would be seen that the cross-sectional areas of the elasto-plastic

TABLE 5.—ELASTIC STANCHIONS AND PLASTIC BEAMS

Factored stanchion load P: tons	Stanchion area A_S: sq. in.	Beam area A_B: sq. in.	Plastic moment of beam $_BM_p$: tons-in.	Stanchion moment M: tons-in.	Beam moment M_1: tons-in.	Beam moment M_2: tons-in.
30	9·2	5·15	225	43·1	225	139
45	11·2	5·15	225	55·4	225	114
60	12·6	5·15	225	62·7	225	100
100	15·6	5·15	225	75·3	225	74

nchions were only about one-half of those of the elastic stanchions, whilst the crosstional areas of the beams had only been increased by 5%.

There was evidence, therefore, that a considerable overall economy could be obtained by signing internal stanchions elasto-plastically. That economy could not, however, be tended to outside stanchions which, because they had no unloaded elastic beams to pport them, must be designed elastically if the beams framing into them were designed stically.

TABLE 6.—ELASTO-PLASTIC STANCHIONS AND PLASTIC BEAMS

Factored stanchion load P: tons	Stanchion area A_S: sq. in.	Beam area A_B: sq. in.	Plastic moment of beam ^BM_p: tons-in.	Stanchion moment M: tons-in.	Beam moment M_1: tons-in.	Beam moment M_2: tons-in.
30	5·0	5·4	241	− 16·7	177	210
45	5·0	5·4	242	− 24·2	164	212
60	6·7	5·4	240	− 14·4	180	209
100	11·1	5·4	243	− 18·9	173	211

Mr A. H. Chilver (Demonstrator in Engineering, University of Cambridge) observed that the Author reduced the stanchion problem to "member instability" and "frame instability"; in discussing the former he studied the general problem of a single stanchion of a framed structure, and the particular problem of the I-section column.

It was a much more difficult matter to generalize on the frame instability aspect; that problem was especially important in the design of multi-storey steel-framed buildings in which the steel skeleton itself must resist all lateral movements of the frame. Solutions of that problem had so far proceeded on the conventional lines of elastic stability analysis. It was relevant perhaps to refer to a feature of frame instability which was not frequently encountered in strut behaviour, and which might be important in some problems. As the Author pointed out, the Euler critical load played an important part in the stability of a single member of a framework; that load corresponded to elastic buckling of an axially loaded pin-ended strut. An interesting feature of strut behaviour was that the "elastic critical load", corresponding to the development of large lateral deflexions in the elastic condition of the material, was not affected by eccentric application of the load, by the presence of lateral forces or end moments, or even by initial imperfections. Some types of rigid frames behaved differently and there was a small, but definite, lowering of the critical load owing to the presence of other disturbing forces. It was impossible to discuss

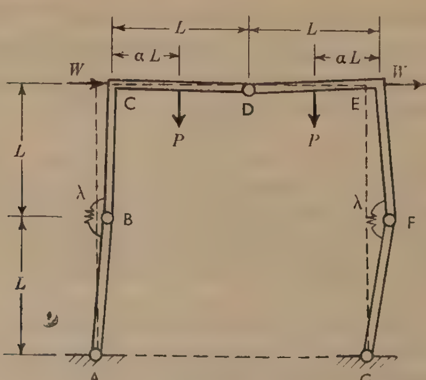

FIG. 37

that problem in a general way, and for that reason it was profitable to resort to a simple elastic model. Fig. 37 showed a model portal frame consisting of hinged rigid members: the feet A and G were pinned to rigid foundations; angular movement of the hinges B and F was resisted by elastic springs of rotational stiffness λ (moment/radian), whilst hinge

was unrestrained. The members BCD, FED, corresponded to the rigid corners of a mple portal frame. In Fig. 37 the frame was shown under the action of vertical loads P nd lateral loads W. In the absence of lateral forces, $W = 0$, and with the loads P pplied to the tops of the stanchions, the frame might buckle in the symmetric form of ig. 38a at a critical load $P = 2\lambda/L$, or in the anti-symmetric mode of Fig. 38b at a critical oad $P = \lambda/L$. If the frame was free of any external lateral restraint the anti-symmetric ode was more important, and it might be said that the relevant buckling load was $P = \lambda/L$. On discussing the stability of the frame under the action of the more general ystem of forces shown in Fig. 37 it was found that lateral buckling of the frame occurred t a value of P given by the root of the quadratic equation:

$$(2-\alpha)\left(\frac{PL}{\lambda}\right)^2 - 6\left(\frac{PL}{\lambda}\right) + 4 = 0,$$

n which α defined the point of application of the vertical loads P. One root of that equaon gave a value of P less than λ/L, suggesting that buckling might occur at a load less han that corresponding to "pure" instability in the anti-symmetric mode (Fig. 38b). ome solutions of the quadratic equation were given below.

α . . .	0·0	0·2	0·4	0·6	0·8	1·0
PL/λ . .	1·00	0·92	0·87	0·83	0·79	0·77

A similar effect had been observed by Chwalla[43] in the buckling of portal frames; he oted a reduction in certain cases of only 3% or so in the buckling load. Although the

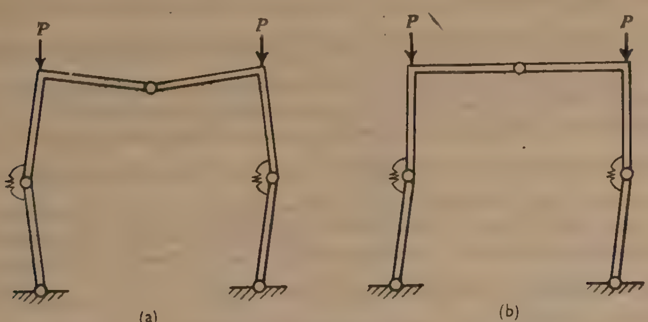

FIG. 38

ffect in those particular cases was small, it was important for the designer to appreciate he possibility of buckling at a rather reduced load. Chwalla's analysis was necessarily omplicated; the model presented, although it had no quantitative value *per se*, had the nerit of indicating in a simple fashion that particular characteristic of portal frames.

The Author, in reply, welcomed the statement from Professor Baker regarding he relationship between the present Paper and the work of the Steel Structures Research Committee on stanchions. The difficulties encountered in attempting the derivation of complete stanchion design method underlined the achievement of the Steel Structures Research Committee in producing a method based entirely on elastic theory, and a great eal could still be learned from the work of that Committee.

The Author had been very interested to receive from Professors Campus and Massonnet n account of their test results for eccentrically loaded columns. Their thorough comparison between those results and the failure loads predicted by the two theories was of

great value in assessing the merits and demerits of the two treatments. It seemed worth while to subject the percentage errors given by the two methods to a statistical investigation by means of an analysis of variance. Had the two series of tests been complete, each would have represented a three-fold factorial experiment with $6 \times 3 \times 3 = 54$ results. Omitting the DIE profile test for $\lambda = 175$, $\frac{e_2}{e_1} = -1$ and $m = 6$, and the PN profile test for $\lambda = 40$ and $m = 0$, the DIE profile results lacked six values and the PN profile results twelve values. The PN profile test for $\lambda = 60$, $\frac{e_2}{e_1} = -1$ and $m = 0{\cdot}8$ could be regarded as sufficiently close to $m = 1{\cdot}0$ to be accepted in the analysis. The missing values were first to be fitted by the method of least squares, so that the residual variance was minimum. The variances finally obtained were shown in Table 7 for the DIE profile and in Table 8 for the PN profile. In each Table, columns 3 to 5 gave the results for the Author's theory, whilst the results for the Campus–Massonnet theory were contained in columns 9 to 11. The first three rows gave the variances obtained by grouping the tests according to slenderness λ, eccentricity ratio $\frac{e_2}{e_1}$ and eccentricity value m respectively. Each of those variances had two components, namely:

1. that due to the inability of the theory to describe completely the effect of the particular factor concerned; and
2. a further element of variance representing the inability of the theory to take account of factors other than the numerical values of λ, $\frac{e_2}{e_1}$, and m.

The fourth row gave the residual variance, and represented entirely the effects of factors other than λ, $\frac{e_2}{e_1}$, and m. Columns 5 and 11 gave the ratios of the first three variances to the residual variance, whilst columns 6, 7, and 8 gave the upper and lower 5%, 1%, and 0·1% significance levels for those variance ratios according to the F test.

Considering first the results obtained from the Author's theory, there seemed to be scope for improving the treatment of the factors λ and $\frac{e_2}{e_1}$ for the DIE profile, since highly significant variance ratios were obtained for those factors. On the other hand, those factors had a very small variance when the PN profile was considered, indicating that for such sections, the treatment was very successful. The treatment for the eccentricity value m appeared capable of improvement for both profiles. The treatment of the $\frac{e_2}{e_1}$ factor by the Campus–Massonnet method appeared consistently good, the treatment of slenderness λ was indifferent for the DIE profile but good for the PN profile, whilst the treatment of the eccentricity value m was very poor for the PN profile.

The individual variances obtained for the Campus–Massonnet theory as ratios of the corresponding variances for the Horne theory were given in column 3 of Tables 9 and 10. The percentage points in the F test were shown in columns 4, 5, and 6. The variances for the Horne theory were smaller than those for the Campus–Massonnet theory except for that due to the eccentricity ratio $\frac{e_2}{e_1}$ for the DIE profile. It was, however, to be noted that none of the variance ratios for the factors λ, $\frac{e_2}{e_1}$, and m were significant at the 5% level. The slightly higher accuracy of the Horne theory as compared with that of Campus and Massonnet had thus to be ascribed, not to the particular methods of dealing with the factors λ, $\frac{e_2}{e_1}$, and m, but rather to the difference in the general approach to the problem. Whereas the Campus–Massonet approach was that of a purely empirical interaction curve, the

TABLE 7.—ANALYSIS OF VARIANCE FOR DIE PROFILE TESTS

1	2	3	4	5	6	7	8	9	10	11
Source of variance	Degrees of freedom	Horne theory			Points of F-distribution			Campus–Massonnet theory		
		Sum of squares	Variance	Variance ratio	5%	1	0·1%	Sum of squares	Variance	Variance ratio
Slenderness λ	5	1,692	338	12·67	2·46 ·224	3·54 ·107	5·19 ·040	3,638	728	6·33
Eccentricity ratio $\frac{e_2}{e_1}$.	2	626	313	11·73	3·24 ·051	5·21 ·010	8·33 ·001	383	191	1·66
Larger eccentricity m .	2	382	191	7·16				1,640	820	7·14
Residual	38	1,014	26·7					4,366	114·9	
Total	47	3,714						10,027		

TABLE 8.—ANALYSIS OF VARIANCE FOR PN PROFILE TESTS

1	2	3	4	5	6	7	8	9	10	11
Source of variance	Degrees of freedom	Horne theory			Points of F-distribution			Campus–Massonnet theory		
		Sum of squares	Variance	Variance ratio	5%	1%	0·1%	Sum of squares	Variance	Variance ratio
Slenderness λ	5	311	62	2·13	2·51 ·223	3·65 ·107	5·43 ·040	519	104	1·82
Eccentricity ratio $\frac{e_2}{e_1}$.	2	2	1	·03	3·29 ·051	5·34 ·010	8·64 ·001	11	6	·10
Larger eccentricity m .	2	570	285	9·73				1,606	803	14·11
Residual	32	939	29·3					1,822	56·9	
Total	41	1,822						3,958		

TABLE 9.—COMPARISON OF THEORIES FOR DIE PROFILE TESTS

1	2	3	4	5	6
Source of variance	Degrees of freedom	(Variance in Campus–Massonnet theory) ÷ (Variance in Horne theory)	Points of F-distribution		
			5%	1%	0·1%
Slenderness λ . . .	5	2·15	5·05 ·198	10·97 ·091	29·75 ·034
Eccentricity ratio $\frac{e_2}{e_1}$.	2	0·61	19·0 ·053	99·0 ·010	999 ·001
Larger eccentricity m	2	4·29			
Residual	38	4·30	1·72 ·582	2·16 ·463	2·80 ·357

TABLE 10.—COMPARISON OF THEORIES FOR PN PROFILE TESTS

1	2	3	4	5	6
Source of variance	Degrees of freedom	(Variance in Campus–Massonnet theory) ÷ (Variance in Horne theory)	Points of F-distribution		
			5%	1%	0·1%
Slenderness λ . . .	5	1·67	5·05 ·198	10·97 ·091	29·75 ·034
Eccentricity ratio $\frac{e_2}{e_1}$.	2	6·33	19·0 ·053	99·0 ·010	999 ·001
Larger eccentricity m	2	2·82			
Residual	32	1·94	1·80 ·554	2·32 ·431	3·09 ·323

Author's theory was based on an attempt to estimate the maximum extreme fibre stress in the section. That did seem to give a significantly better agreement with the test results, at the cost of somewhat greater complexity.

Mr Eickhoff's contribution was valuable as an indication of the economical designs which could be achieved for internal stanchions by using the restraining influence of the beams. The difficulty was, of course, in obtaining a convenient design method based on that conception, since stanchion behaviour was then most complicated. The difficulties would become even more pronounced if an attempt were made to design a stanchion with approximately equal principal moments of inertia on the same basis, and a solution to that problem did not seem to be within sight.

Mr Eickhoff had considered the case of alternate beams loaded. The stanchions could then take axial loads which exceeded the Euler critical loads for pin-ended members, because of the restraint offered by the lightly loaded beams which remained elastic. It appeared, however, that the elastic restraint might disappear if all the beams were fully loaded, and the stanchion would then fail at a load below the Euler value. The loading condition considered by Mr Eickhoff might thus not, in all cases, be the most critical.

Mr Chilver's contribution illustrated an aspect of frame instability which had been neglected in dealing with elastic structures, but which might become of importance in structures loaded beyond the elastic limit because of the larger deflexions introduced

The main lesson to be learned was that it could be misleading to approach instability problems intuitively as an extension of elastic pin-ended strut behaviour. The only safe course was first to assume real disturbing forces or imperfections or both, and then to follow the actual behaviour of the structure. If interest were centred on the condition of no disturbing forces and no imperfections, then that should be approached as the limiting case of the problem in which such elements were present. Had that been realized earlier, the confusion over reduced and tangent modulus loads for members of a strain-hardening material would not have arisen.

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CORRIGENDUM

Proceedings, Part III, April 1956, p. 214 :

Delete asterisk and footnote.